Multi-D Relativistic Boltzmann-Hydro Code for Core Collapse Supernovae (CCSNe)

Hiroki Nagakura (YITP)

collaborators

K. Sumiyoshi (Numazu), W. Iwakami (YITP, Waseda), S. Furusawa (NAOJ), S. Yamada (Waseda)

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Outline

1. Introduction

- Ultimate Goal in the field of CCSNe.
- Current Status of Neutrino-Hydro Simulations for CCSNe.

2. Numerical Algorithm for the 3D Relativistic-Boltzmann-Hydro Code

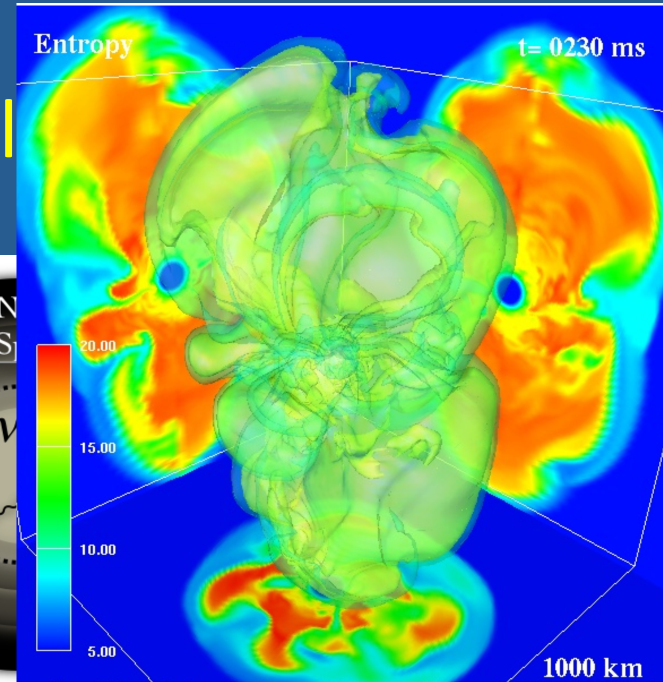
- The importance and difficulty of handling relativistic effects
- Code Tests (Collision term, advection term, 1D CCSNe simulation)

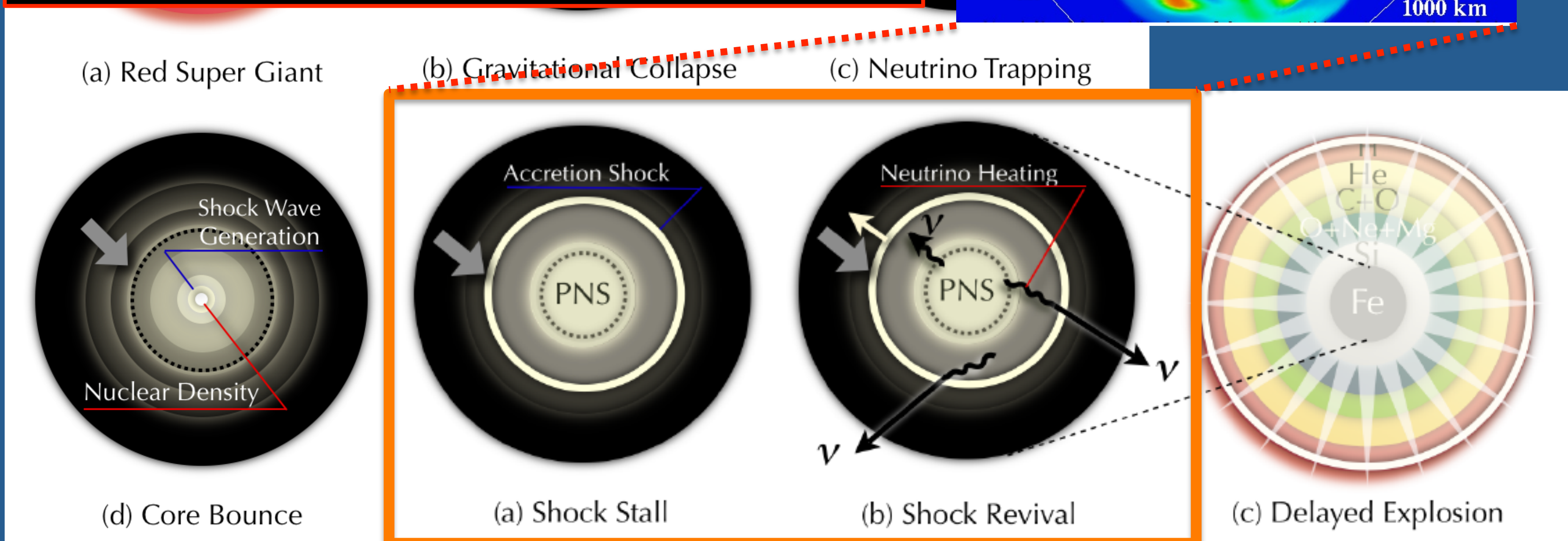
3. Summary and Conclusion

Key Physics

Multi-D Fluid Instabilities
+
Neutrino Heating

We have recently witnessed some successful of shock revival in most advanced numerical simulations (see e.g., Takiwaki et al. 2014, Lentz et al. 2013, Muller et al. 2012)





Cartoon From Iwakami D thesis

Supernova Physics

Micro- and Macro- Physics complexly interplay in CCSNe.

- $$(\rho_0 u^\mu)_{;\mu} = 0$$: Continuity Equation
- $$T^{\mu\nu}_{(\mathrm{hd});\nu} + (T^{\mu\nu}_{(\mathrm{em});\nu}) = G^\mu$$: Energy Momentum Conservation
- $$(n_e u^\mu)_{;\mu} = \Gamma$$: Lepton number Conservation
- $$(F^{\mu\nu}{}_{;\nu} = 4\pi J^\mu)$$: Maxwell Equation
- $$G_{\mu\nu} = 8\pi T_{\mu\nu}$$: Einstein Equation
- $$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = \left(\frac{\delta f}{\delta \tau}\right)_{\mathrm{col}}$$: Boltzmann Equation (Neutrino Transfer)

EOS (Nuclear Physics)

Weak Interaction

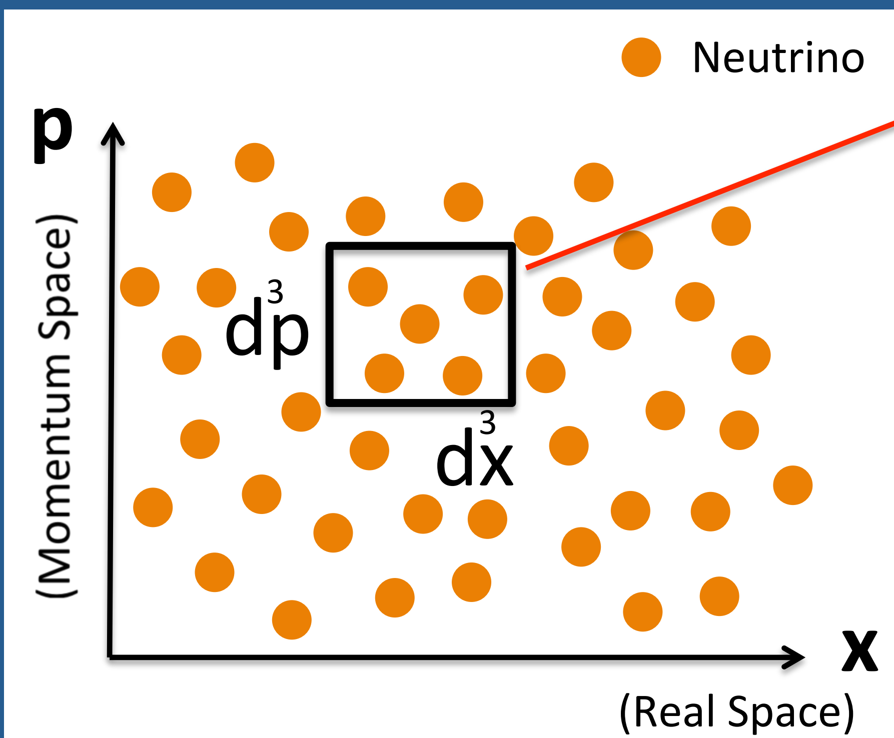
We have not yet accomplished numerical studies under the fully consistent treatments.

Ab initio approach: Solving GR Boltzmann equation

$$p^{\mu}\frac{\partial f}{\partial x^{\mu}} + \frac{dp^{i}}{d\tau}\frac{\partial f}{\partial p^{i}} = \left(\frac{\delta f}{\delta \tau}\right)_{\rm col},$$

(Time evolution + Advection Term) (Collision Term)

6 Dimensional Phase Space



$$dN = f(t, \boldsymbol{p}, \boldsymbol{x}) d^3p d^3x$$

Conservation form of GR Boltzmann eq.

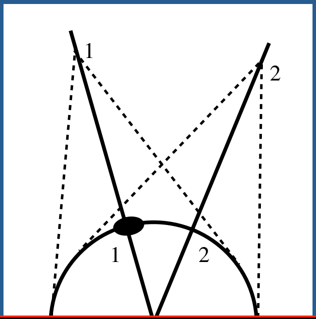
$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\nu^{-1}p^{\alpha}f)}{\partial x^{\alpha}}\Bigg|_{q_{(i)}} + \frac{1}{\nu^2}\frac{\partial}{\partial \nu}\left(-\nu f p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(0)}\right)$$
$$+\frac{1}{\sin\bar\theta}\frac{\partial}{\partial\bar\theta}\left(\nu^{-2}\sin\bar\theta f\sum_{j=1}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar\theta}\right)$$
$$+\frac{1}{\sin^2\bar\theta}\frac{\partial}{\partial\bar\varphi}\left(\nu^{-2}f\sum_{j=2}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar\varphi}\right) = S_{\rm rad},$$

Shibata, Nagakura, Sekiguchi, Yamada (2014)

Various Approximations for Multi-D Neutrino Transfer

- **Ray-by-Ray Approach** (MPA, Oak Ridge, Kotake-Takiwaki-Suwa)

  Neutrino-Advection is essentially considered under spherical symmetry.



- **Isotropic Diffusion Source Approximation (IDSA)**
  (Basel, Kotake-Takiwaki-Suwa)

Almost every approach employs O(v/c) expansion except for several numerical relativity simulations.

Fully consistent SR treatment is an important step towards full GR Boltzmann simulation, which has been implemented in our newly developed code.

in the higher moment.

- **Multi-Group Flux-Limited-Diffusion (MGFLD)**
  (Oak Ridge, Princeton, Caltech)

  Neutrino Transports are treated as the Energy-Dependent Diffusion Equation.

Recent Progress in our group

- Sumiyoshi & Yamada (2012) succeeded to solve the 3D Boltzmann Transfer equation.

  All O(v/c) terms are omitted.

  Back reactions to matter are neglected.

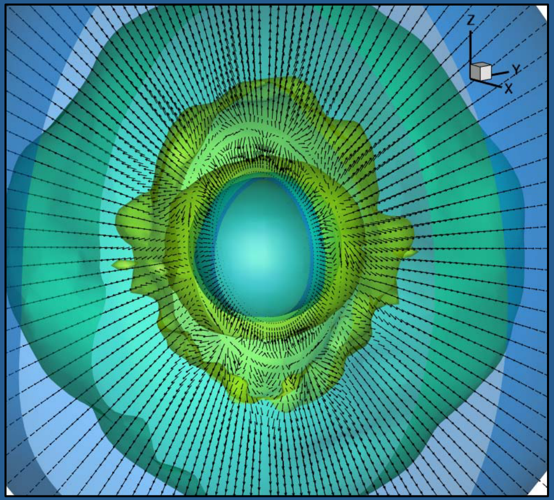
- 3D SR-Boltzmann-Hydro Code has been developed (Nagakura et al. 2014).

  The success of coupling Boltzmann Transfer with Hydro.

  Handling SR effects in a non-conventional way, taking into account of all order of v/c.

- Numerical Study of 2D Post-bounce evolutions by Boltzmann-Hydro Code (Iwakami et al. in prep).

  The success of demonstrations for SASI and neutrino driven convections.



Color contour for neutrino number density (Sumiyoshi et al. 2014)

2. Numerical Algorithm for the 3D SR-Boltzmann-Hydro Code

Basic Equations

ring Scheme)

V $(\rho_0 u$ — on

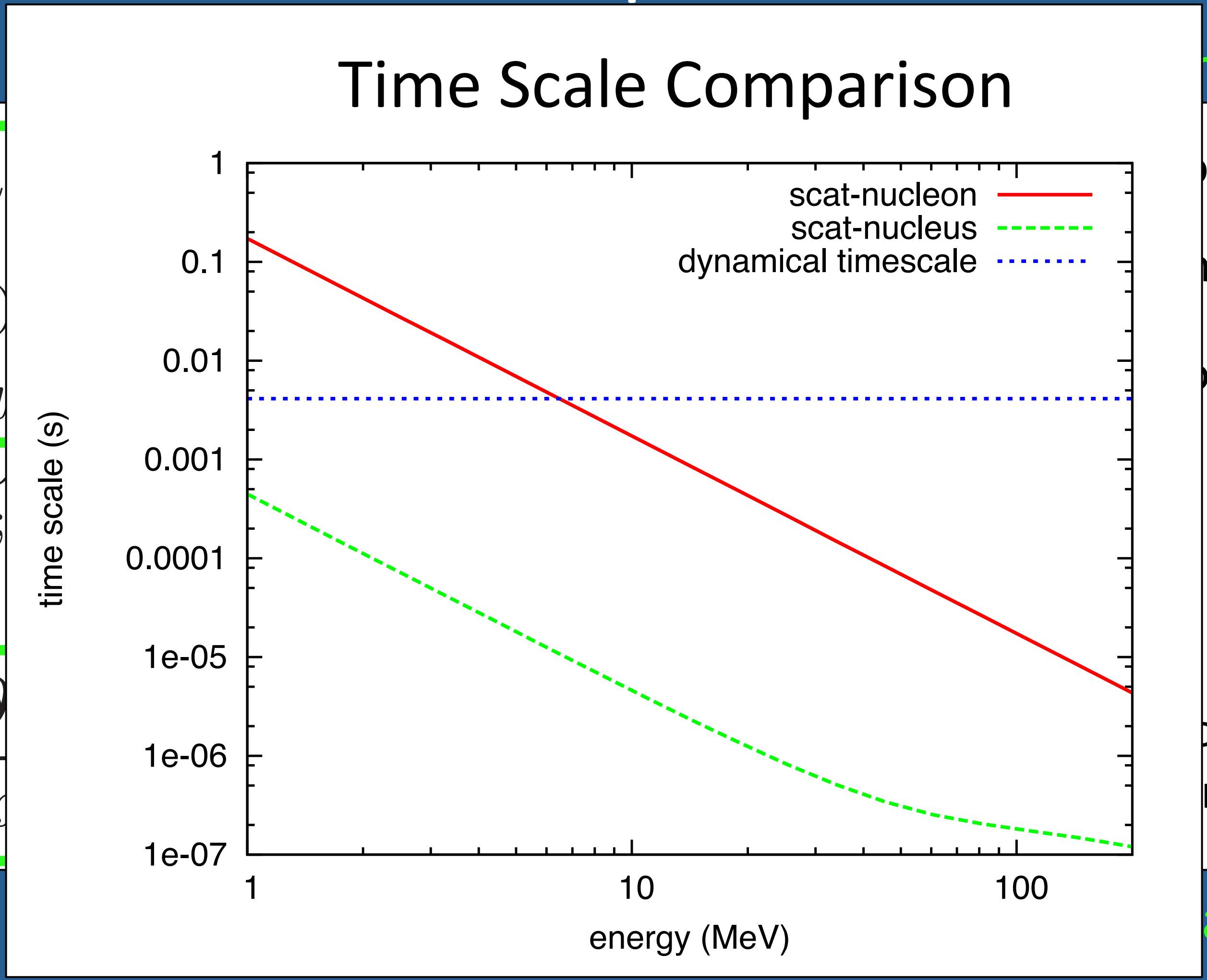
V $T^{\mu\nu}_{(\mathrm{hd})}$ — n Conservation

V $(n_e u$ — onservation

V $(F^{\mu\nu}_{;}$

V $G_{\mu\nu}$

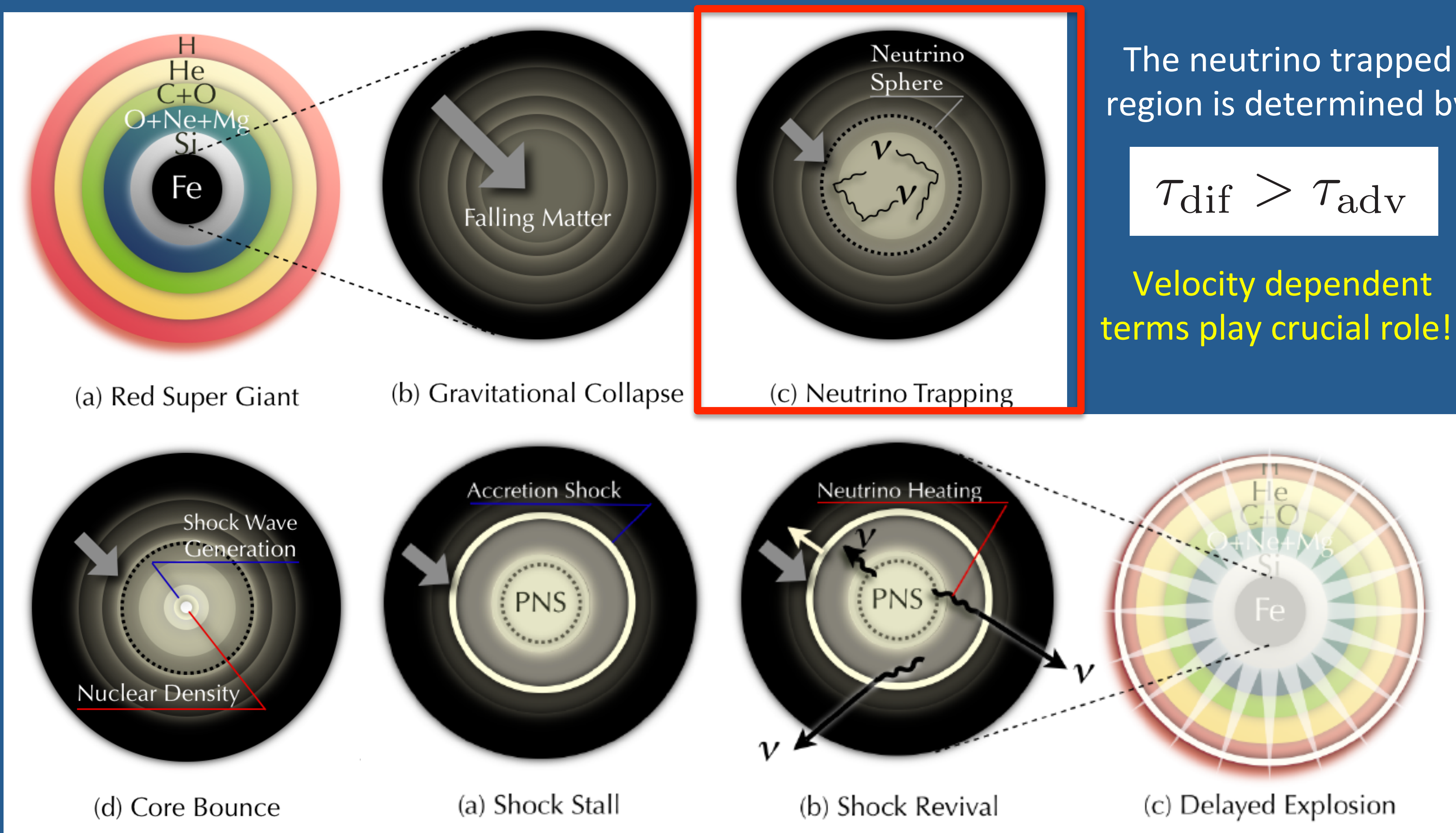
V $p^{\mu}\frac{\partial}{\partial x}$ — on r)



ation)

The characteristic time scale of weak Interaction is much shorter than the dynamical time scale.

The importance of SR effects (velocity dependent terms)

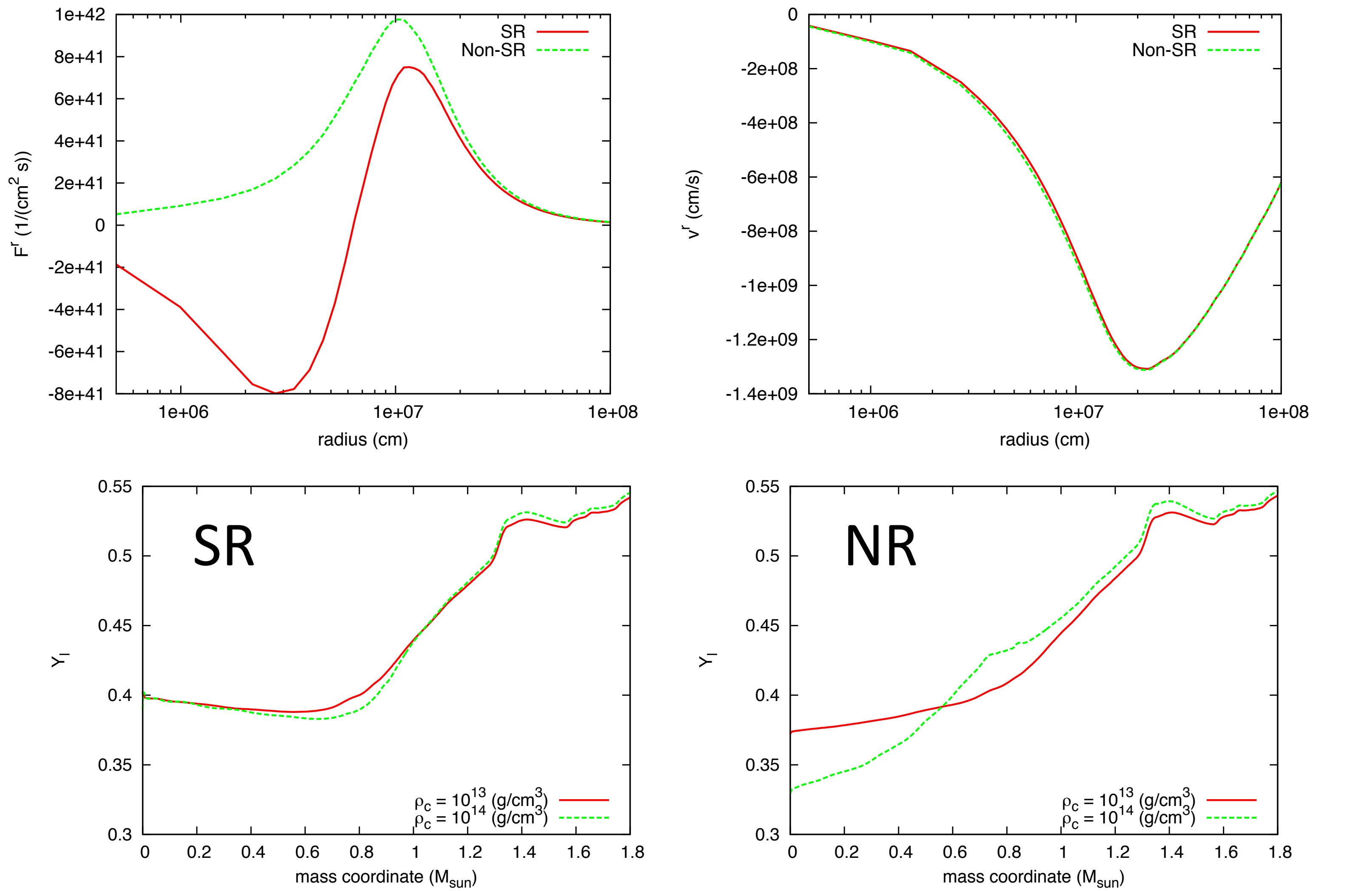


The neutrino trapped region is determined by

$$\tau_{\rm dif} > \tau_{\rm adv}$$

Velocity dependent terms play crucial role!!

Neutrino flux and the evolution of lepton fraction in CCSNe

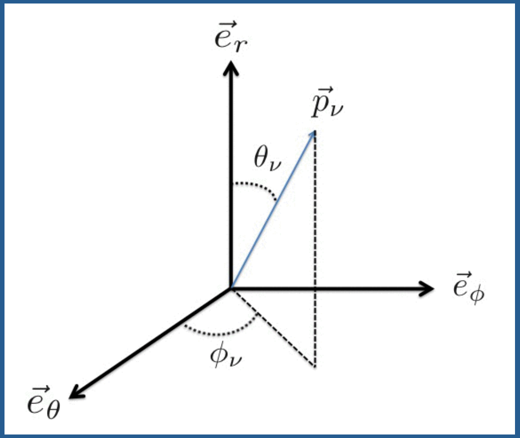


Two independent ways to involve velocity dependent terms (SR treatment)

Eulerian Approach

Energy and angular direction of neutrinos are defined in the laboratory frame.

Velocity dependent terms are appeared in the treatment of collision term.



Lagrangian Approach

Energy and angular direction of neutrinos are defined in the fluid-rest frame.

Velocity dependent terms are appeared in the advection term.

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\,\nu^{-1}p^{\alpha}f)}{\partial x^{\alpha}}\bigg|_{q_{(i)}} + \frac{1}{\nu^{2}}\frac{\partial}{\partial\nu}\left(-\nu f p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(0)}\right)$$
$$+\frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\left(\nu^{-2}\sin\bar{\theta}f\sum_{j=1}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\theta}}\right)$$
$$+\frac{1}{\sin^{2}\bar{\theta}}\frac{\partial}{\partial\bar{\varphi}}\left(\nu^{-2}f\sum_{j=2}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\varphi}}\right) = S_{\rm rad},$$

Drawbacks and Advantages
(Eulerian Approach)

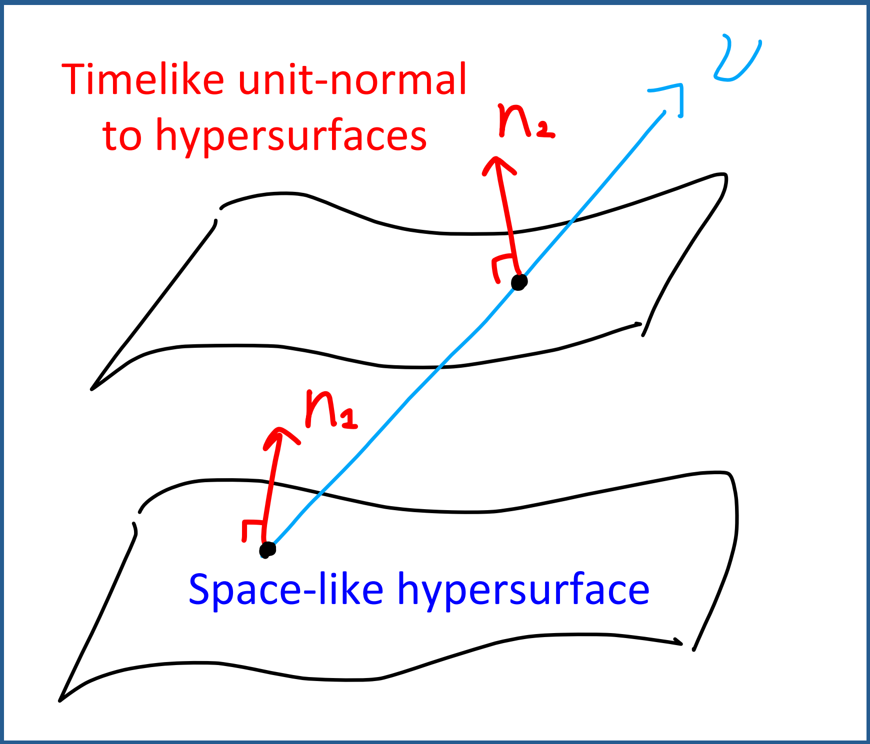
∨ Advantages

Easy to treat Advection Terms

$$\frac{\partial f}{\partial t} + \frac{\mu_\nu}{r^2}\frac{\partial}{\partial r}(r^2 f) + \frac{\sqrt{1-\mu_\nu^2}\,\cos\,\phi_\nu}{r\sin\,\theta}\frac{\partial}{\partial\theta}(\sin\,\theta f)$$
$$+\frac{\sqrt{1-\mu_\nu^2}\,\sin\,\phi_\nu}{r\sin\,\theta}\frac{\partial f}{\partial\phi} + \frac{1}{r}\frac{\partial}{\partial\mu_\nu}[(1-\mu_\nu^2)f]$$
$$-\frac{\sqrt{1-\mu_\nu^2}}{r}\frac{\cos\,\theta}{\sin\,\theta}\frac{\partial}{\partial\phi_\nu}(\sin\,\phi_\nu f) = \left(\frac{\delta f}{\delta t}\right)_{\rm col}^{\rm lb},$$

For flat spacetime, no energy derivative terms in advection are appeared.
(Even for GR, there are no serious problems for the treatment of advection terms.)

Neutrino energy shift in the curved spacetime (from the perspective of 3+1 decomposition)



GR Boltzmann Equation

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\nu^{-1}p^{\alpha}f)}{\partial x^{\alpha}}\Bigg|_{q_{(i)}} + \frac{1}{\nu^2}\frac{\partial}{\partial \nu}(-\nu f p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(0)}) + \frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\left(\nu^{-2}\sin\bar{\theta}f\sum_{j=1}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\theta}}\right) + \frac{1}{\sin^2\bar{\theta}}\frac{\partial}{\partial\bar{\varphi}}\left(\nu^{-2}f\sum_{j=2}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\varphi}}\right) = S_{\rm rad},$$

Neutrino energy is defined as

$$\nu = -p_a e^a_{(0)}$$

Even if "n" vector is chosen as the time-like tetrad , the neutrino energy is shifted. (since n vectors are spatially inhomogeneous and dynamical.)

Drawbacks and Advantages
(Eulerian Approach)

✓ Advantages

Easy to treat Advection Terms

$$\frac{\partial f}{\partial t} + \frac{\mu_\nu}{r^2}\frac{\partial}{\partial r}(r^2 f) + \frac{\sqrt{1-\mu_\nu^2}\cos\phi_\nu}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f)$$
$$+\frac{\sqrt{1-\mu_\nu^2}\sin\phi_\nu}{r\sin\theta}\frac{\partial f}{\partial\phi} + \frac{1}{r}\frac{\partial}{\partial\mu_\nu}[(1-\mu_\nu^2)f]$$
$$-\frac{\sqrt{1-\mu_\nu^2}}{r}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\phi_\nu}(\sin\phi_\nu f) = \left(\frac{\delta f}{\delta t}\right)_{\rm col}^{\rm lb},$$

For flat spacetime, no energy derivative terms can be appeared.
(Even for GR, there are no serious problems for the treatment of advection terms.)

✓ Drawbacks

Handling the frame transformations for evaluating collision terms

f itself is invariant

$$\left(\frac{\delta f}{\delta t}\right)_{\rm col}^{\rm lb} = D^{\rm lb}\left(\frac{\delta f}{\delta \tilde{t}}\right)_{\rm col}^{\rm fr}$$

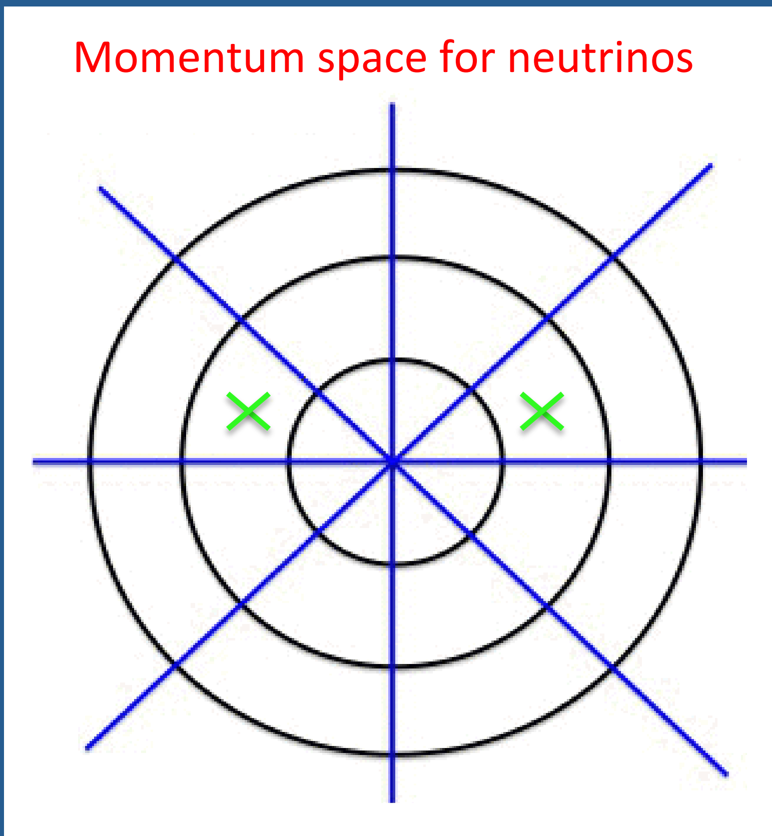
$$\boldsymbol{n}^{\rm fr} = \frac{1}{D^{\rm lb}}[\boldsymbol{n}^{\rm lb} + \{-\gamma + (\gamma-1)\frac{\boldsymbol{n}^{\rm lb}\cdot\boldsymbol{v}}{v^2}\}\boldsymbol{v}]$$

$$\varepsilon^{\rm fr} = \varepsilon^{\rm lb}\gamma(1-\boldsymbol{n}^{\rm lb}\cdot\boldsymbol{v}),$$

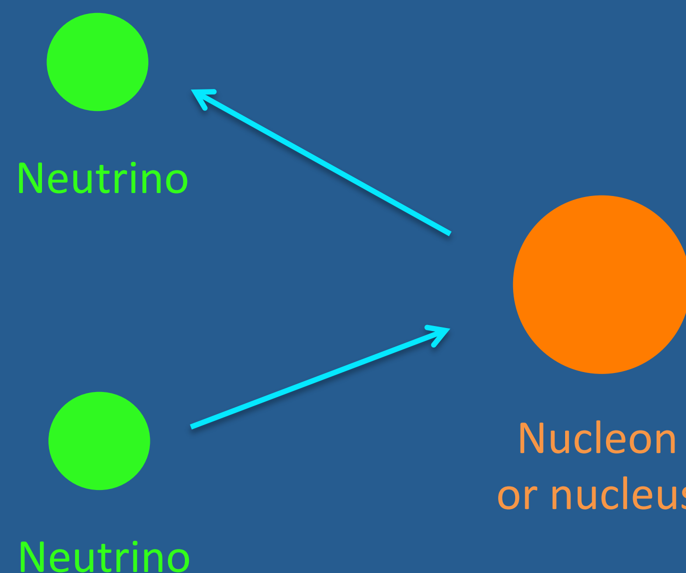
The treatment of iso-energy scattering is practically very difficult !!!

Iso-Energy Scattering

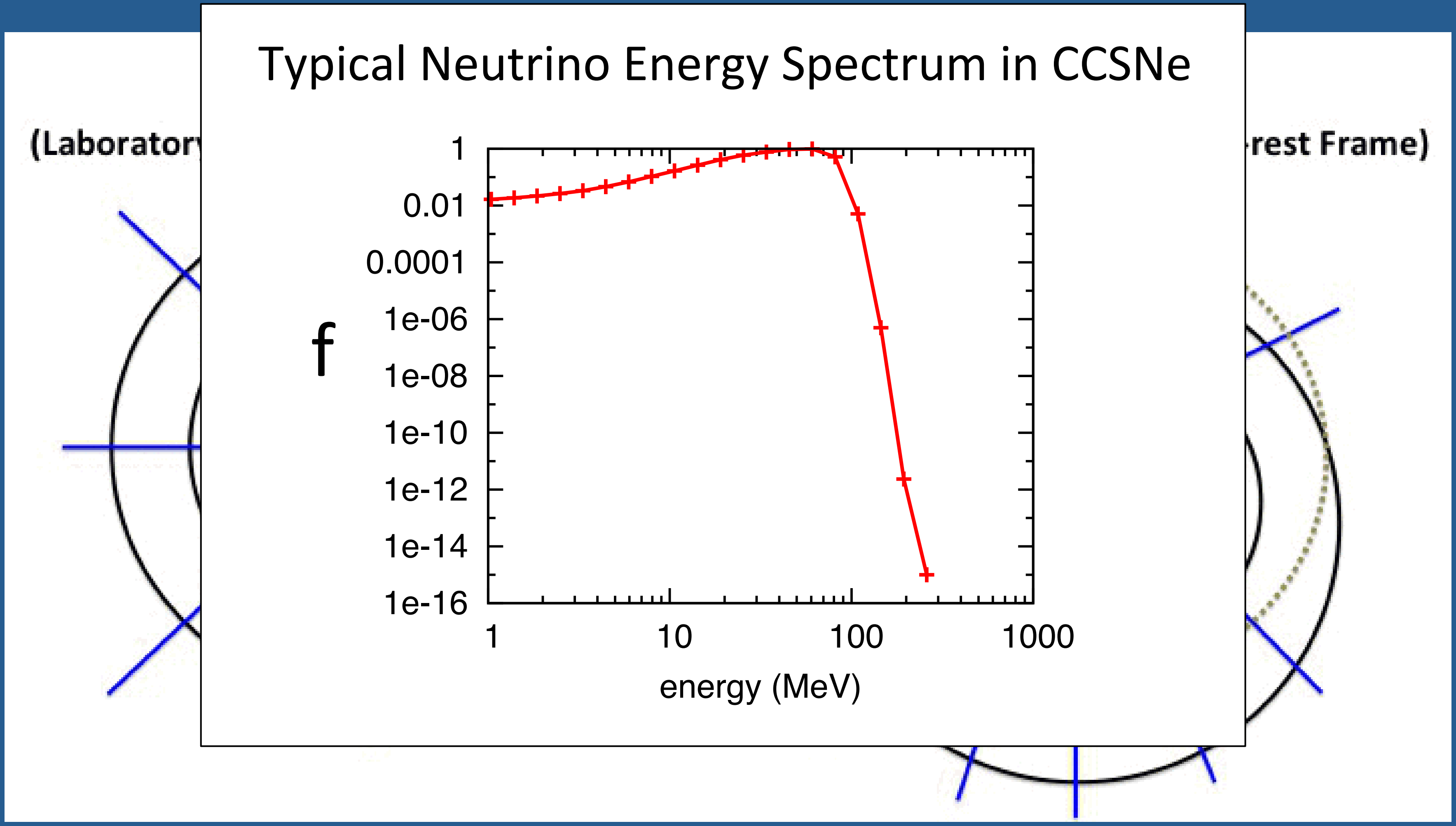
The dominant opacity during the collapsing phase.
It should be treated with the appropriate manner!



$$\nu + N \longleftrightarrow \nu + N \quad ,$$
$$\nu + A \longleftrightarrow \nu + A \quad .$$



Momentum meshes are distorted due to frame transformations



Drawbacks and Advantages
(Lagrangian Approach)

∨ Advantages

Easy to treat Collision Terms

Not necessary to handle the frame transformation.

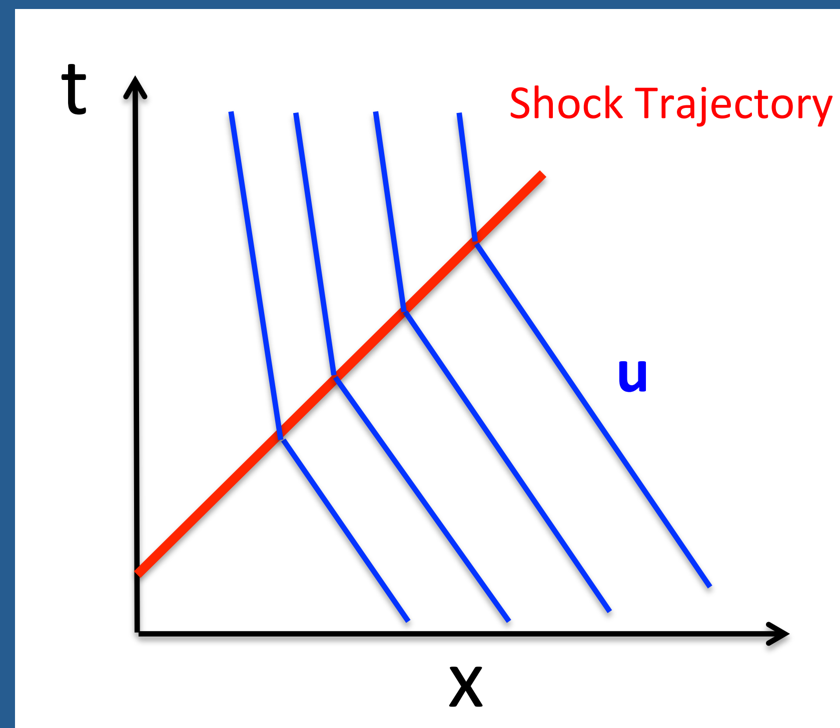
Easy to treat neutrino trapping

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\,\nu^{-1}p^{\alpha}f)}{\partial x^{\alpha}}\bigg|_{q_{(i)}} + \frac{1}{\nu^{2}}\frac{\partial}{\partial\nu}\left(-\nu f p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(0)}\right)$$
$$+ \frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\left(\nu^{-2}\sin\bar{\theta} f\sum_{j=1}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\theta}}\right)$$
$$+ \frac{1}{\sin^{2}\bar{\theta}}\frac{\partial}{\partial\bar{\varphi}}\left(\nu^{-2} f\sum_{j=2}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\varphi}}\right) = S_{\rm rad},$$

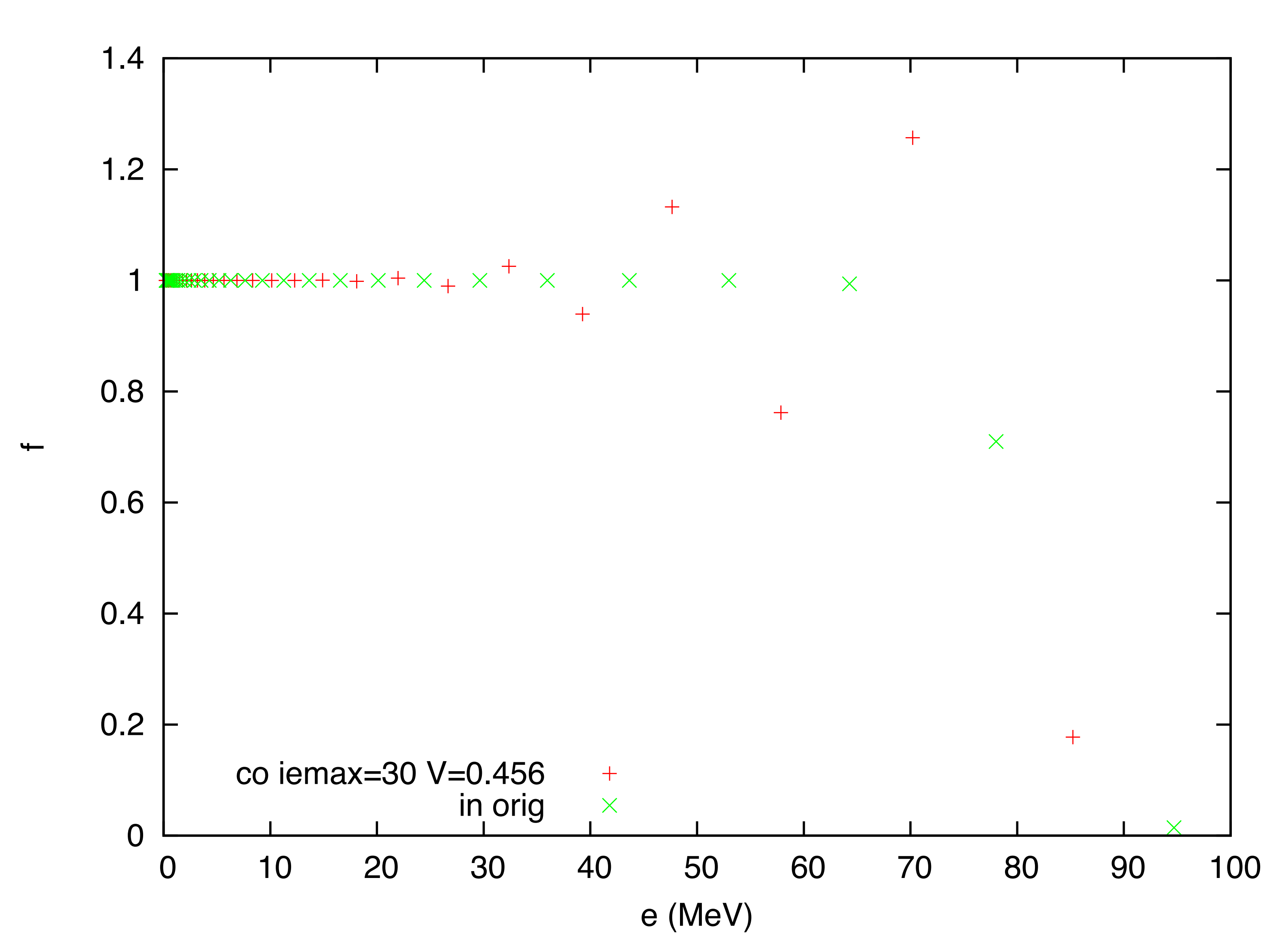
$= u^{\beta}$

∨ Drawbacks

Discrete energy shifts at the shock location (Very hard to handle it)

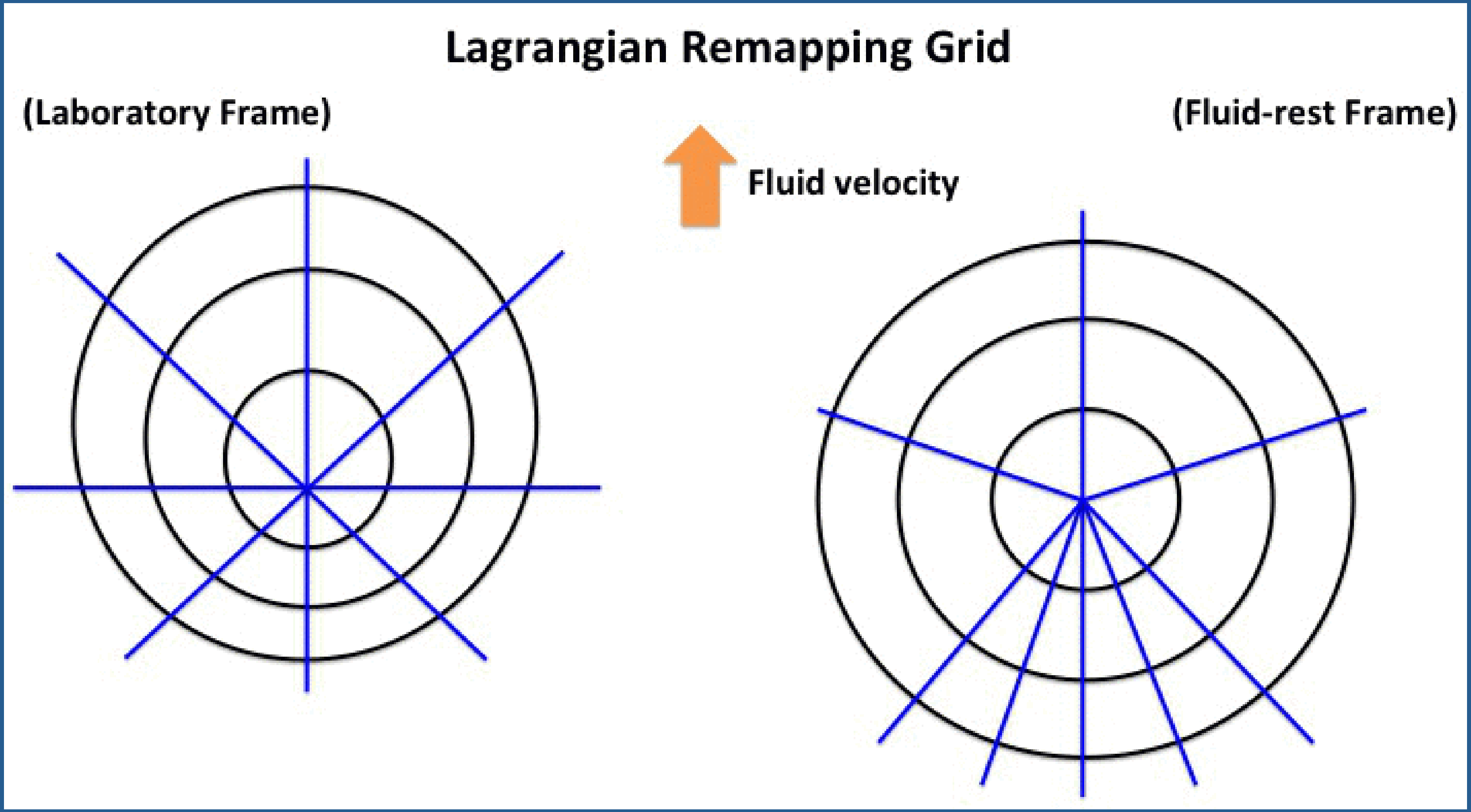


Struggling with numerical instabilities for 3 years ….

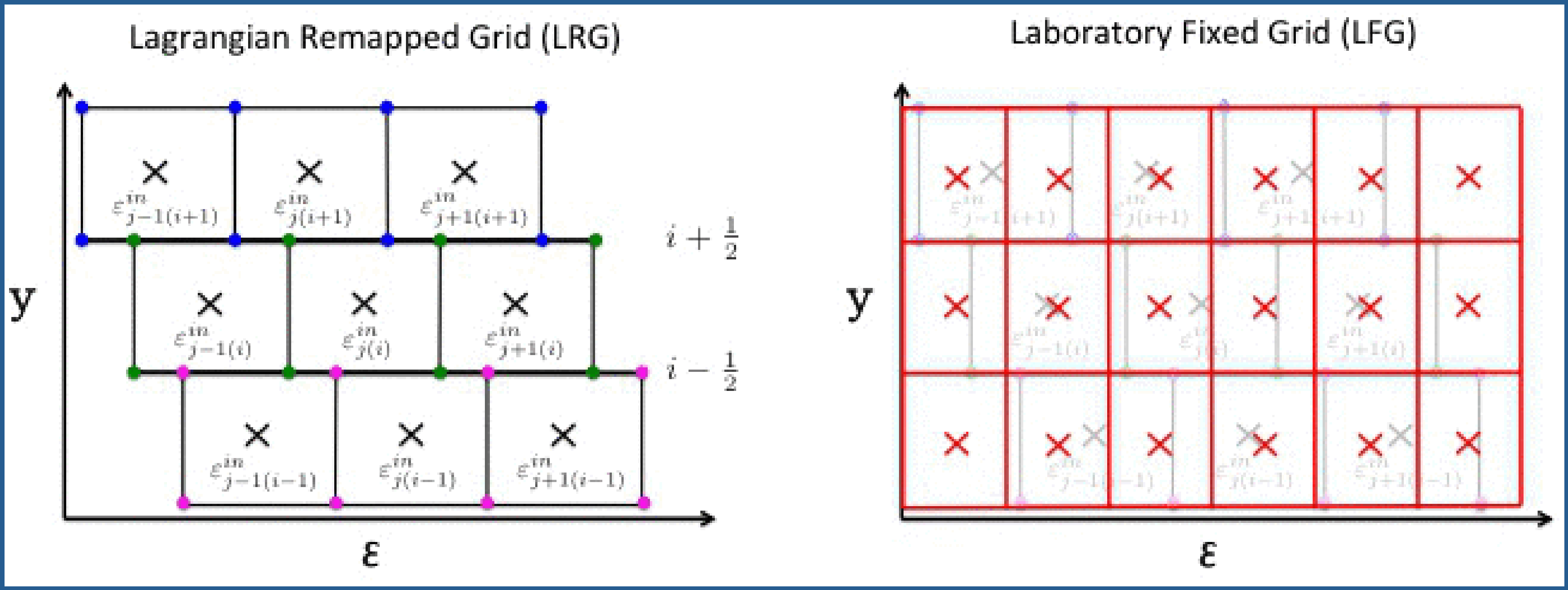


Break through !!
(Lagrangian remapping and Laboratory fixed grids)

We solve the Boltzmann equation by Eulerian approach, but energy grid is dynamical so as to keep isotropic in the fluid-rest frame.



Break through !!
(Lagrangian remapping and Laboratory fixed grids)

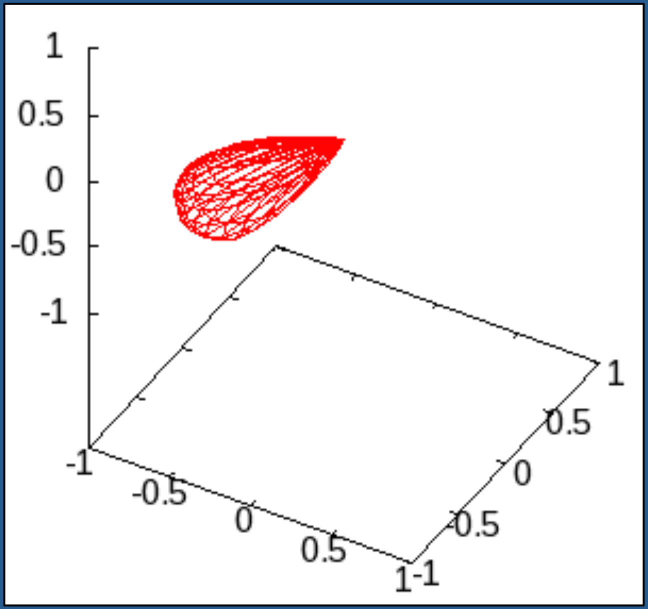


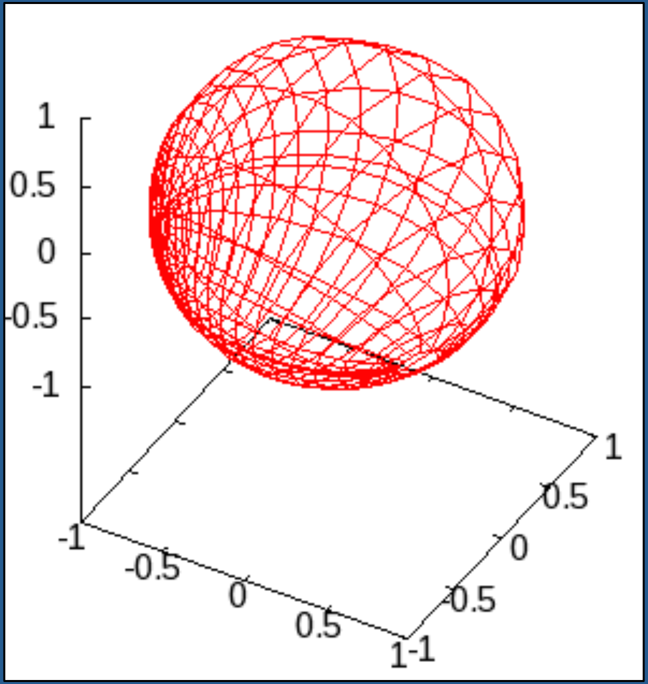
The advection terms are evaluated by using Laboratory Fixed Grids.

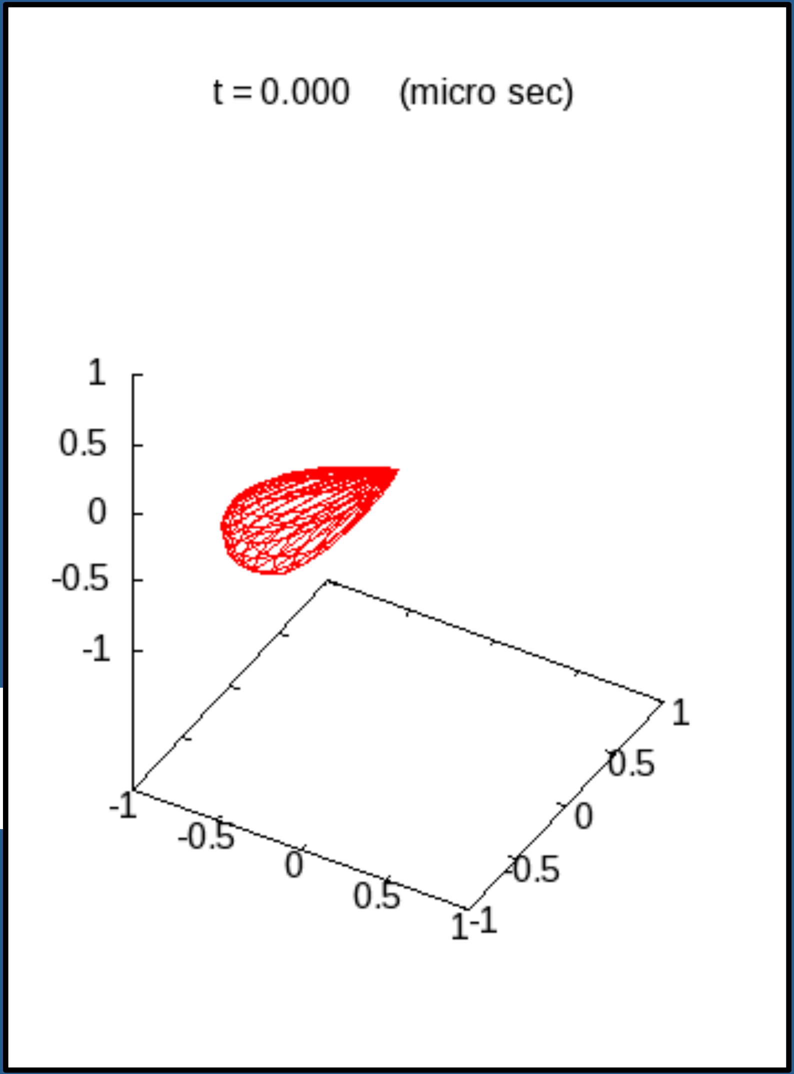
$$
\begin{aligned}
&\frac{\partial f}{\partial t} + \frac{\mu_\nu}{r^2}\frac{\partial}{\partial r}(r^2 f) + \frac{\sqrt{1-\mu_\nu^2}\,\cos\phi_\nu}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f) \\
&+ \frac{\sqrt{1-\mu_\nu^2}\,\sin\phi_\nu}{r\sin\theta}\frac{\partial f}{\partial\phi} + \frac{1}{r}\frac{\partial}{\partial\mu_\nu}[(1-\mu_\nu^2)f] \\
&- \frac{\sqrt{1-\mu_\nu^2}}{r}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\phi_\nu}(\sin\phi_\nu f) = \left(\frac{\delta f}{\delta t}\right)^{\mathrm{lb}}_{\mathrm{col}},
\end{aligned}
$$

Iso-energy scattering test

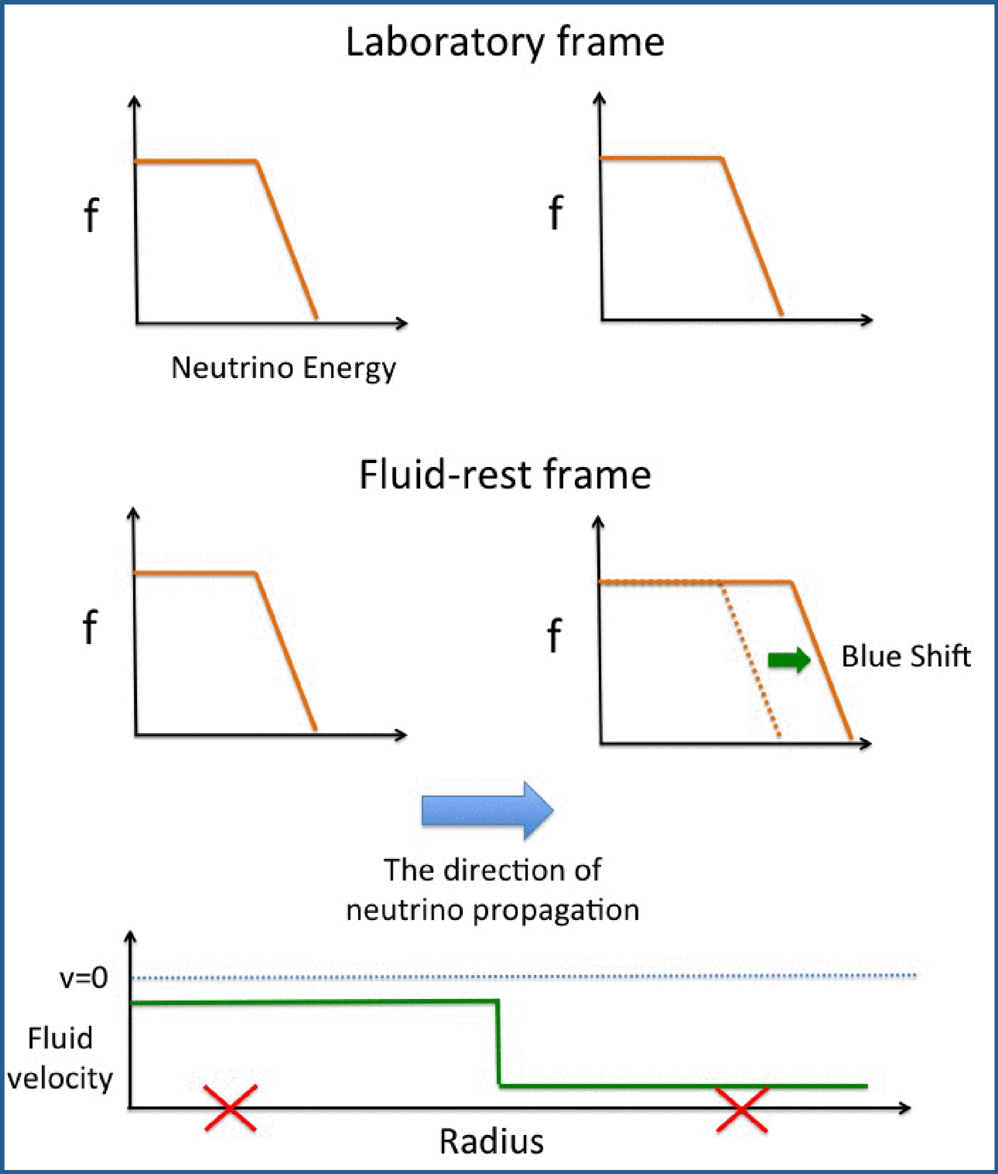
(Scattering with nucleons and nuclei)

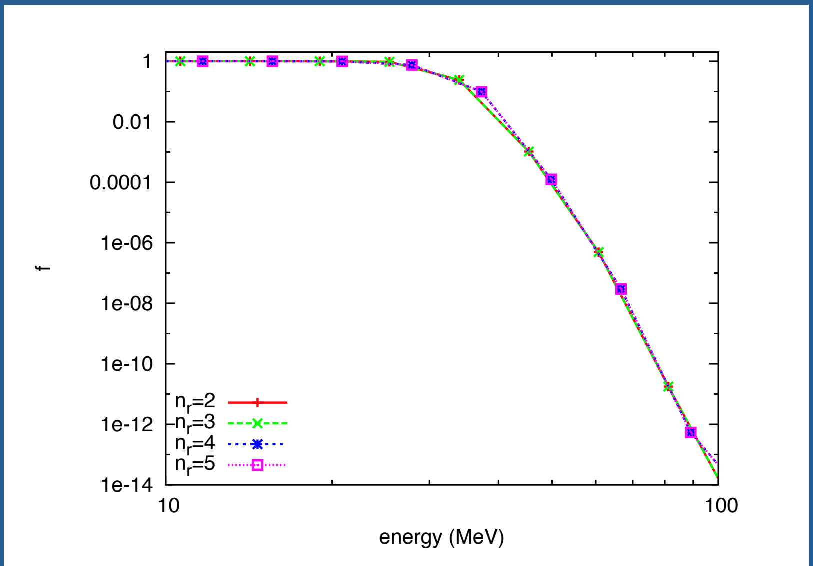


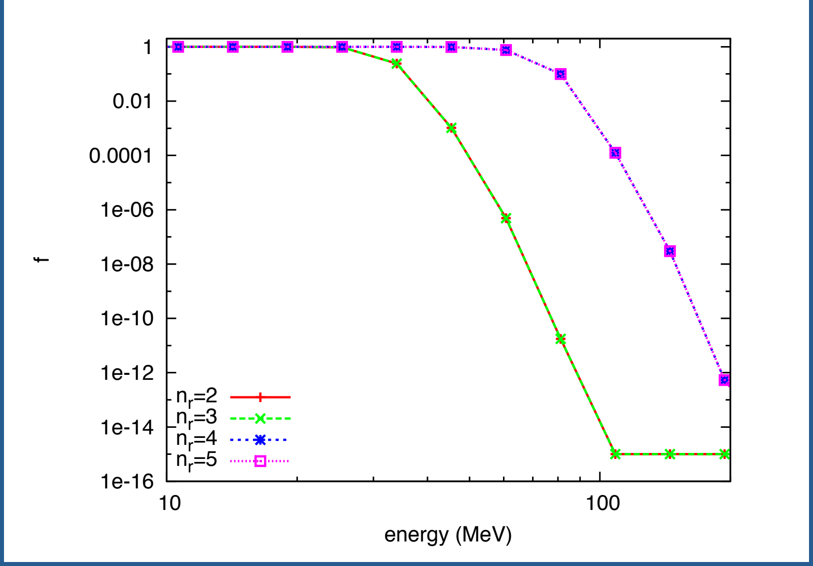




The advection term test (Shock region)

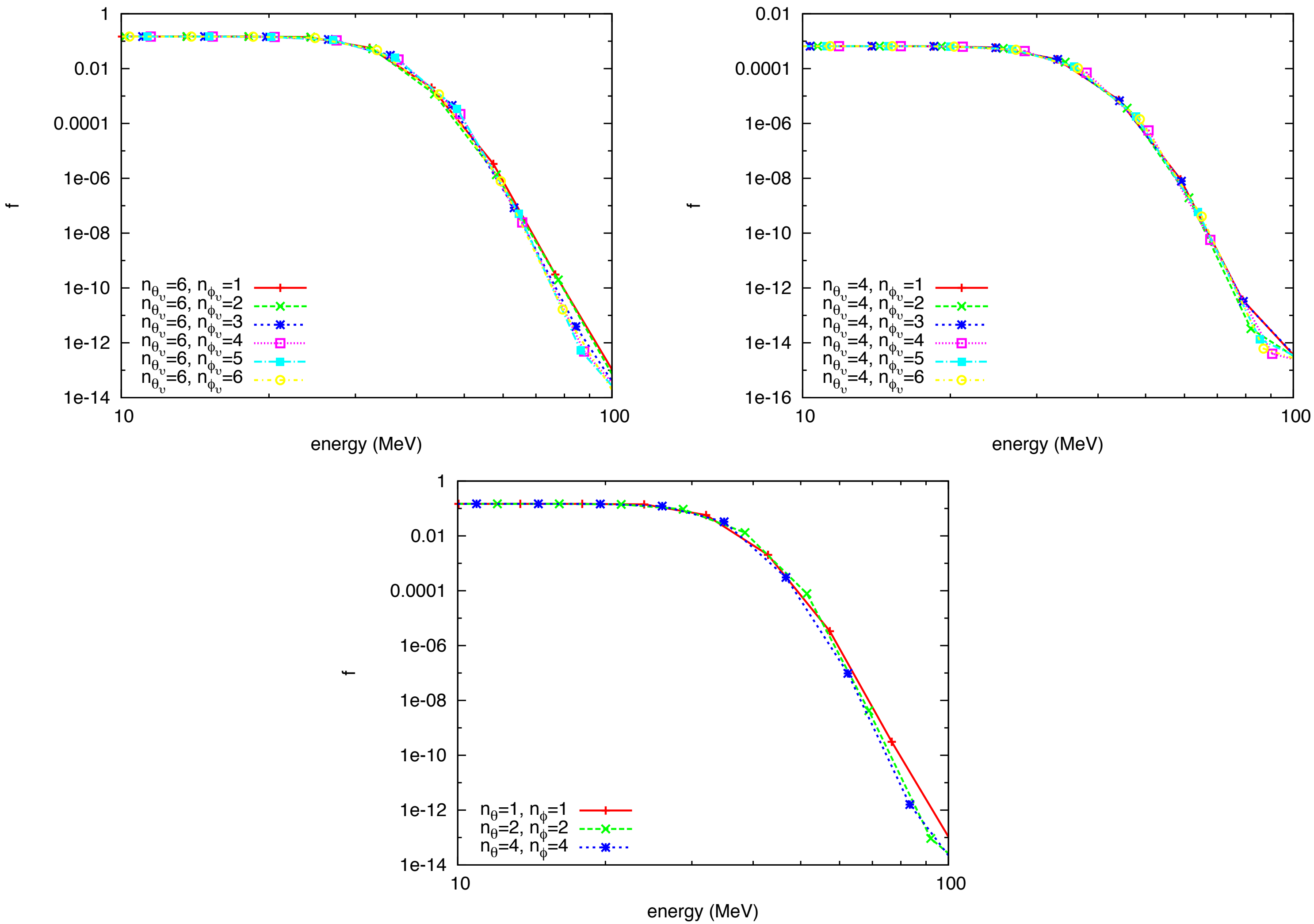






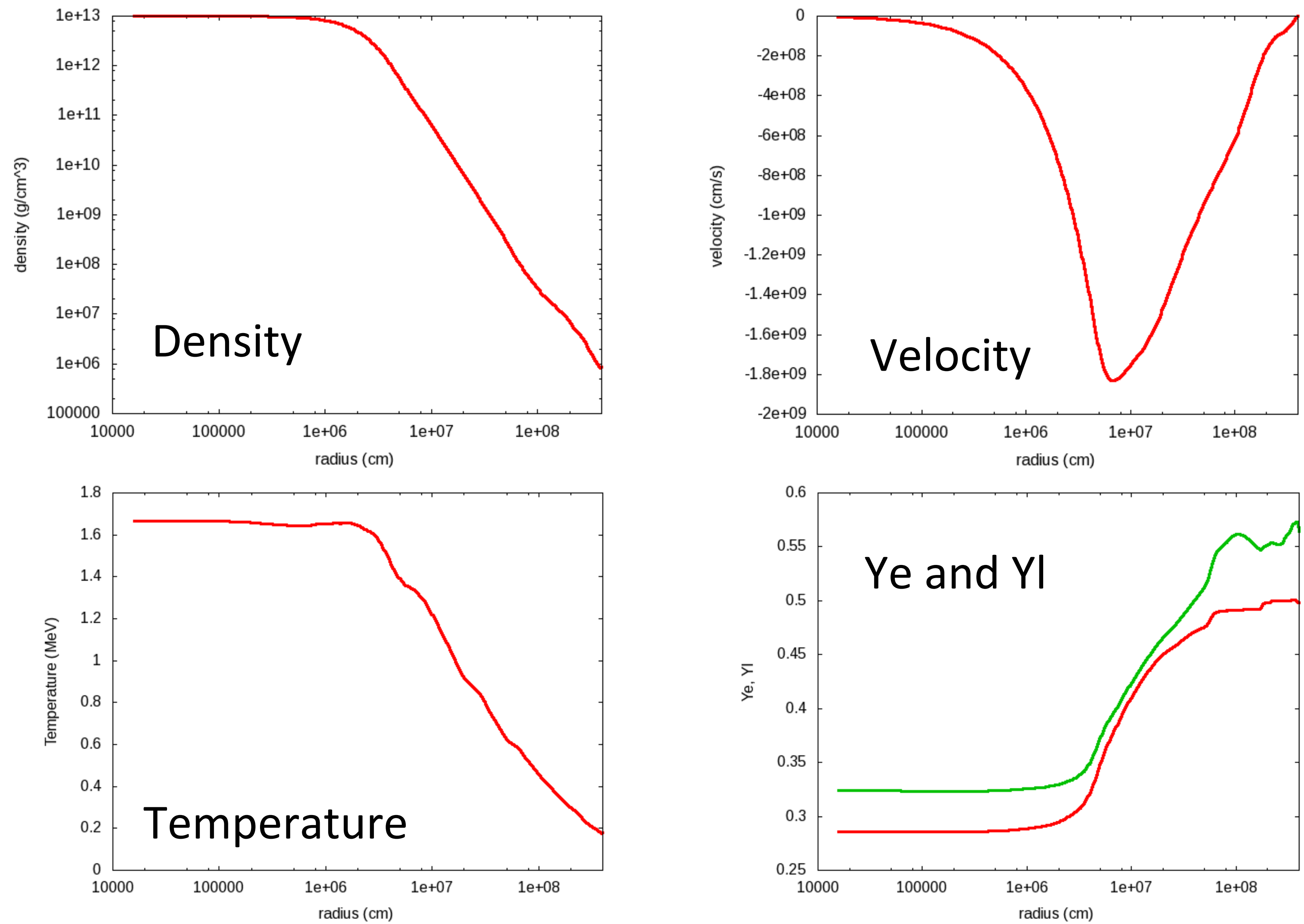
The advection term test

(3D Transfer in 1D optically thin medium)



Most recent numerical simulations

(1D 15M collapse with GSI electron capture data)



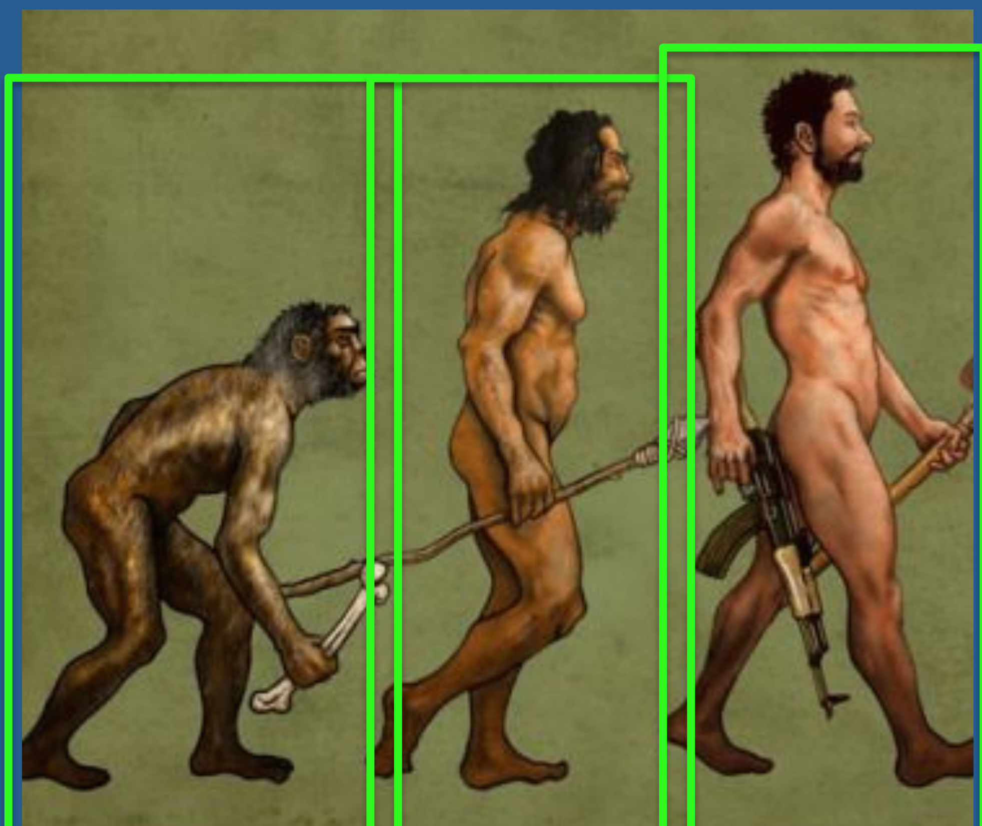
2D Post-bounce Boltzmann-Hydro Simulations

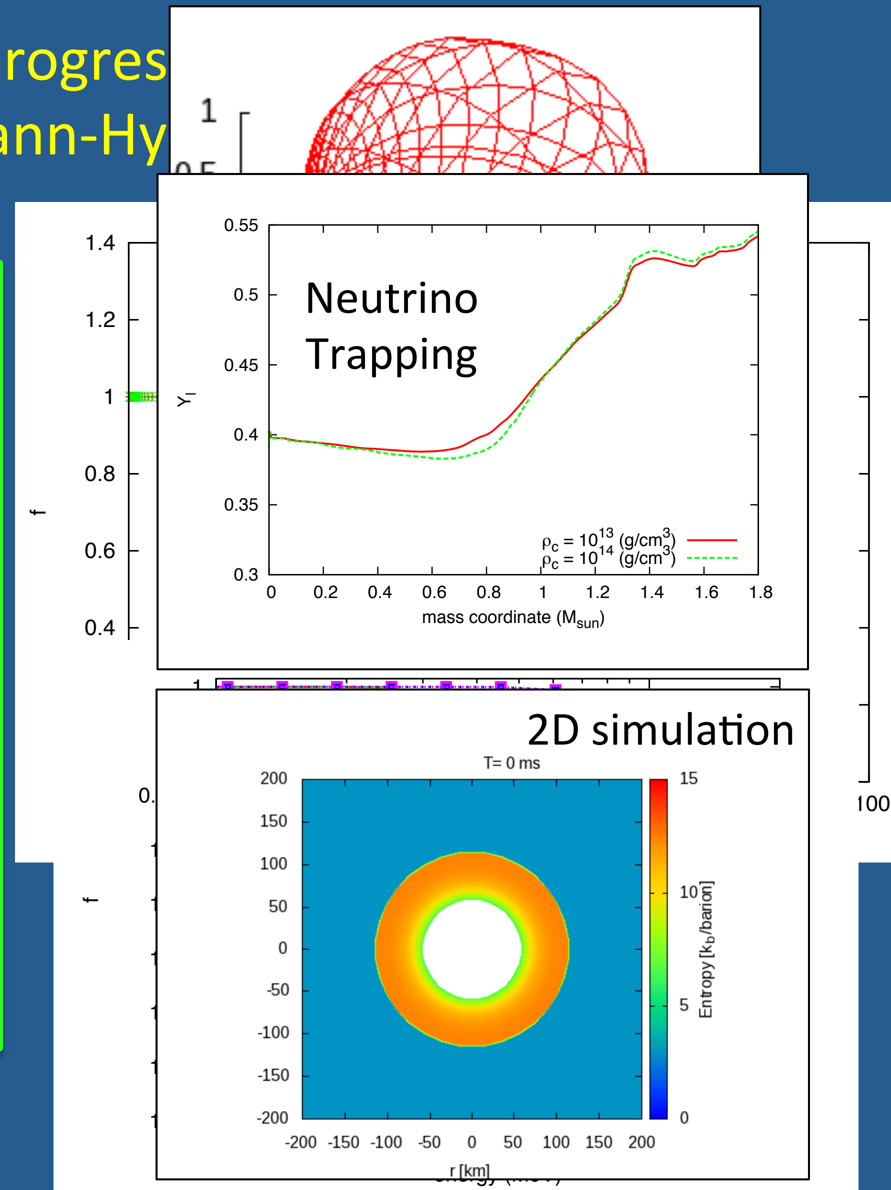
Iwakami et al. 2014 in prep

Summary and Conclusion

We achieve the steady progres

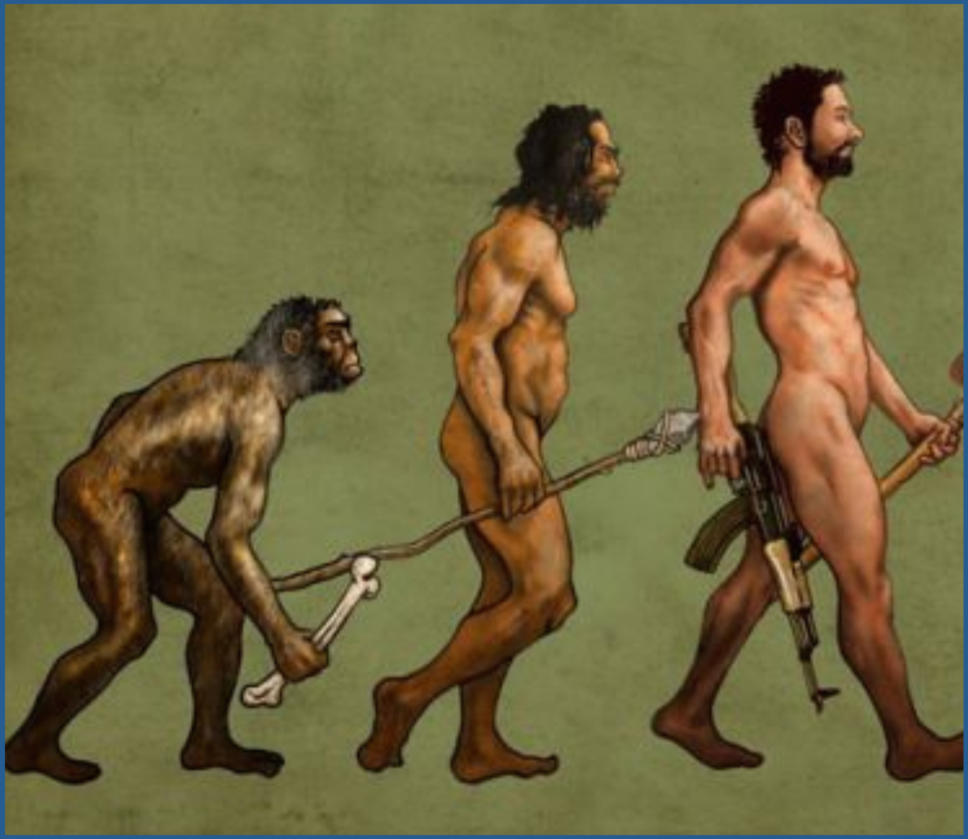
Multi-D SR Boltzmann-Hy





Summary and Conclusion

Never become a Lazybones !!!!!



Multi-D Simulations

Full GR

Improve Weak Interactions and Nuclear Physics

Jet propagation and its collimation in the ejecta of double neutron star merger

Hiroki Nagakura (YITP)

collaborators

K. Hotokezaka (Hebrew Univ.), Y. Sekiguchi (YITP), M. Shibata (YITP), K. Ioka (KEK)

Gamma-Ray Bursts in the Multi-Messenger Era, Paris, June 2014

NS-NS Merger Simulations (by Numerical Relativity)

Large Mass Ejection (～0.01Msun) around the pole

GR results are Qualitatively different from Newtonian Simulations !!!!

What is wrong in Newtonian Simulations?

Newtonian gravity is weaker than GR one

↓

Weak Shock Heating at the merger

↓

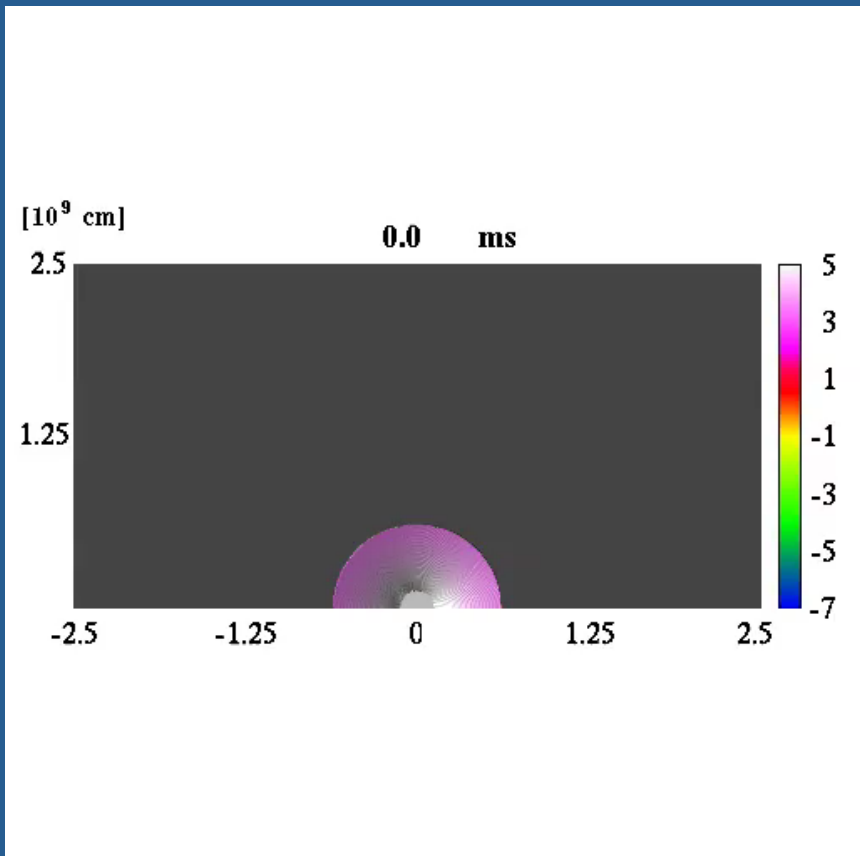
Little mass ejection around the pole

Animation by Hotokezaka

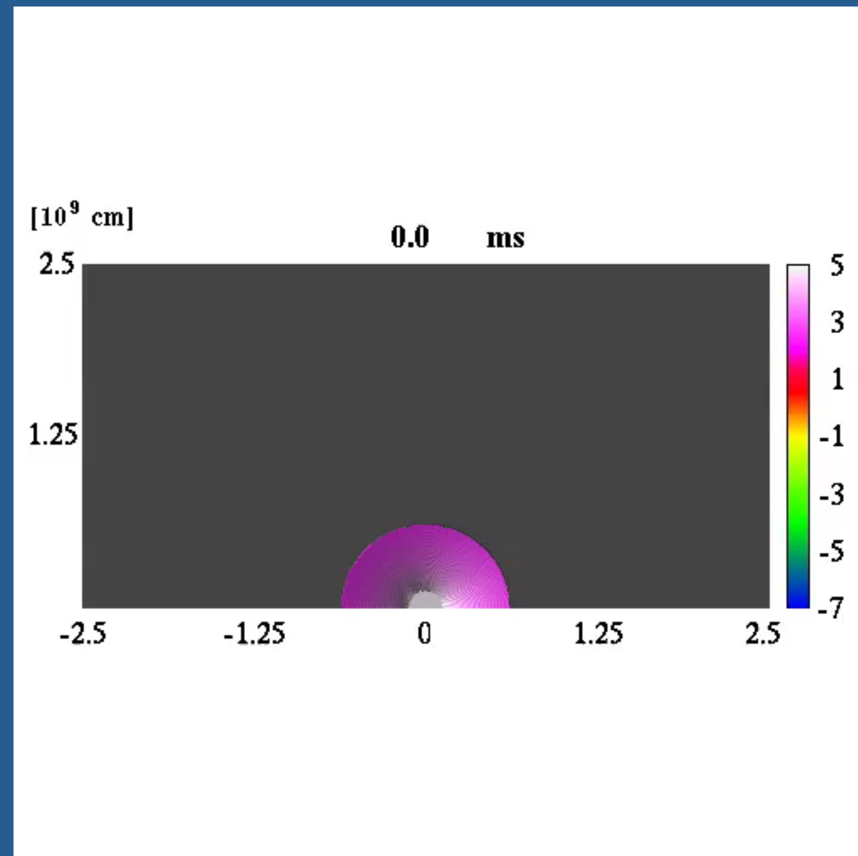
Relativistic Jet Simulations in the post merger phase

Nagakura et al. 2014 ApJL

M = 0.01 Msun



M = 0.001 Msun



Other jet parameters: L=2.e50 erg/s (inspired by GRB 130603B), Θ=15°

Jet injection timing: 50ms after the merger