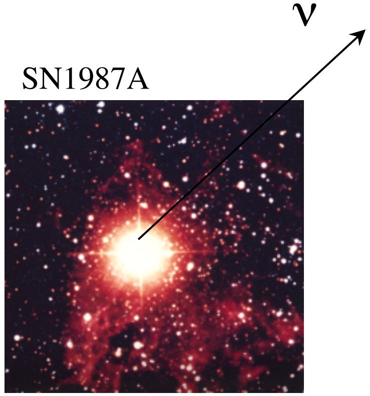
Numerical simulations of core-collapse supernovae: Neutrino transfer by 6D Boltzmann equation

SN1987A



Mann "Shadow of a Star"

K. Sumiyoshi

Numazu College of Technology
Japan

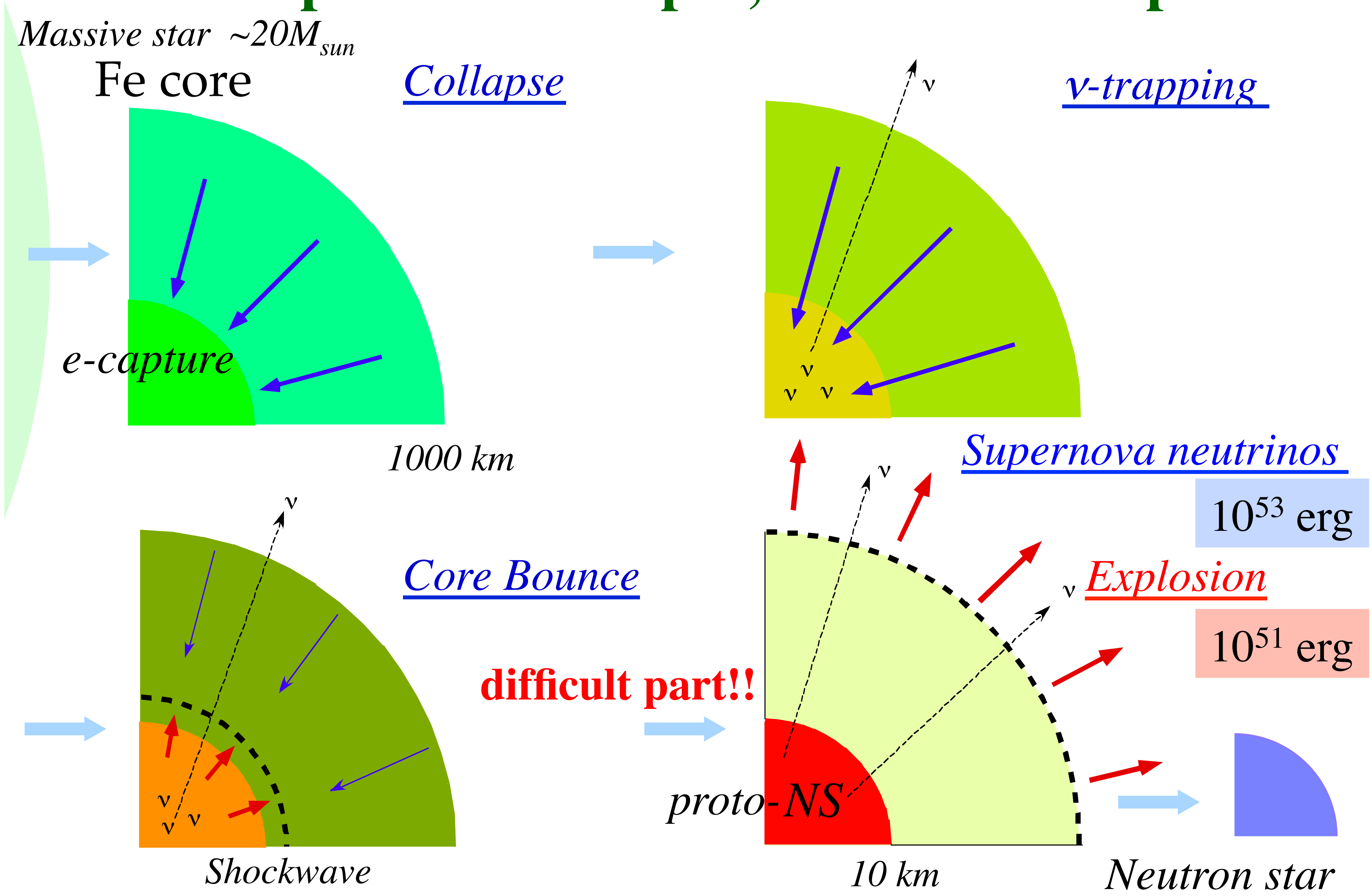
Supercomputers

KEK SR16000

Neutrino transfer in 3D supernova core
Some updates in nuclear physics

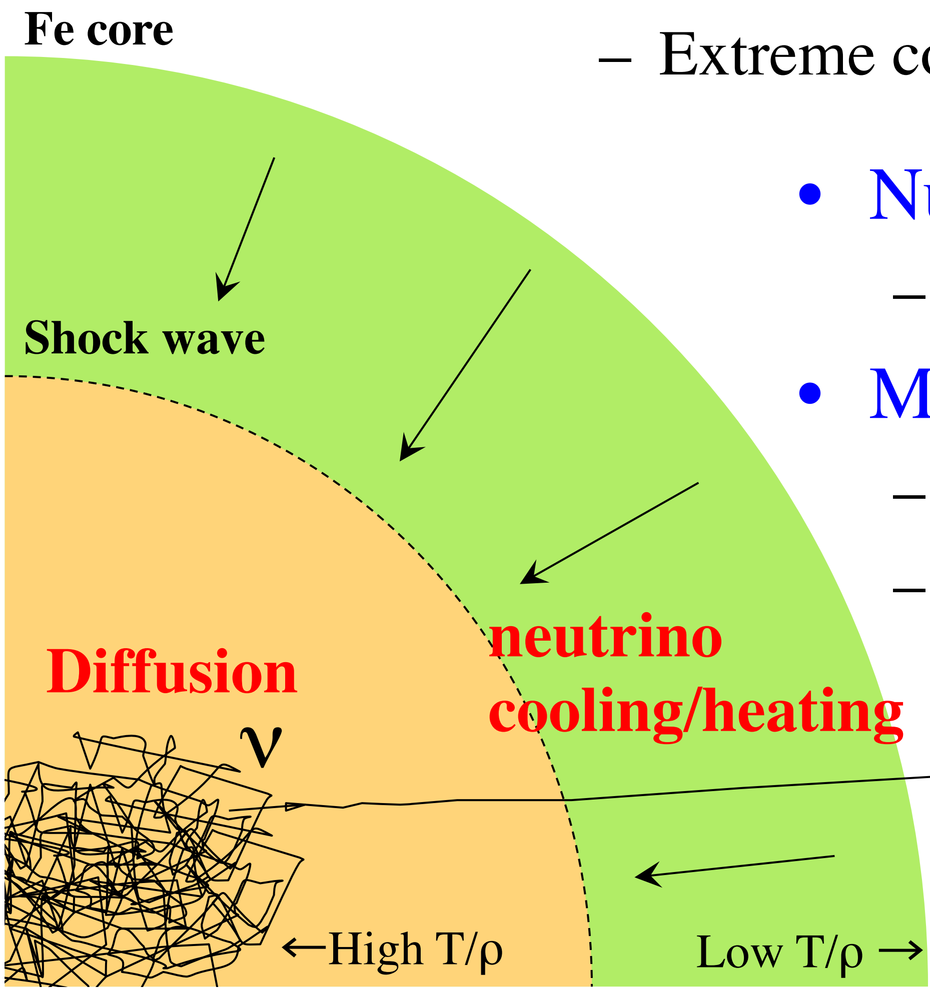
- Neutrino-radiation hydrodynamics -> Nagakura
- 3D supernova explosions by hydro instabilities -> Takiwaki

Core-collapse SNe: collapse, bounce and explosion



Longstanding issues of core-collapse SNe

- Explosion mechanism
  - Revival of shock wave, Explosion energy
- Birth place of neutron star / black hole
  - Extreme condition of matter
- Nucleosynthesis
  - Heavy elements
- Multi-messengers
  - Neutrino bursts
  - Grav. waves



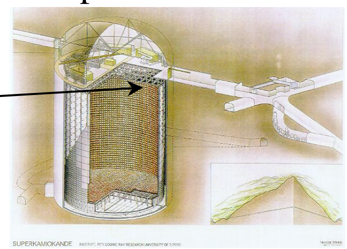
ν

Supernova neutrinos

Prof. Koshiba

http://nobelprize.org/

Super-Kamiokande



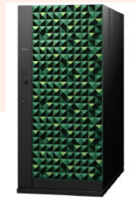
http://www-sk.icrr.u-tokyo.ac.jp/

From nuclear physics to astrophysics

- Equation of state
- Neutrino reactions
- Nuclear data

- Hydrodynamics
- Neutrino transfer
- Stellar models

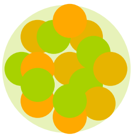
- Numerical simulations of core-collapse supernovae
  - Supercomputing technology



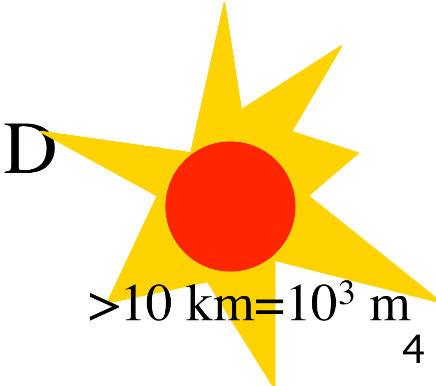
KEK

K computer

- Challenges:
  - Nuclear physics at high ρ and T
  - Neutrino-radiation hydrodynamics in 3D



~ fm=10^{-15} m



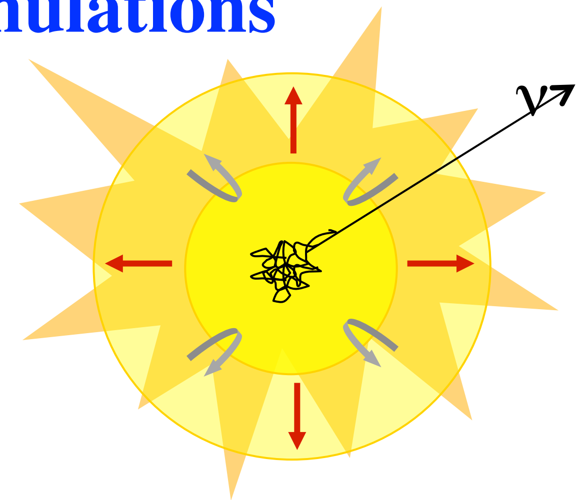
>10 km=10^3 m

Development of EOS tables

Inputs from nuclear physics

Properties of dense matter at extreme conditions

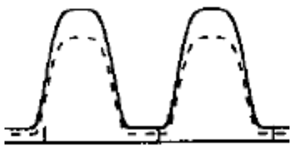
- **Necessary inputs for numerical simulations**



1. Pressure-Density
   - Stellar structure, Dynamics, Maximum Mass
2. Temperature & chemical potentials
3. Composition (proton, neutron, nuclei)
   - ν-energy distribution, ν-reaction

- **Equation of state (EOS) in supernova core**

  - Dense more than nuclei: $\rho > \rho_0 = 3\text{x}10^{14}\text{g/cm}^3$
  - Neutron-rich: $Y_p < Z/A = 0.46$ for ^{56}Fe
  - Very Hot: $T > 10$ MeV



- Unified framework to cover wide range of ρ, Y_p, T
- Check by experimental data (ex. RIKEN Nishina C.)

Brief history of supernova EOS for simulations

1980's • Systematic studies to explore EOS effects

Analytic formulae	*Takahara-Sato, Baron-Cooperstein-Kahana*

1990's • Data sets for supernova EOS: benchmark

Skyrme-Hartree-Fock	*Hillebrandt-Wolff* (HW)
Extended liquid drop models	*Lattimer-Swesty* (LS)
Relativistic Mean Field	*Shen-Toki-Oyamatsu-KS* (Shen)

2001~ • Improvement of EOS tables

3D, mixture of nuclei	*G. Shen, Furusawa*
Interactions	*Hempel, Steiner*
Nuclear many body theory	*Togashi-Takano, Constantinou*

• Extension to exotic phases: strangeness and quarks

Mixture of Λ, Σ, Ξ-particles	*Hempel-Schaeffner, Ishizuka*
Quark-hadron phase transition	*Sagert, Nakazato*

Shen equation of state for supernovae

H. Shen, Toki, Oyamatsu & Sumiyoshi NPA, PTP(1998), ApJS (2011)

- Relativistic mean field theory+ local-density approx.
  - Based on relativistic Brueckner Hartree-Fock (RBHF) theory
  - Checked by exp. data of n-rich unstable nuclei: TM1
    - Nuclear structure: mass, charge radius, neutron skin,…

- Covers wide range of
  - Density: $10^{5.1} \sim 10^{16}$ g/cm^3
  - Proton fraction: 0 ~ 0.65
  - Temperature: 0 ~ 400 MeV
- Data table ~140 MB (110 x 66 x 92 points)
  - Quantities: ε, p, S, μ_i, X_i, m*
- Extensions with hyperons & quarks

  Ishizuka et al. (2006), Nakazato et al. (2008)

Shen-EOS

```
cccccccccccccccccccccccccccccccccccccccc
 Temperature=  1.000000E-01

5.100000E+00 7.581421E-11 -2.000000E+00 1
5.200000E+00 9.544443E-11 -2.000000E+00 1
5.300000E+00 1.201574E-10 -2.000000E+00 1
5.400000E+00 1.512692E-10 -2.000000E+00 1
5.500000E+00 1.904367E-10 -2.000000E+00 1
5.600000E+00 2.397456E-10 -2.000000E+00 1
5.700000E+00 3.018218E-10 -2.000000E+00 1
5.800000E+00 3.799711E-10 -2.000000E+00 1
5.900000E+00 4.783553E-10 -2.000000E+00 1
6.000000E+00 6.022137E-10 -2.000000E+00 1
6.100000E+00 7.581421E-10 -2.000000E+00 1
6.200000E+00 9.544443E-10 -2.000000E+00 1
6.300000E+00 1.201574E-09 -2.000000E+00 1
6.400000E+00 1.512692E-09 -2.000000E+00 1
6.500000E+00 1.904367E-09 -2.000000E+00 1
6.600000E+00 2.397456E-09 -2.000000E+00 1
```

http://user.numazu-ct.ac.jp/~sumi/eos

2001~ : Recent progress of supernova EOS

→ Numerical simulations

- Improvement of EOS tables

Finite system, mixture of nuclei	*G. Shen, Furusawa*	See also CompOSE
Interactions, energy functions	*Hempel, Steiner*	
Nuclear many body theory	*Togashi-Takano, Constantinou*	

- Extension to exotic phases: strangeness and quarks

Mixture of Λ, Σ, Ξ-particles	*Hempel-Schaeffner, Ishizuka*
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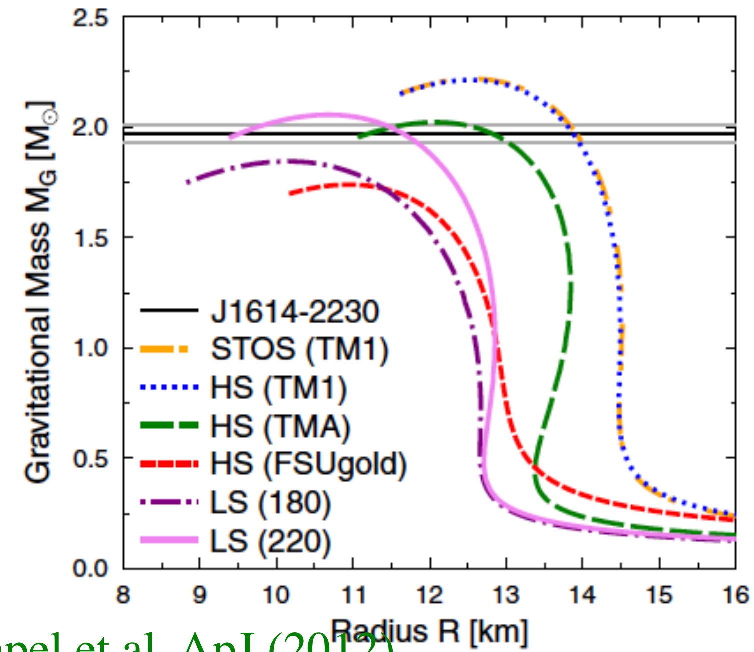
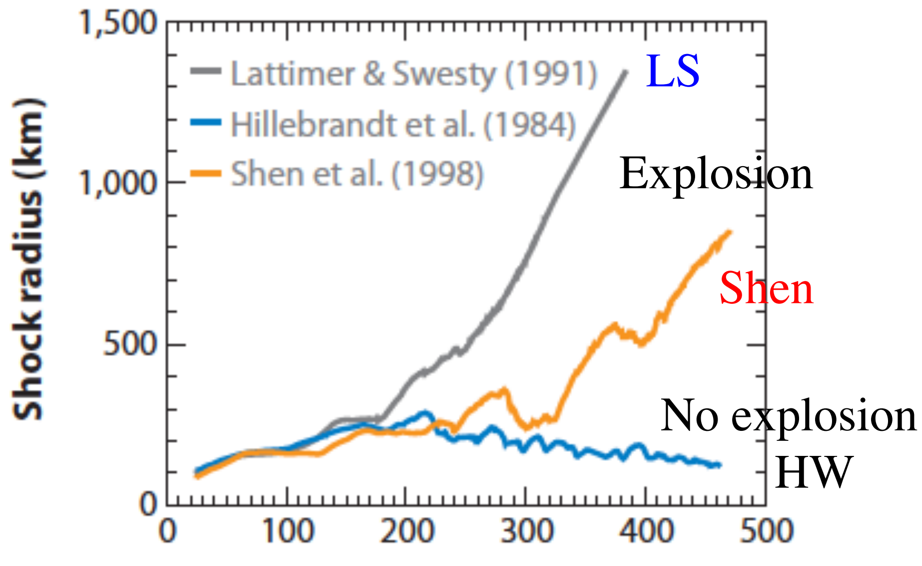


Hempel et al. ApJ (2012)

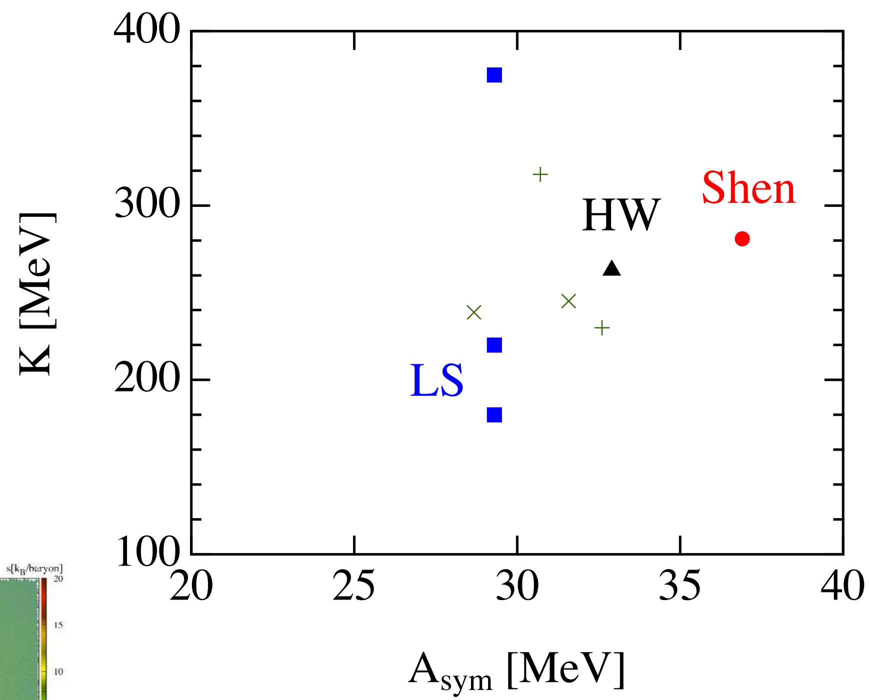
- Nuclear data: A_{sym}, K
- Observation of NS: $2M_{sun}$, 8-16 km
- Extreme conditions for BH cases, NS mergers, GRB
- Systematic EOS to examine 2D/3D

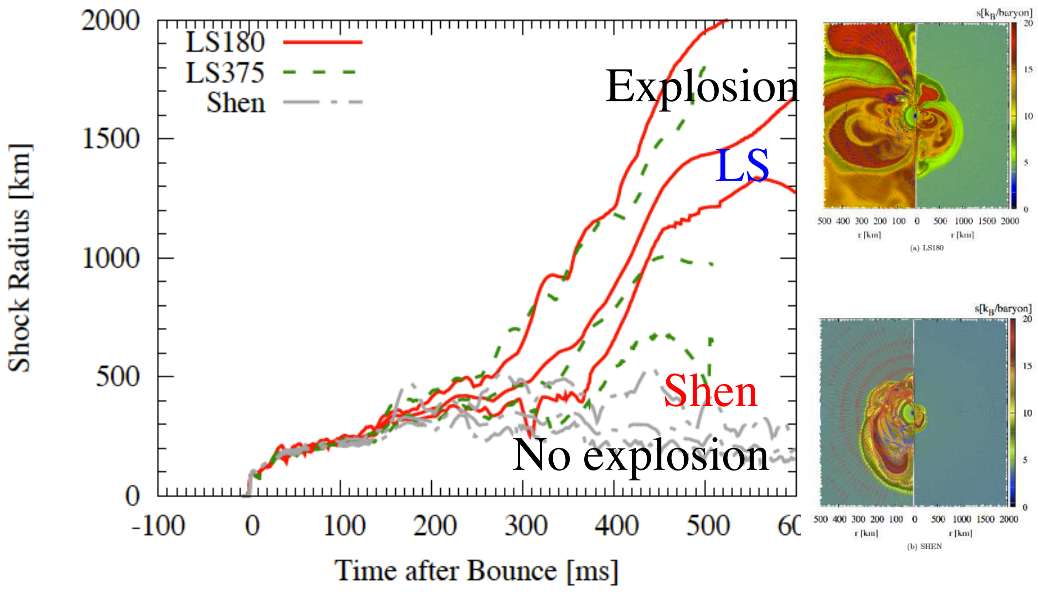
EOS effects in multi-D supernovae



Janka ARNPS (2012)

Need more systematic studies





Suwa et al. ApJ (2013)

EOS affects also
- Neutrino signals
- Gravitational waves

Kotake et al. PRD (2004),
Marek et al. ApJ (2009)

Neutrino transfer is important

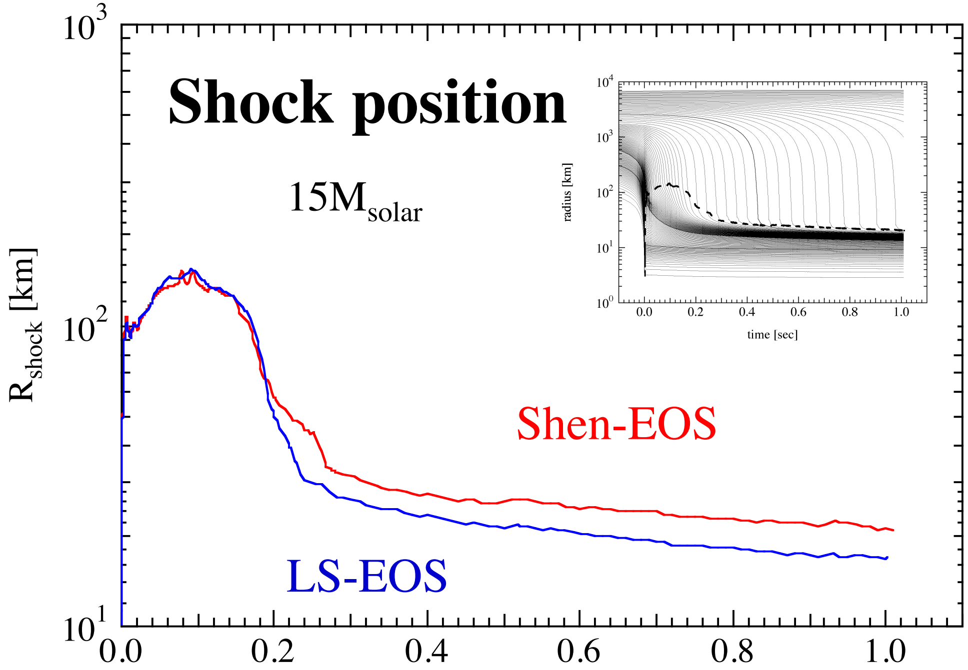
Evaluate neutrino heating

No explosion in 1D simulations

spherical

US, Germany, Japan (2001-)

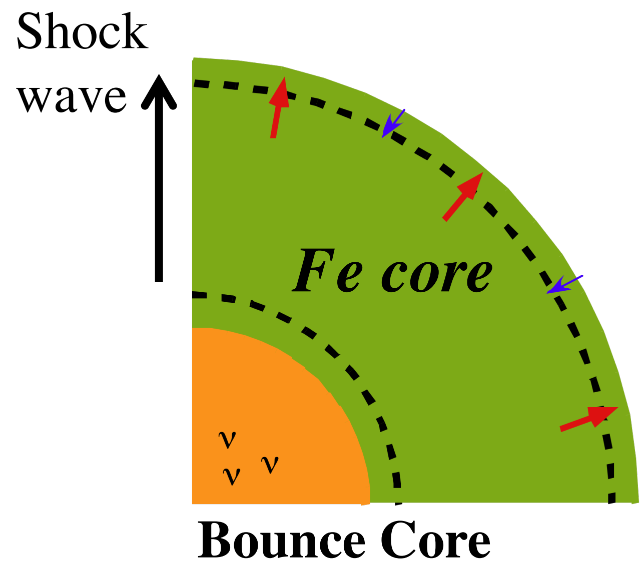
First principle calculation: ν-radiation hydrodynamics



Sumiyoshi et al. ApJ (2005)

time [sec]

time



Shock wave stalls on the way

Initial shock energy

$$E_{shock} \sim \frac{GM_{inner}^2}{R_{inner}} = several \times 10^{51} erg$$

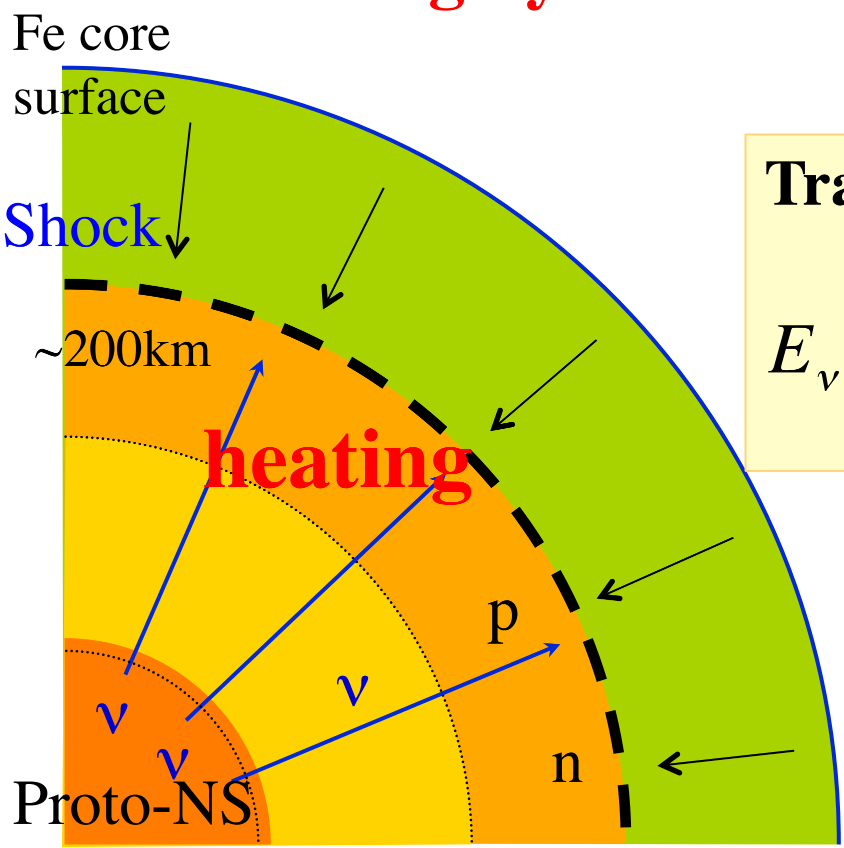
Energy loss due to Fe dissociation

$$E_{loss} \sim -1.6 \times 10^{51} \left(\frac{M_{outer}}{0.1 M_{solar}} \right) erg$$

Neutrino heating mechanism for revival of shock

Heating by neutrino absorption

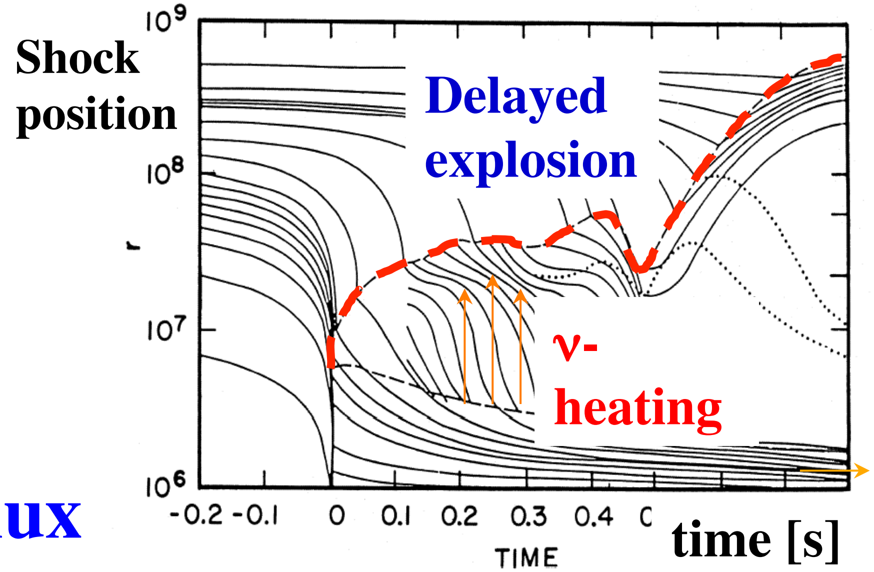
$$\nu_e + n \rightarrow e^- + p,$$
$$\bar{\nu}_e + p \rightarrow e^+ + n$$



100ms after bounce

Transfer of energy from ν Janka A&A (1996)

$$E_{\nu-heat} \sim 2.2\times10^{51}\left(\frac{\Delta M}{0.1M_{solar}}\right)\left(\frac{\Delta t}{0.1s}\right)erg$$



Bethe & Wilson ApJ (1985)

Depends on neutrino energy/flux targets

ν-heating occurs in the intermediate region

between diffusion
& free-streaming

- Need neutrino-transfer
for energy, angle distribution

$f(E_\nu, \theta_\nu)$

ex. Diffusion approx. is not enough

- Even ~10 % change of ν-heating
may affect the outcome: explosion

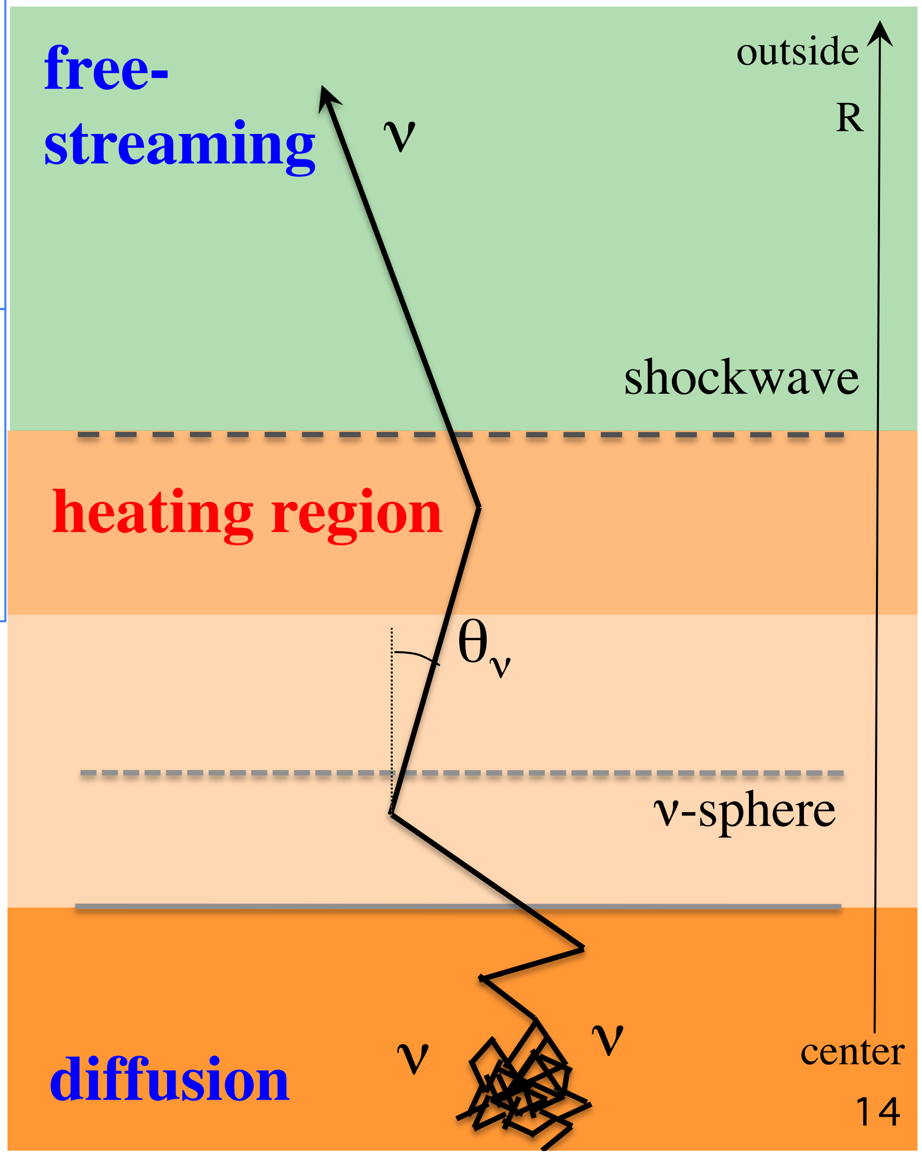
Competing with other effects

ν-heating rate   Janka A&A (1996)

$$Q_\nu^i \approx 110 \frac{MeV}{s \cdot N} \left(\frac{L_\nu E_\nu^2}{R_7^2 < \mu >} X_i \right)$$

average energy, flux: E_ν, L_ν

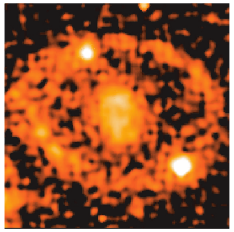
flux factor: $<\mu> = <\cos\theta_\nu> = 0 \sim 1$



Neutrino heating and hydro instabilities

- Convection, SASI, rotation, magnetic etc
- Observations

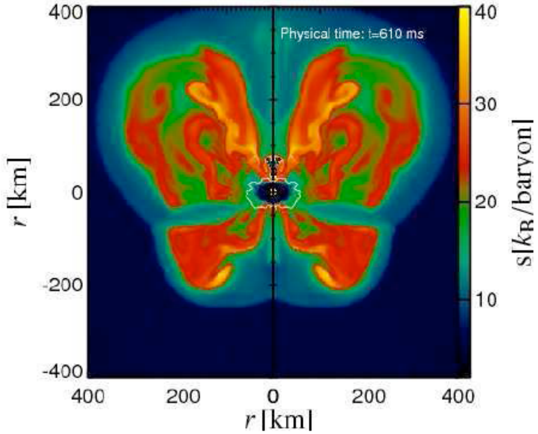
SN1987A



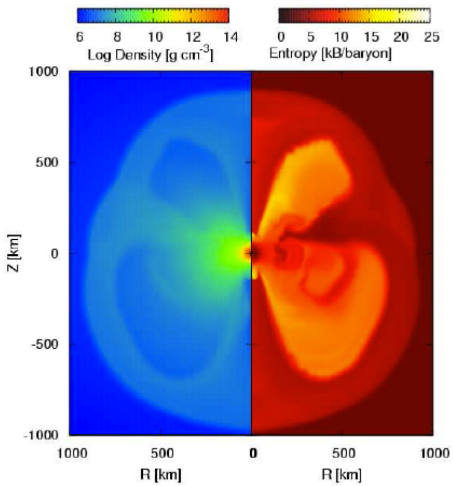
Wang (2002)

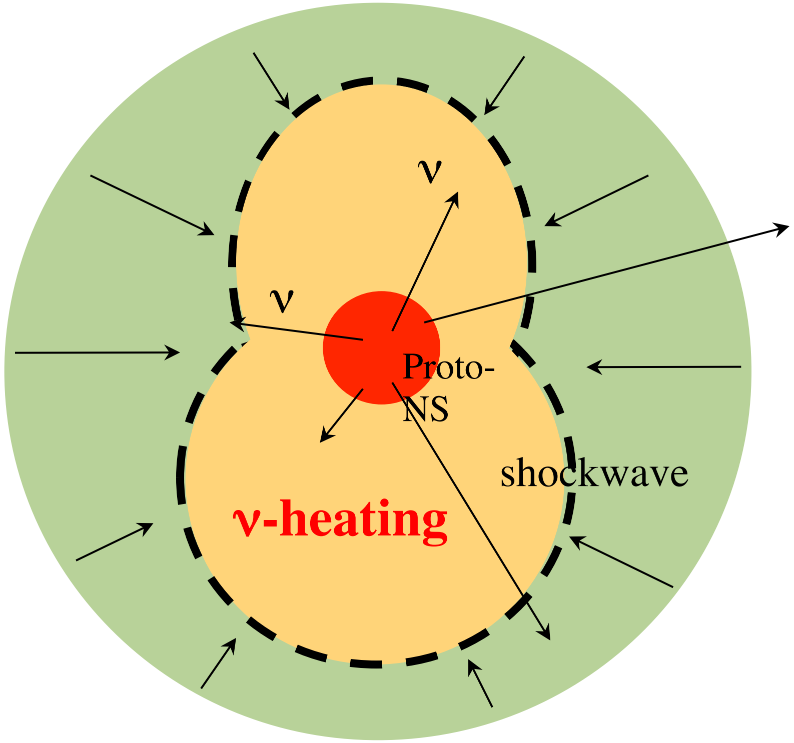
→ neutrino-transfer in multi-dimensions

Marek et al, ApJ (2009)



Suwa et al. (2010) PASJ





To obtain enough ν-heating

Progress of neutrino-transfer

Mezzakappa-Bruenn, Liebendoerfer, Thompson-Burrows,…
Yamada-Sumiyoshi, Kotake-Takiwaki, Rampp-Marek-Janka,…

- 1D: first principle calculations

  Boltzmann eq., Moment method

- 2D, 3D: approximate treatment

– Diffusion (with flux limiter) /IDSA

  Suitable in central part

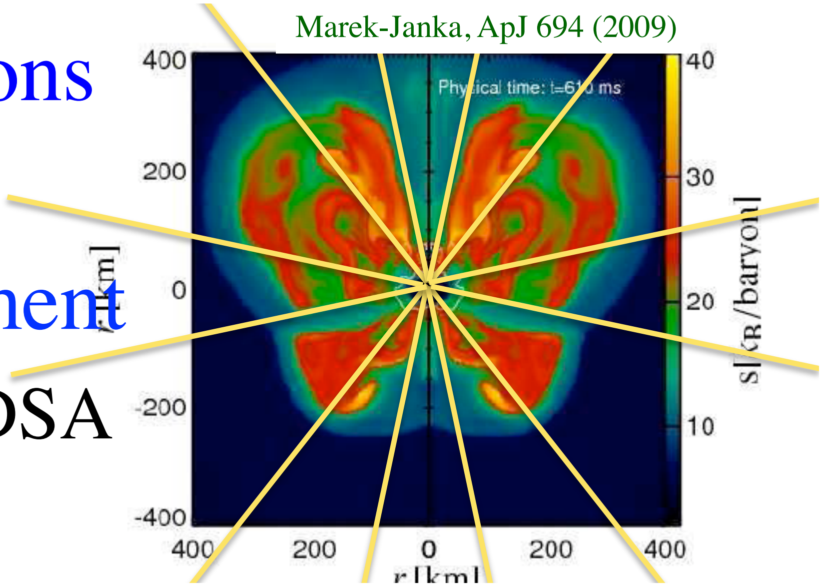
– Ray-by-ray (radial transport)

  Dropping lateral transport

S_n-method in 2D *Ott et al. ApJ(2008)*

Ray-by-ray method

Marek-Janka, ApJ 694 (2009)



1D-transport independently

- Need full 3D calculations: toward the grand challenge
- New code to solve 3D neutrino-transfer

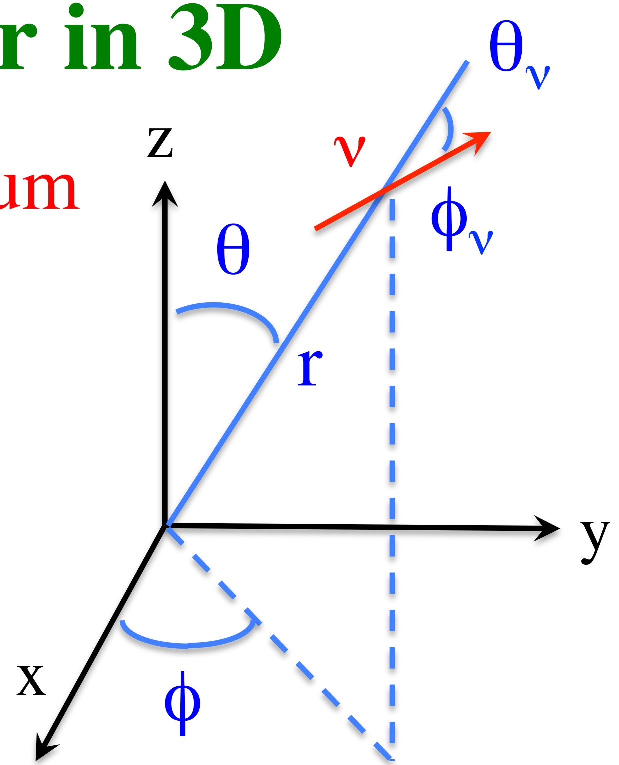
Sumiyoshi & Yamada, ApJS (2012)

Solving neutrino-transfer in 3D space

Challenge: Boltzmann equation in 6D

Sumiyoshi & Yamada, ApJS 199 (2012) 17

To solve neutrino transfer in 3D



- Work in 6D: 3D space + 3D momentum

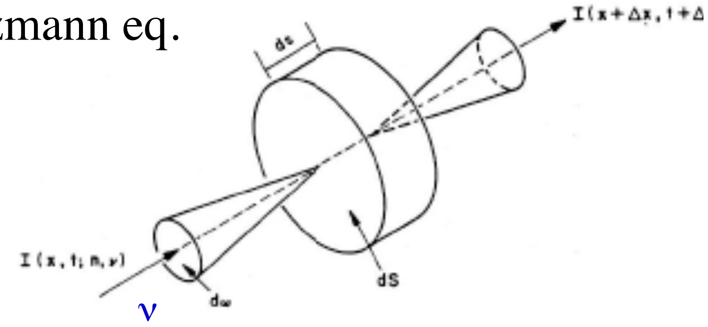
$$f_{\nu}(r,\theta,\phi;\ \varepsilon_{\nu},\theta_{\nu},\phi_{\nu};\ t)$$

  – Neutrino energy (ε_{ν}), angle (θ_{ν}, ϕ_{ν})

- Time evolution of 6D-distribution

$$\frac{1}{c}\frac{\partial f_{\nu}}{\partial t} + \vec{n}\cdot\vec{\nabla} f_{\nu} = \frac{1}{c}\left(\frac{\delta f_{\nu}}{\delta t}\right)_{collision}$$

Boltzmann eq.



  – Left: Neutrino number change
  – Right: Change by neutrino reactions

- Energy, angle-dependent reactions
  – Compositions in dense matter (EOS table)

Boltzmann eq. in spherical coordinate

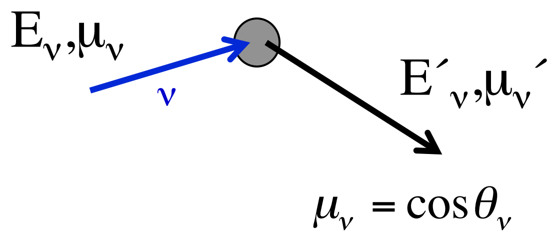
Sumiyoshi & Yamada, ApJS (2012)

$$\frac{1}{c}\frac{\partial f_\nu}{\partial t} + \frac{\mu_\nu}{r^2}\frac{\partial}{\partial r}(r^2 f_\nu) + \frac{\sqrt{1-\mu_\nu^2}\cos\phi_\nu}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f_\nu) + \frac{\sqrt{1-\mu_\nu^2}\sin\phi_\nu}{r\sin\theta}\frac{\partial f_\nu}{\partial\phi}$$

$$+\frac{1}{r}\frac{\partial}{\partial\mu_\nu}[(1-\mu_\nu^2)f_\nu] + \frac{\sqrt{1-\mu_\nu^2}\cos\theta}{r\sin\theta}\frac{\partial}{\partial\phi_\nu}(\sin\phi_\nu f_\nu) = \frac{1}{c}\left(\frac{\delta f_\nu}{\delta t}\right)_{collision}$$

- Discrete in conservative form (S_n method)
- Implicit method in time
  - stability, time step, equilibrium
- Collision term for ν-reactions
  - Different time scales: Stiff eq.

Multi-energy, angle



$$\frac{1}{c}\left(\frac{\delta f_\nu}{\delta t}\right)_{collision} = j_{emission}(1-f_\nu) - \frac{1}{\lambda_{absoption}}f_\nu + C_{inelastic}[\int f_\nu(E'_\nu,\mu'_\nu)dE'_\nu]$$

- absorption, emission, scattering and …

Neutrino reactions in collision term

Basic sets for supernova simulations   Bruenn (1985) +Shen

- Emission & absorption:

  $e^- + p \leftrightarrow \nu_e + n$ $e^- + A \leftrightarrow \nu_e + A'$

  $e^+ + n \leftrightarrow \bar{\nu}_e + p$

- Scattering:

  $\nu_i + N \leftrightarrow \nu_i + N$ $\nu_i + A \leftrightarrow \nu_i + A$

- Pair-process:

  $e^- + e^+ \leftrightarrow \nu_i + \bar{\nu}_i$

  $N + N \leftrightarrow N + N + \nu_i + \bar{\nu}_i$

  3 species: $\nu_e, \bar{\nu}_e, \nu_\mu$

For current computing resources: only with iso-energy scattering & limited relativistic effects

Main computational load: matrix solver

- Linear equation

$$A\vec{f}_{\nu} = \vec{d}$$

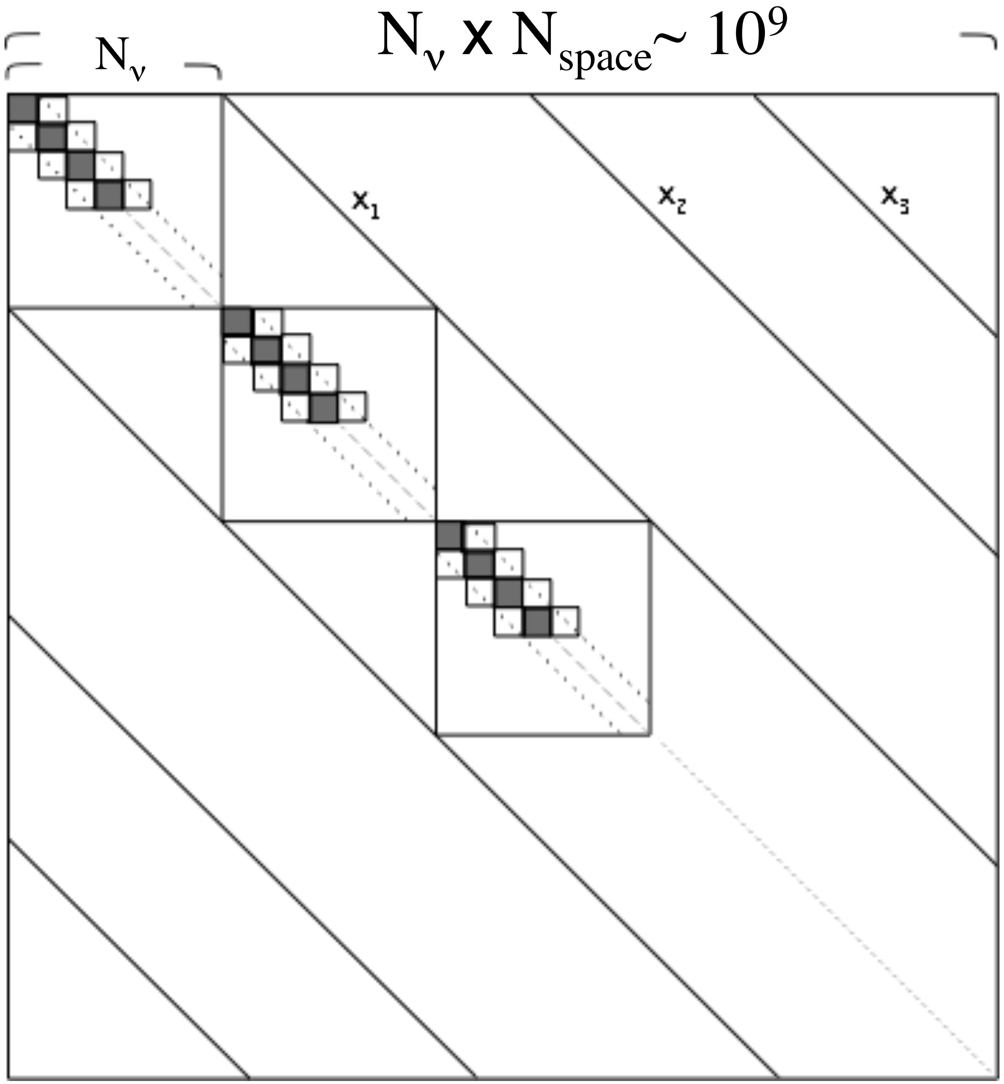
- Neutrino distribution
  - $N_{space}=n_r \times n_\theta \times n_\phi$
  - $N_\nu=n_\varepsilon \times n_{\theta\nu} \times n_{\phi\nu}$

  $N_{vector}\sim10^6 \times 10^3$
- Memory size

  ν-distribution : >10GB

  matrix: >1TB
- Iterative method
  - Pre-conditioner

Imakura et al. JSIAM (2012)



Kotake et al. PTEP (2012)

6D Boltzmann solver works indeed

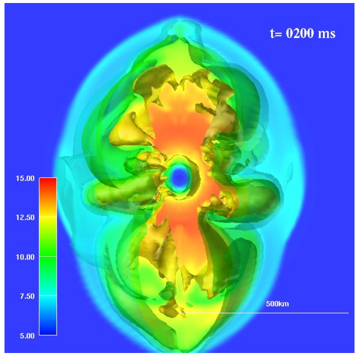
Applications to 3D supernovae

Sumiyoshi, Takiwaki, Matsufuru & Yamada,
arXiv:1403.34476

Neutrino transfer in 3D supernova core

Sumiyoshi et al. (2013,2014)

Takiwaki (2012)



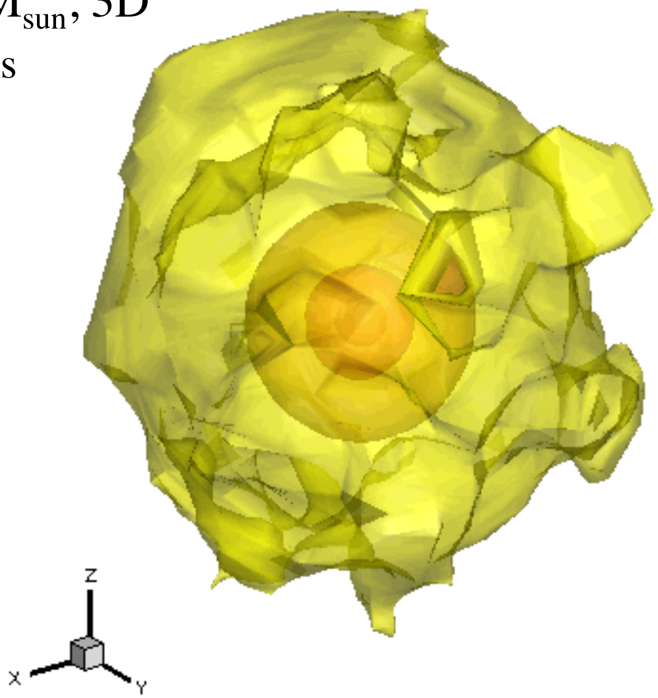
$11.2M_{sun}$, 3D

200ms

Fixing 3D profiles of the supernova core

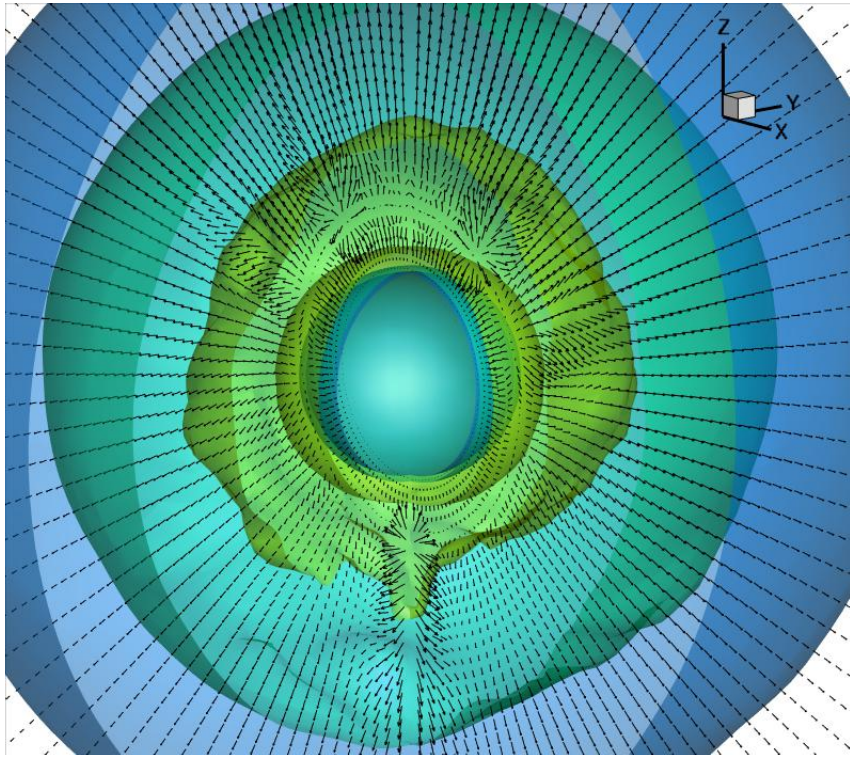
Solve 6D Boltzmann eq. to obtain f_ν

Neutrino density, moments, spectra, heating rate



$\bar{\nu}_e$
density iso-surface

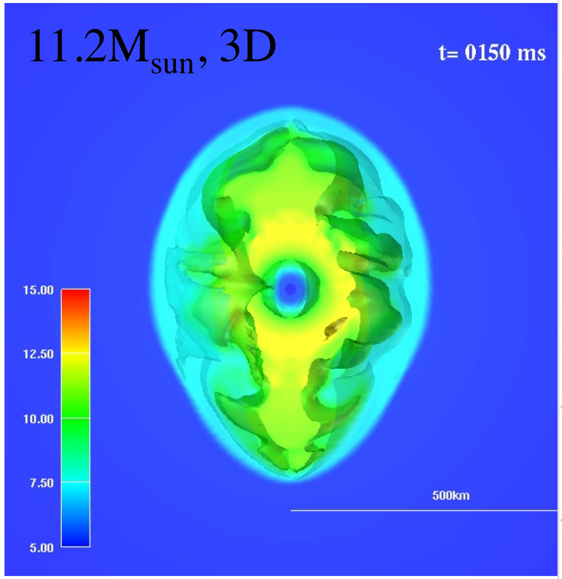
$\bar{\nu}_e$ density iso-surface

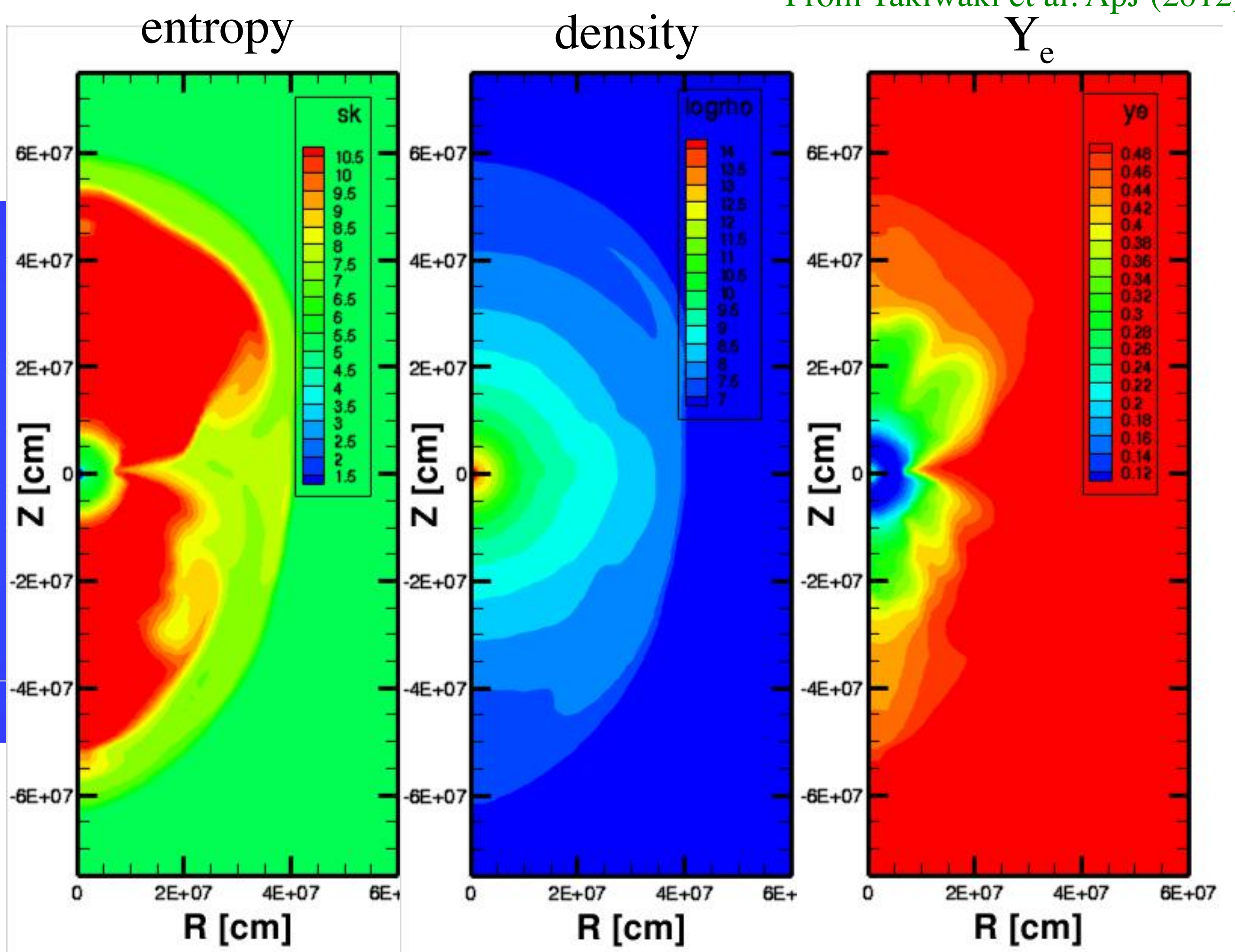


Example: 3D supernova core ($11M_{sun}$) at 150ms

From Takiwaki et al. ApJ (2012)

entropy





150 msec after bounce

R_{shock}~250-400km

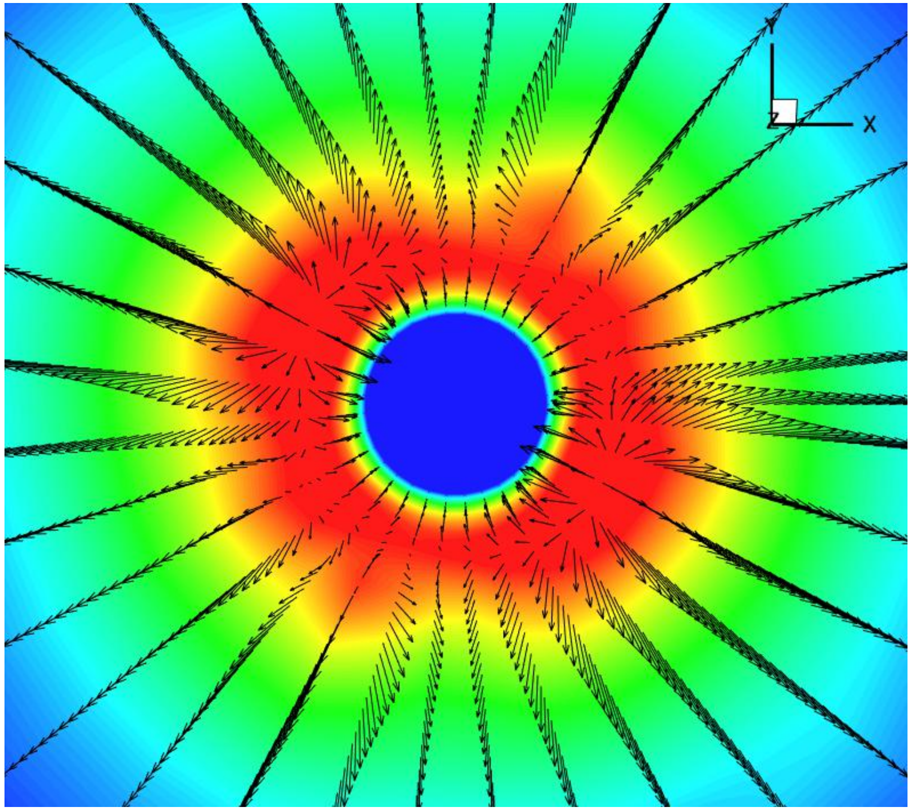
Fix the background profile, evolution by 6D Boltzmann eq.

→ obtain stationary state of the neutrino distributions in 6D

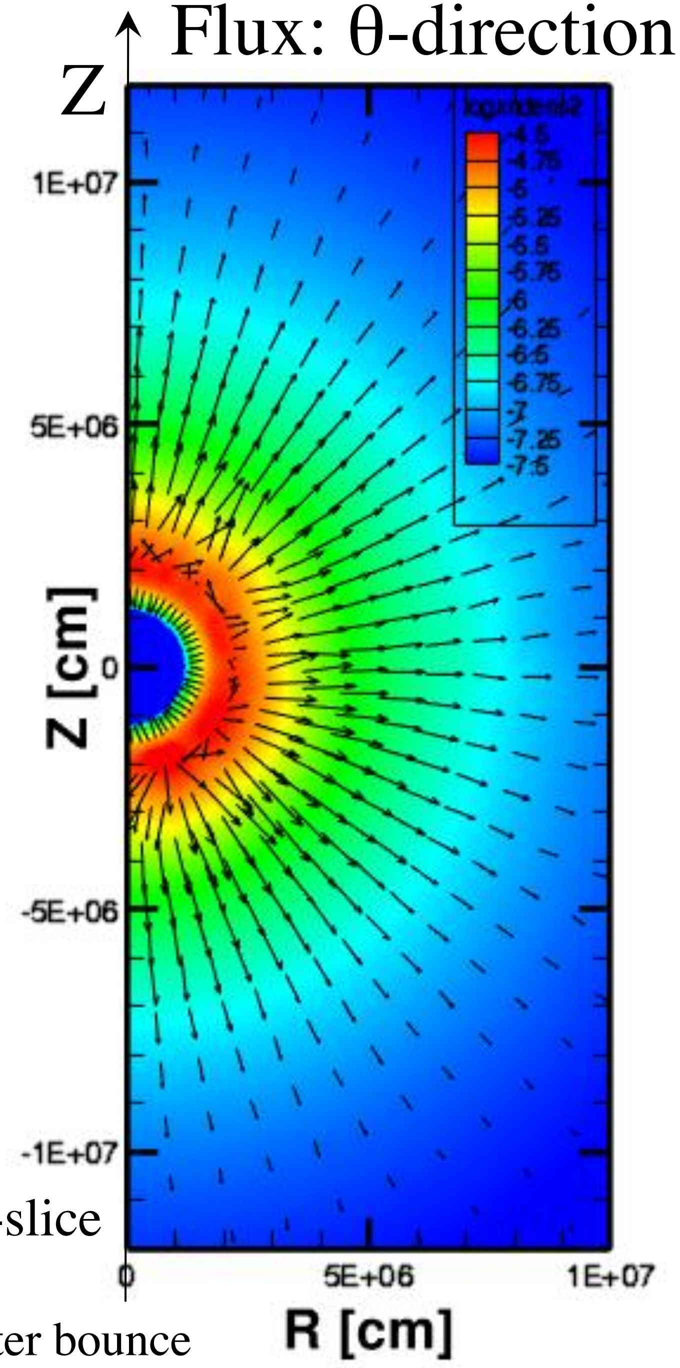
6D Bolzmann in 3D SN core

Describes non-radial transport

Flux: ϕ-direction



Flux: θ-direction



$\bar{\nu}_e$ density: color (flux: arrow)

View from north-pole

View from side: ϕ-slice

Sumiyoshi et al. (2013,2014) 3D Supernova core 150msec after bounce

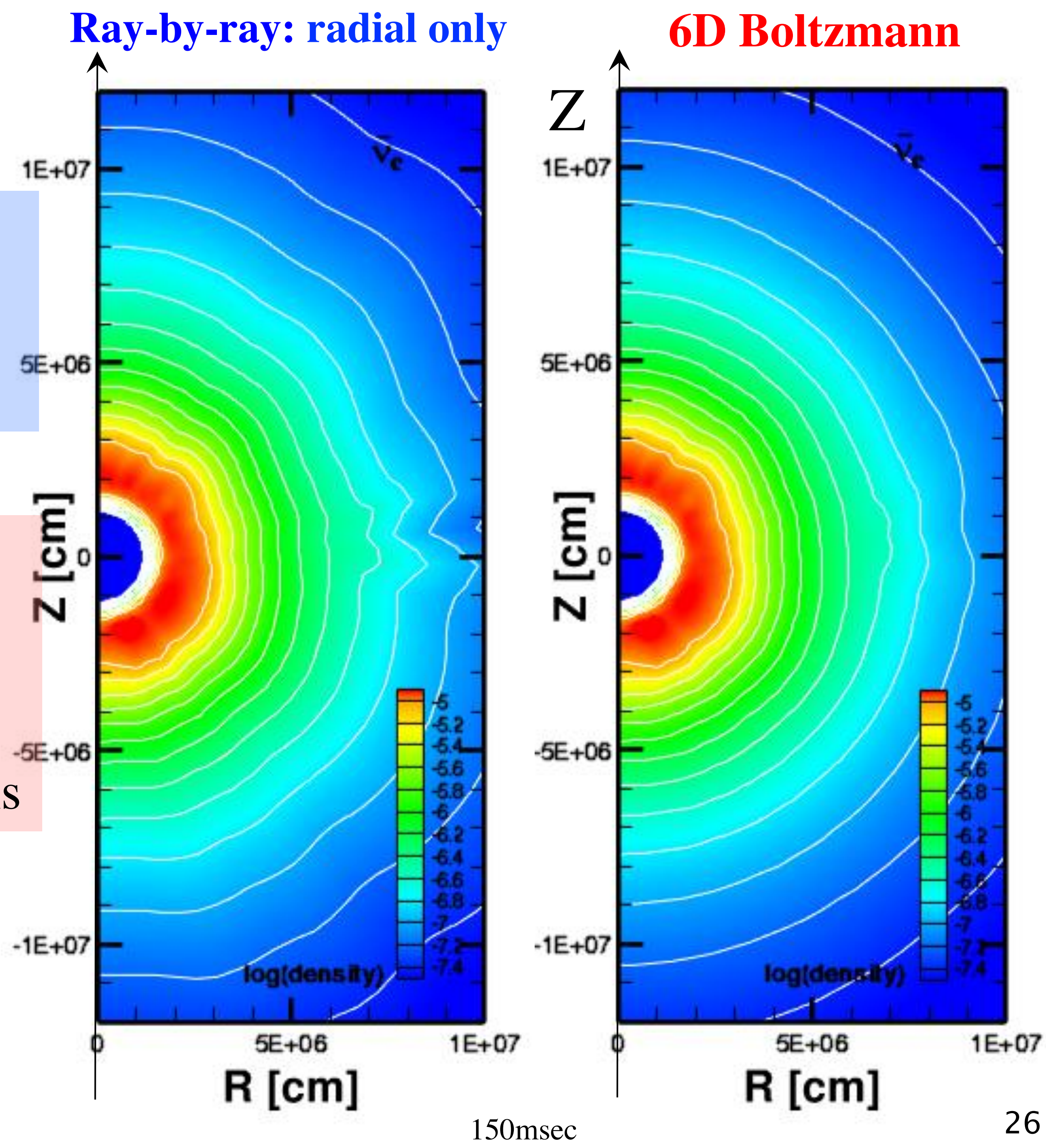
Comparison with approximation

- **Ray-by-ray**
- Only radial transfer
- Anisotropy enhanced

- **6D Boltzmann**
- Non-radial transfer
- Integrated values from various directions

$\bar{\nu}_e$ density: color

View from side: ϕ-slice

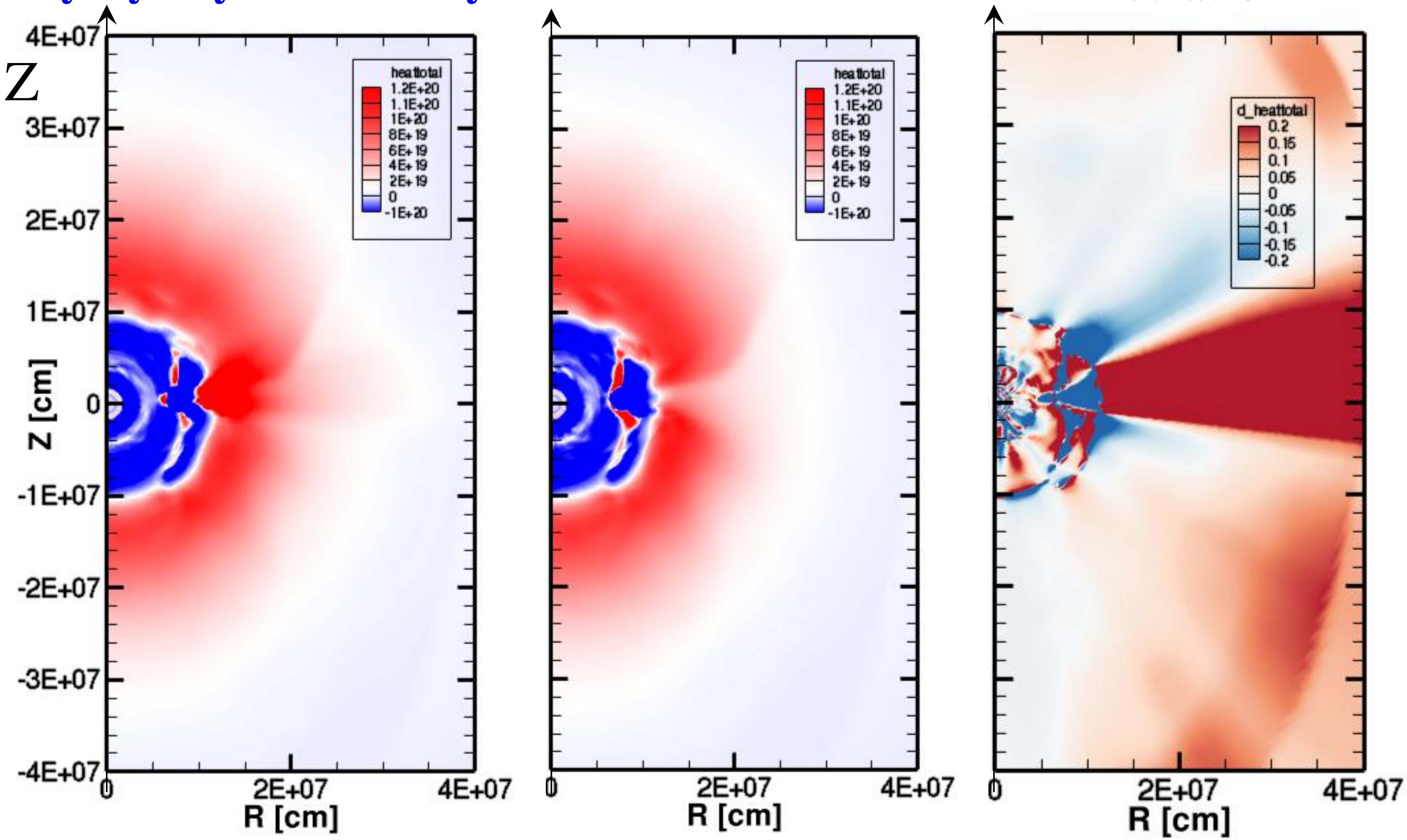


Sumiyoshi et al. (2013,2014)

Comparison: ν-heating rate

$$\delta = \frac{Q_{rbr} - Q_{6D}}{Q_{6D}}$$

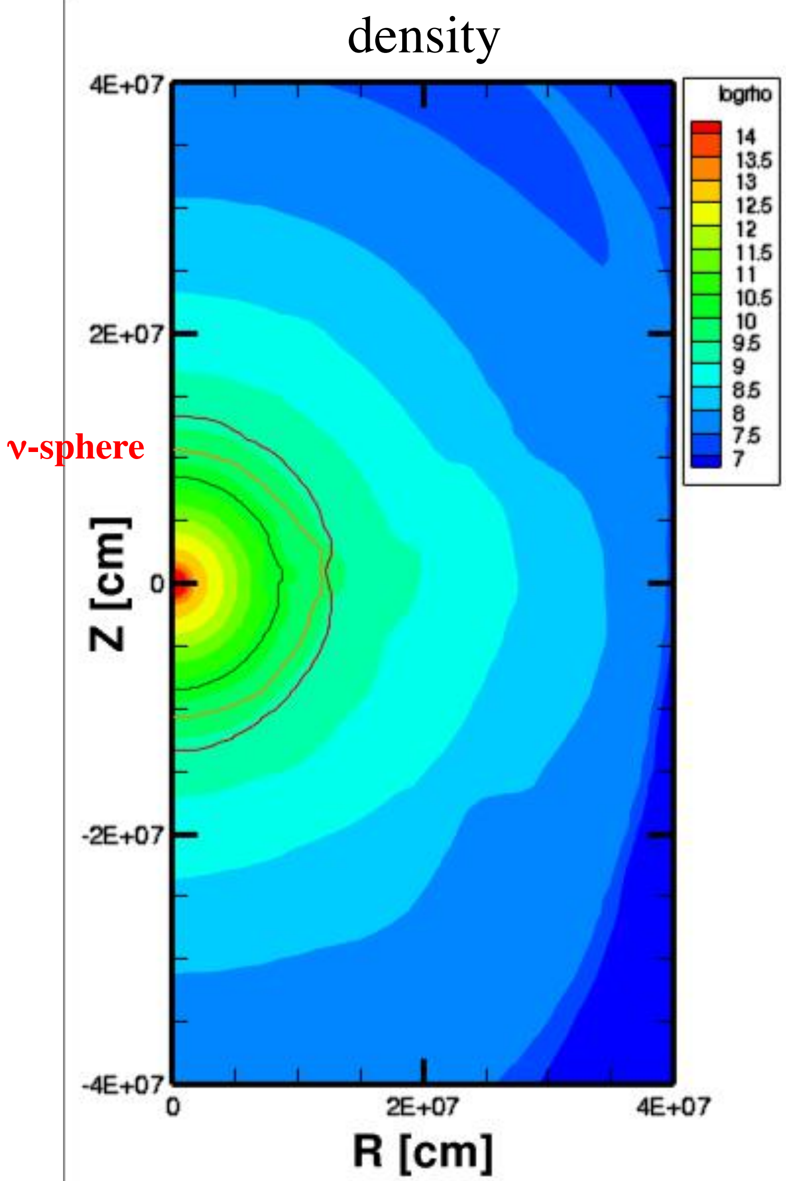
Ray-by-ray: radial only | **6D Boltzmann** | **Deviation**

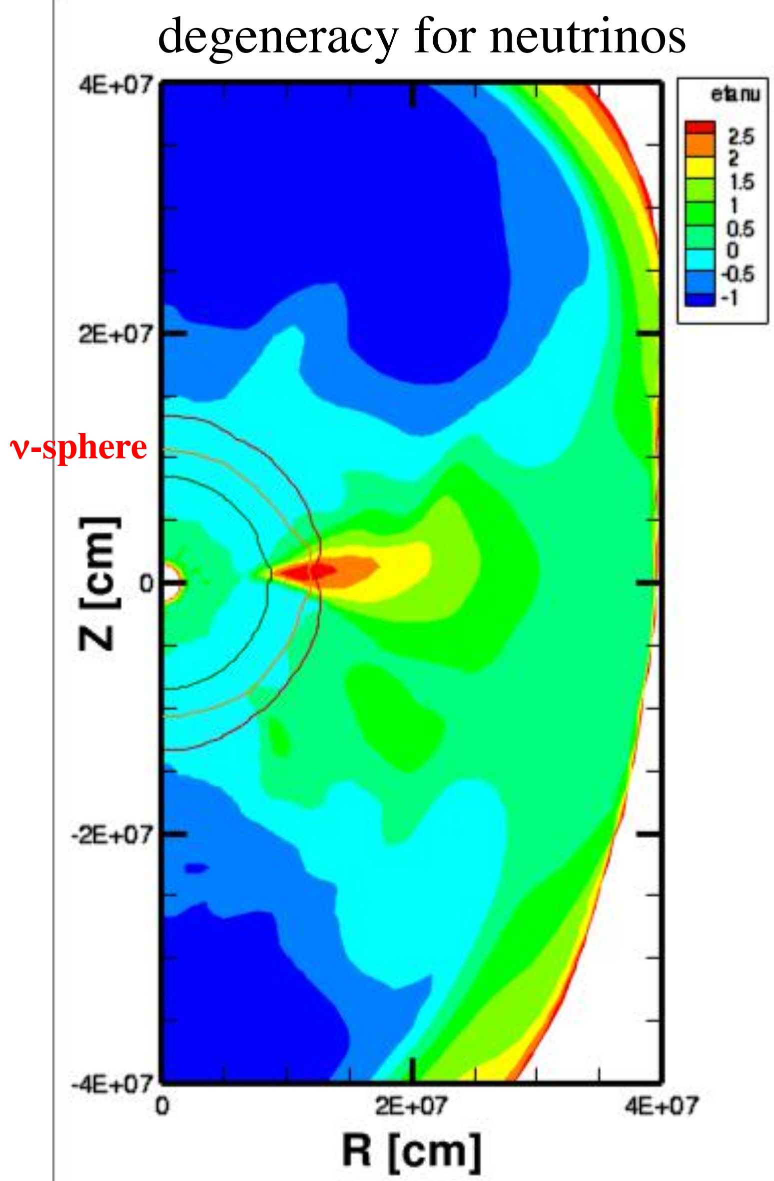


Sumiyoshi et al. (2013,2014)

Red: heating, Blue: cooling 150msec

Local fluctuations of neutrino degeneracy: hotspot



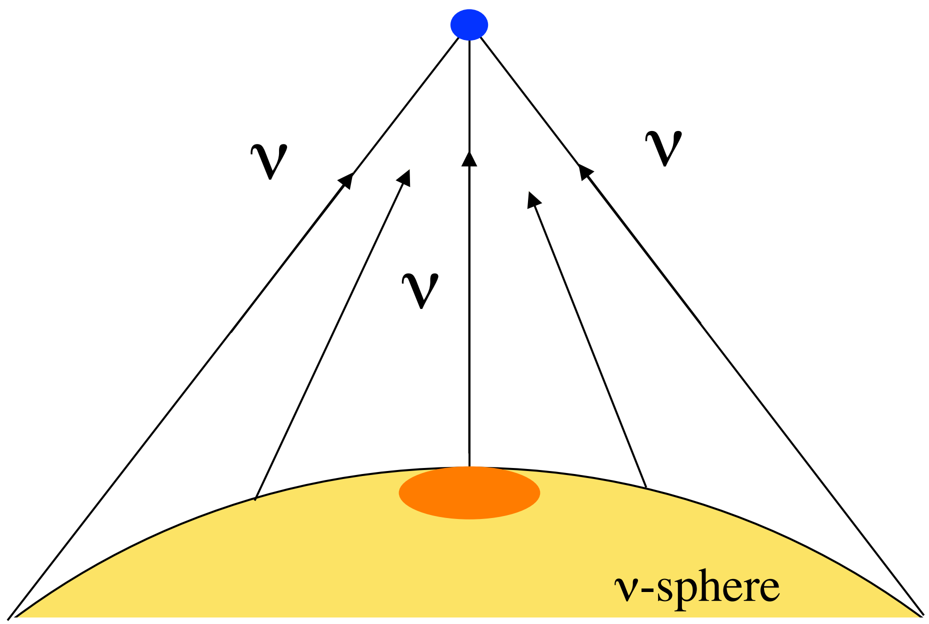


Sumiyoshi et al. (2013,2014)

Evaluation of neutrino fluxes

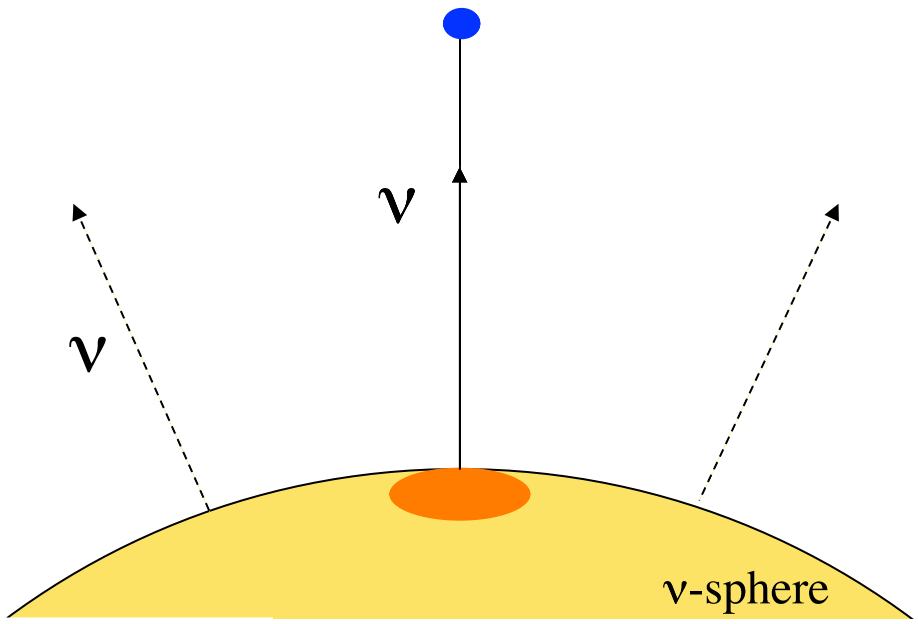
- 6D Boltzmann

Integration from many directions



- Ray-by-ray (RbR)

Contribution from 1 radial direction



Toward 3D supernovae by 6D Boltzmann

- EOS tables and tools available

- Neutrino transfer in 3D supernovae
  - Neutrino heating mechanism for explosion

    Need to determine effects precisely around threshold
- New aspects by 6D Boltzmann solver
  - Non-radial transport, heating rates, angle moments
  - Comparisons with approximate methods
- Toward Exa-scale computing
  - Full 6D Boltzmann & hydrodynamics
  - Need EOS and neutrino reactions rates

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