Development of multidimensional relativistic radiative transfer codes

Nozomu Tominaga (Konan Univ./Kavli IPMU)

Sanshiro Shibata (Konan Univ.)
Sergei I. Blinnikov (ITEP)

KONAN UNIVERSITY

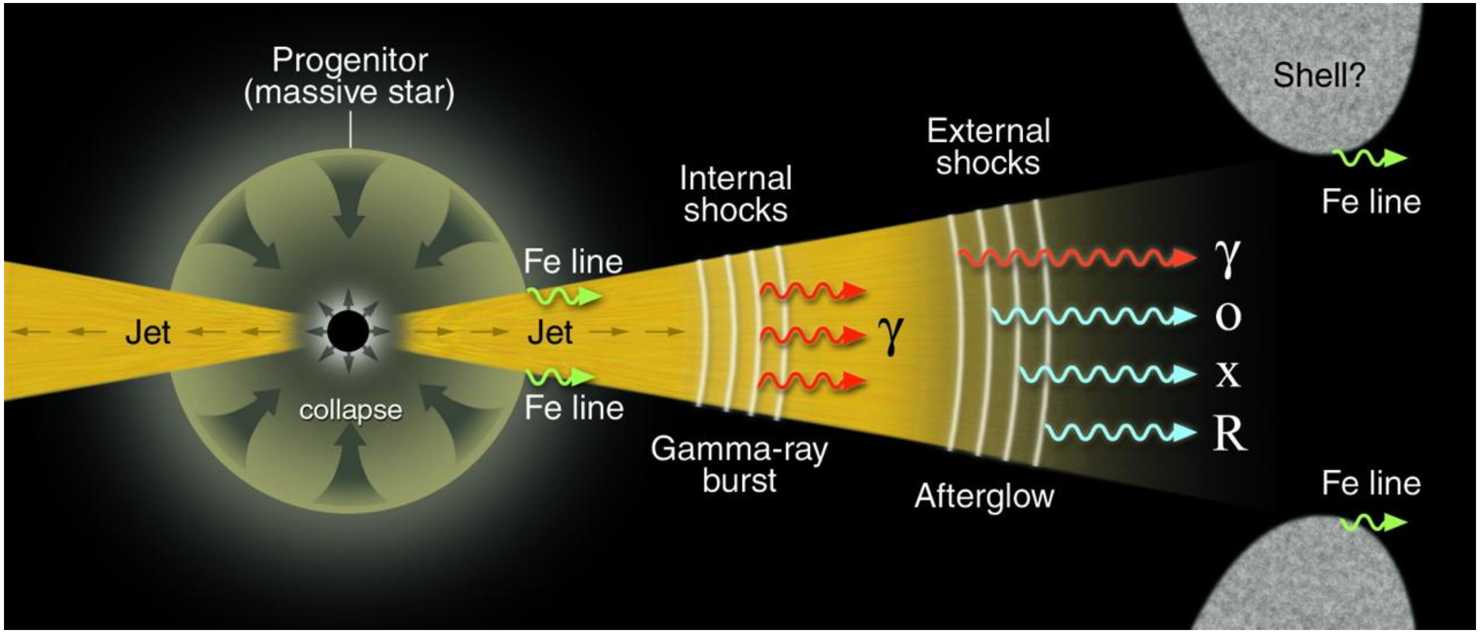
8th Jul 2014
RIKEN IPMU RESCEU Joint Meeting

Outline

* Radiation in gamma-ray bursts
* Multidimensional relativistic RT cal.
  * Monte Carlo method (Shibata & NT in prep.)
    * Photon production site
    * GRB spectrum
  * Spherical Harmonic Discrete Ordinate Method (SHDOM) (NT, Shibata, Blinnikov in prep.)
    * Implementation
    * Test problems
* Summary

Gamma-ray bursts (GRBs)

- Gamma-ray emission from relativistic jets



- The emission mechanism is under debate.
- We need to study it with quantitative comparisons between obs. and theory.

Previous numerical studies

* Relativistic Monte Carlo simulation (on steady flow)
  E.g., Giannois06; Pe'er08; Beloborodov10; Ito+13; Ruffini+13
* Relativistic hydrodynamics + a superposition of black body radiation from a surface (e.g., τ_{sc}=1)
  E.g., Lazzati+11; Mizuta+11; Nagakura+11
* Spherical relativistic radiation hydrodynamics
  Tolstov+13 (rel. rad. transfer: Beloborodov11)

However, GRBs involve **relativistic jets with E_r~E_m**.
Therefore, a multid. rela. rad. hyd. cal. is necessary.
Since even a **multid. rela. rad. trans. cal.** is not available, we develop it as the 1st step toward GRB rad. cal..

Multidimensional multigroup radiative transfer equation

- 6-dimensional Boltzmann equation

$$\frac{1}{c}\frac{\partial I\left(t,s,\nu,\Omega\right)}{\partial t}+\boldsymbol{n}\cdot\nabla I\left(t,s,\nu,\Omega\right)$$
$$=\eta\left(t,s,\nu,\Omega\right)-\sigma_{\mathrm{a}}\left(t,s,\nu,\Omega\right)I\left(t,s,\nu,\Omega\right)$$
$$+\int_0^\infty d\nu'\int d\Omega'\left[\frac{\nu}{\nu'}\sigma_{\mathrm{s}}\left(t,s,\nu'\to\nu,\Omega'\to\Omega\right)I\left(t,s,\nu',\Omega'\right)\right.$$
$$\left.-\sigma_{\mathrm{s}}\left(t,s,\nu\to\nu',\Omega\to\Omega'\right)I\left(t,s,\nu,\Omega\right)\right]$$

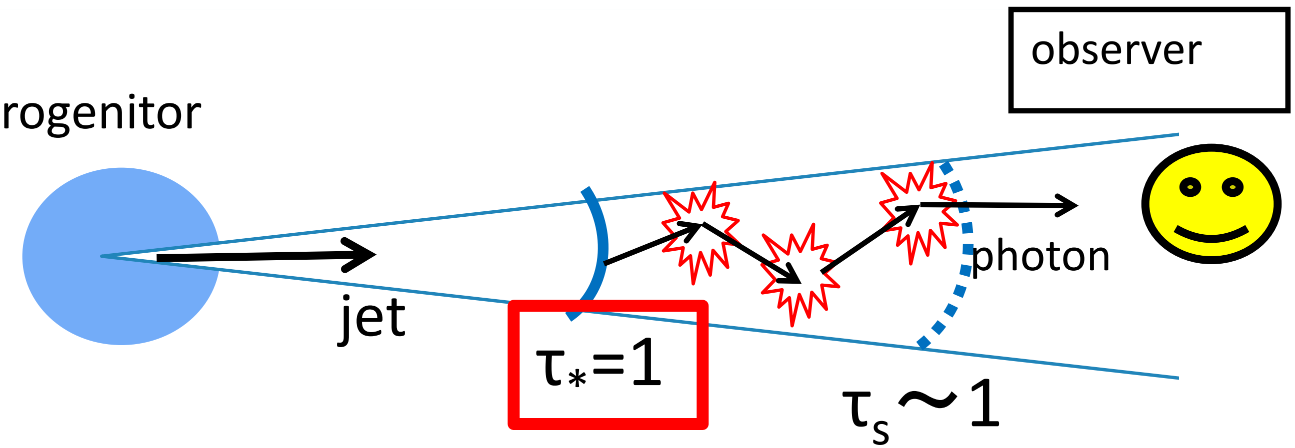
Pomraning73

Our works

- We are developing two codes.
- Monte Carlo method (Shibata & NT in prep.)
  - Pros: easy implementation and calculation
  - Cons: numerical noise and may need large number of photons for time evolution
  - Already applied for spectra synthesis of GRBs
- Spherical Harmonic Discrete Ordinate Method (SHDOM) (NT, Shibata, & Blinnikov in prep.)
  - Pros: solving the RTE for the intensity
  - Cons: needs large memory
  - Under development and checked with test problems

Monte Carlo Method

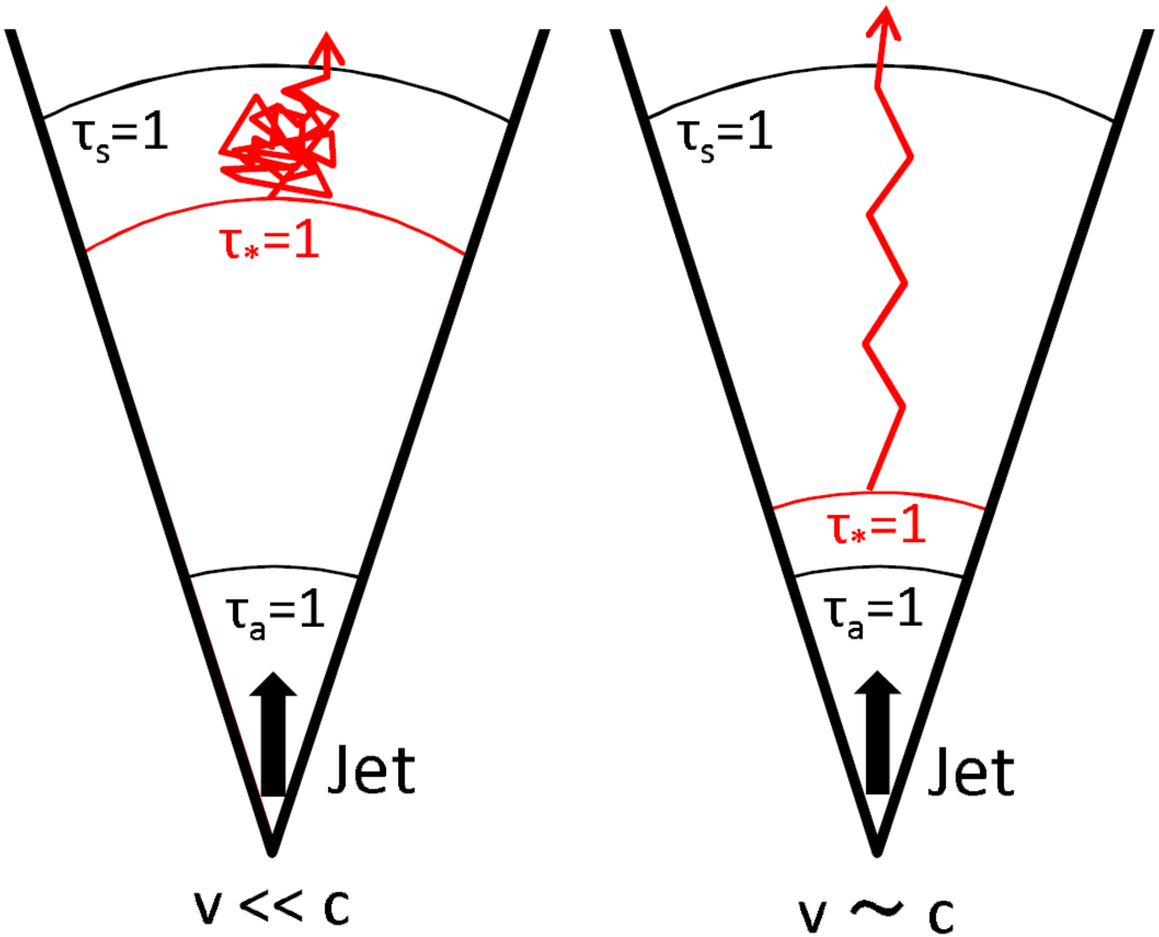
Monte Carlo method



✓ Numerical code

- Monte Carlo method
- Calculate Lorentz transformation and Compton scattering
- Photons are injected at the photon production site.

Photon production site



- Photons are produced deep inside in the relativistic flow, compared with the non-rel. flow.
- The site is located at much deeper than the photosphere for scattering.

Shibata, NT, Tanaka 2014

Effective optical depth

- The effective optical depth τ_*

For the static medium (Rybicki & Lightman 79)

$$\tau_*^{\rm NR} \sim \sqrt{\tau_{\rm a}(\tau_{\rm a}+\tau_{\rm s})}$$

For the relativistic medium

$$\tau_*^{\rm R} = \left\{ \frac{\Gamma^2}{3}(\beta^2+3) + (\Gamma\beta)^2 \frac{\tau_{\rm s}}{\tau_{\rm a}} \right\}^{-1/2} \frac{\sqrt{\tau_{\rm a}(\tau_{\rm a}+\tau_{\rm s})}}{\Gamma(1-\beta\cos\theta_{\rm v})}$$

$$\tau_{\rm a} = \Gamma(1-\beta\cos\theta_{\rm v})\alpha' L \quad , \quad \tau_{\rm s} = \Gamma(1-\beta\cos\theta_{\rm v})\sigma' L$$

In the non-relativistic limit, $\tau_*^{\rm R} \rightarrow \tau_*^{\rm NR}$

In the relativistic limit, $\tau_*^{\rm R} \rightarrow 2\,\tau_{\rm a}$ for $\Theta=0$

The photon production site is located at $\tau_*^{\rm R}=1$.

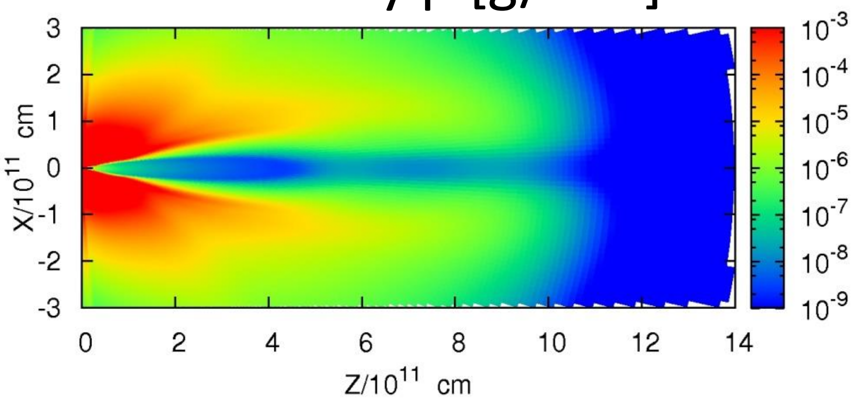
Shibata, NT, Tanaka 2014

Hydrodynamical simulation

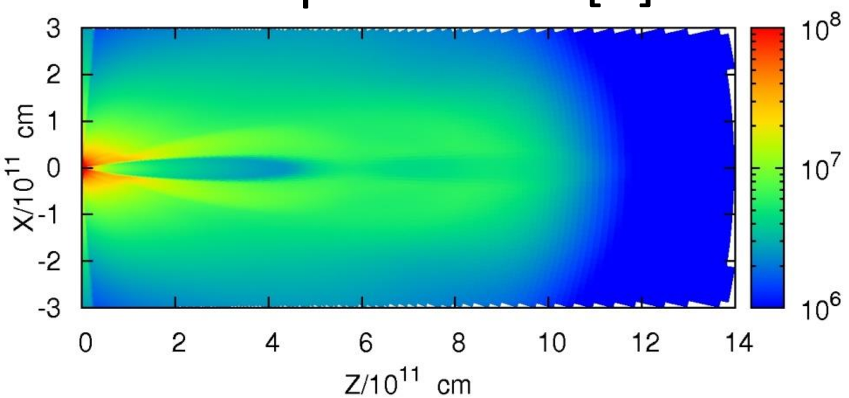
- 2D relativistic hydrodynamics cal. (NT 09)
- Relativistic jets w/ Γ=5 and f_{th}=0.9925 are injected at R=10^9cm.

A snapshot at t=40s is adopted for the MC calculation.

Density ρ [g/cm³]

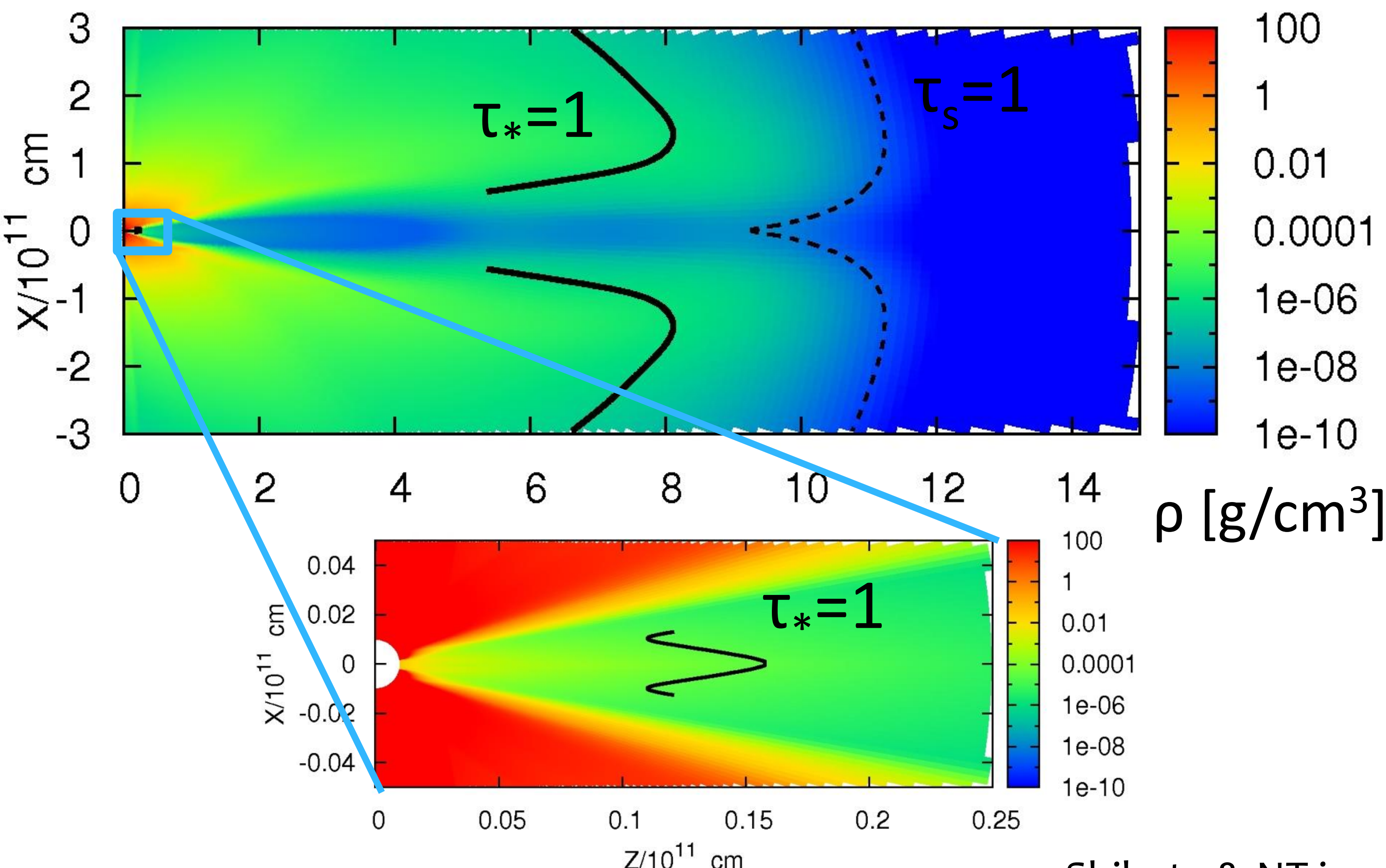


Temperature T [K]



Shibata & NT in prep.

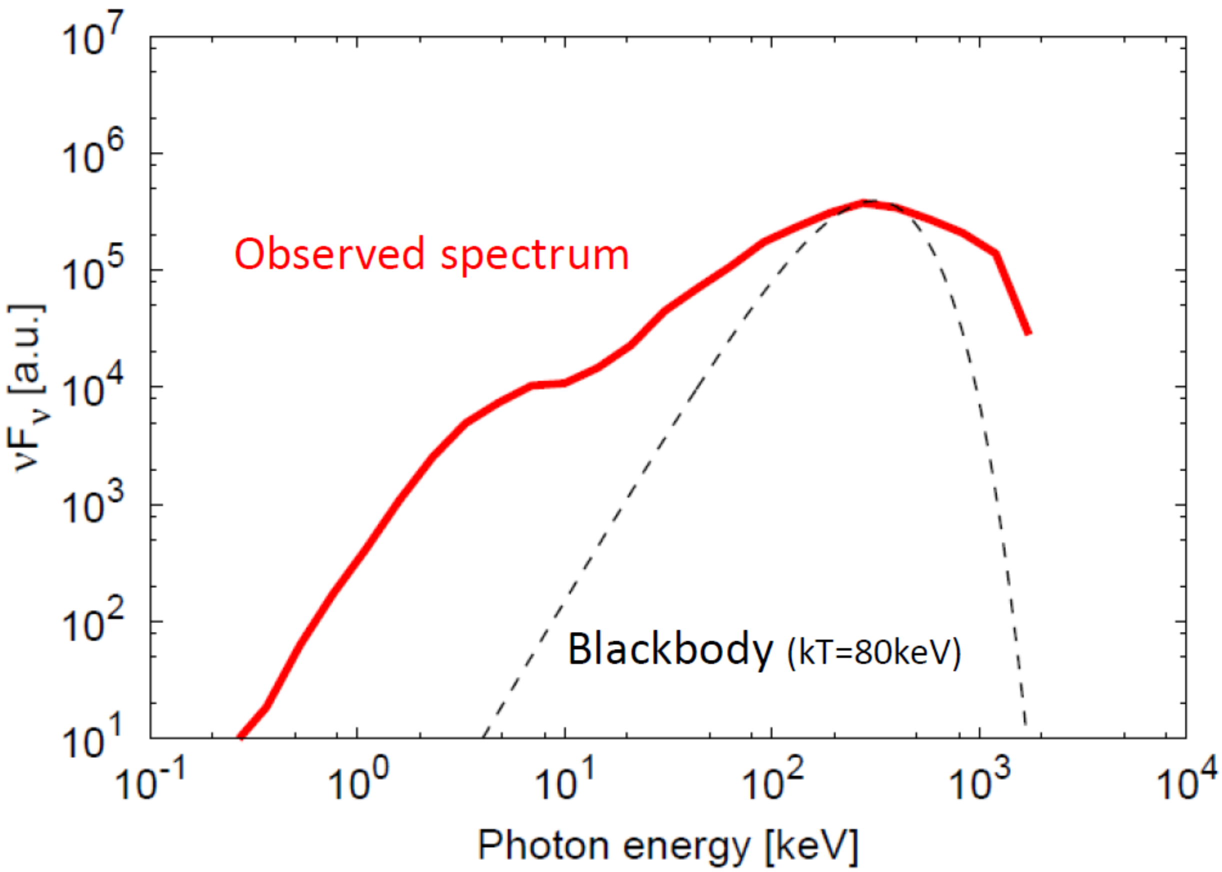
Photon production site



Shibata & NT in prep.

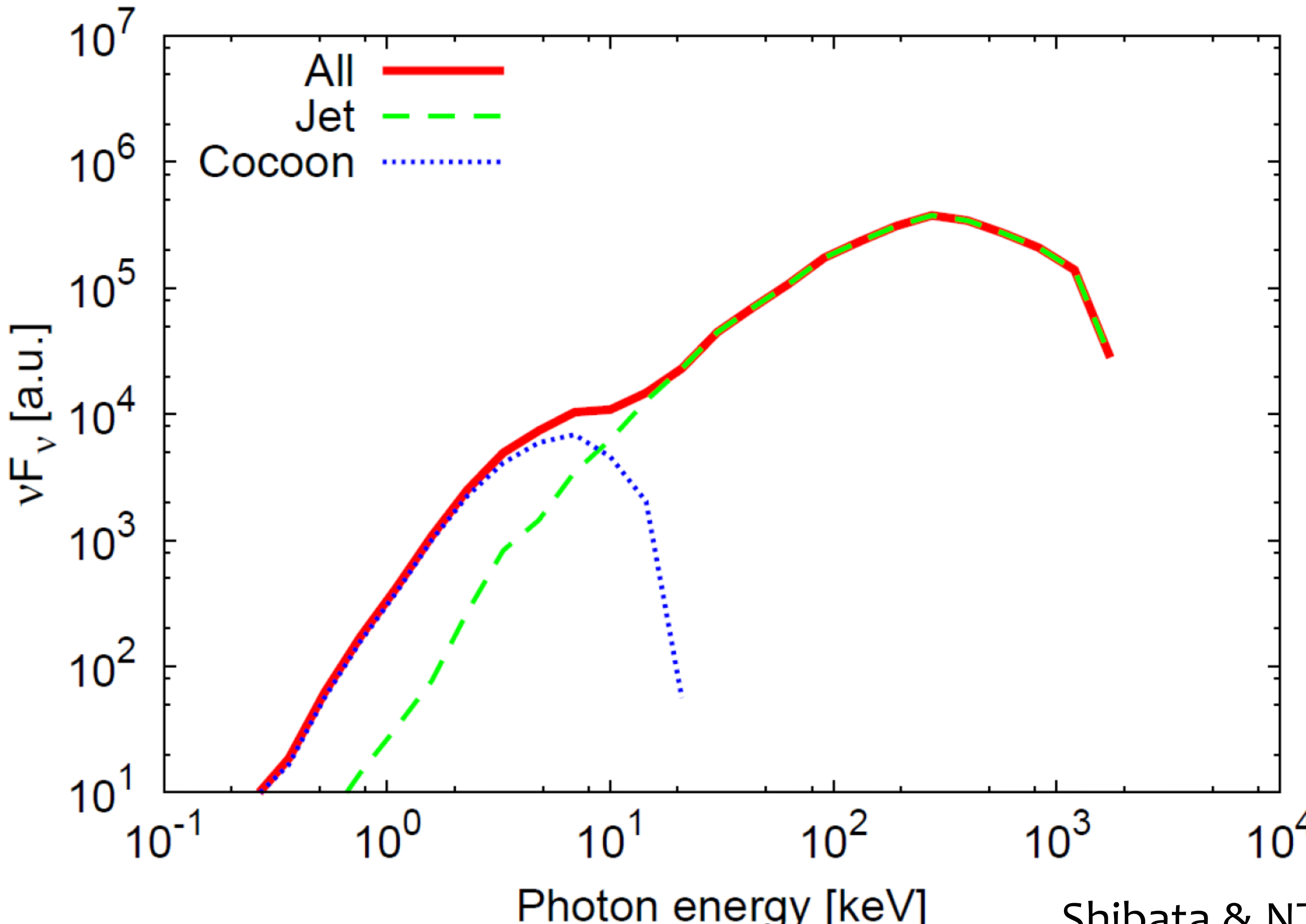
Spectrum

- E_{peak}〜300keV
- NOT a blackbody
  - wider than B.B.
- A bump at low energies



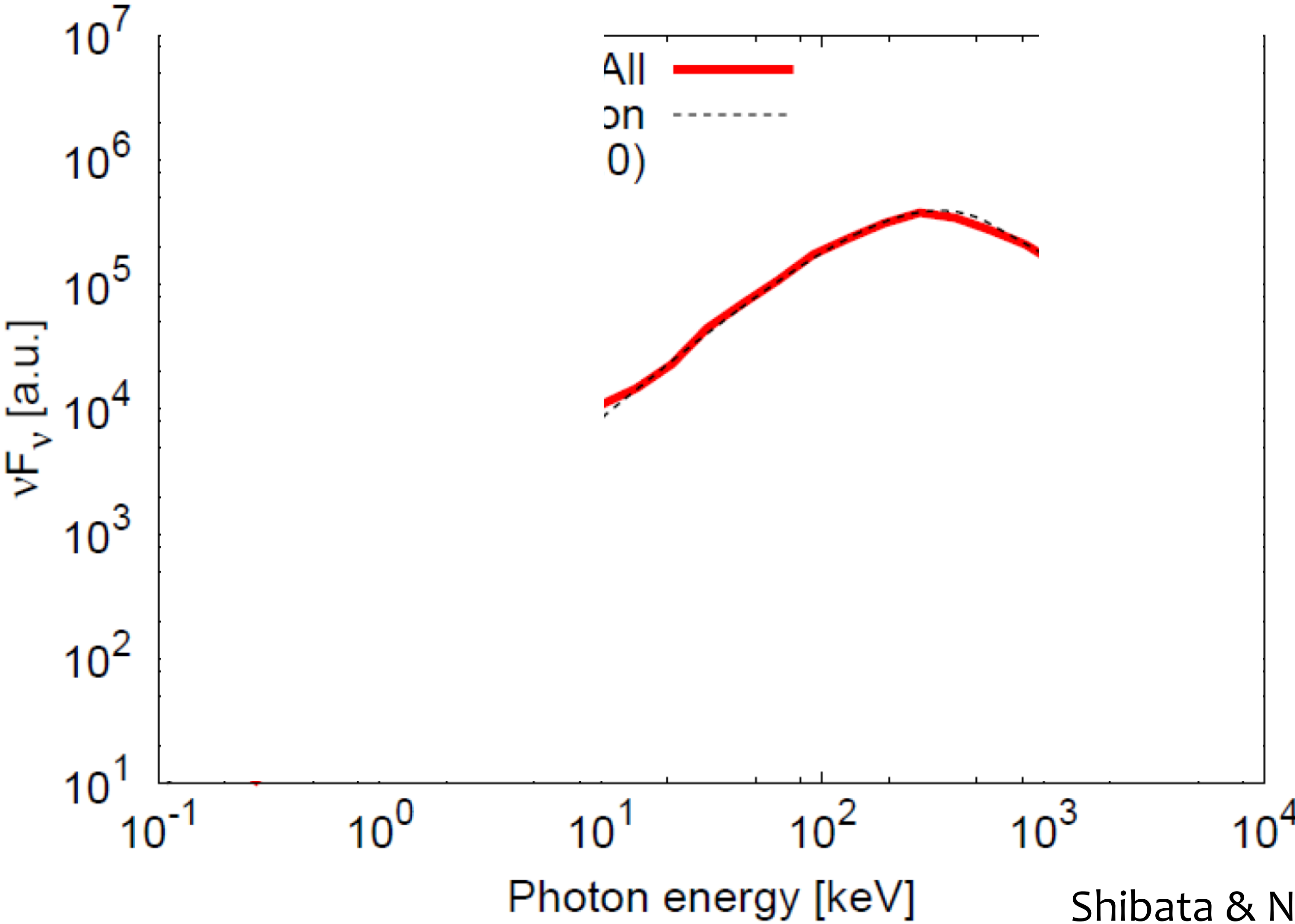
Shibata & NT in prep.

Origin of the bump



Shibata & NT in prep.

Comparison with the Band func.



Shibata & NT in prep.

Spherical Harmonic Discrete Ordinate Method

Multidimensional multigroup radiative transfer equation

* 6-dimensional Boltzmann equation

$$\frac{1}{c}\frac{\partial I\left(t,s,\nu,\Omega\right)}{\partial t}+\boldsymbol{n}\cdot\nabla I\left(t,s,\nu,\Omega\right)$$
$$=\eta\left(t,s,\nu,\Omega\right)-\sigma_{\mathrm{a}}\left(t,s,\nu,\Omega\right)I\left(t,s,\nu,\Omega\right)$$
$$+\int_0^\infty d\nu'\int d\Omega'\left[\frac{\nu}{\nu'}\sigma_{\mathrm{s}}\left(t,s,\nu'\rightarrow\nu,\Omega'\rightarrow\Omega\right)I\left(t,s,\nu',\Omega'\right)\right.$$
$$\left.-\sigma_{\mathrm{s}}\left(t,s,\nu\rightarrow\nu',\Omega\rightarrow\Omega'\right)I\left(t,s,\nu,\Omega\right)\right]$$

Pomraning73

* Hereafter, a spatially 2-dimensional equation (3-dimension for photon direction) is solved.

Spherical Harmonic Discrete Ordinate Method (SHDOM)

* Solve static monochromatic radiative transfer eq.

$$\boldsymbol{n} \cdot \nabla I\left(s, \Omega\right) = \eta_{\mathrm{all}}\left(s, \Omega\right) - \alpha\left(s, \Omega\right) I\left(s, \Omega\right)$$

Evans+98, Pincus&Evans09

* Ray tracing in a discrete ordinate space

$$I\left(s\right) = \exp\left[-\int_0^s \alpha\left(s'\right) ds'\right] I(0) + \int_0^s \exp\left[-\int_{s'}^s \alpha\left(s''\right) ds''\right] S\left(s'\right) \alpha\left(s'\right) ds'$$

* Deriving source function with spherical harmonics expansion

$$S\left(\Omega\right) = \sum_{lm} Y_{lm}\left(\Omega\right) S_{lm} \quad P\left(\cos\Theta\right) = \sum_{l=0}^{N_{\mathrm{L}}} \chi_l \mathscr{P}_l\left(\cos\Theta\right)$$

$$S_{lm} = \frac{\omega \chi_l}{2l+1} I_{lm} + T_{lm}$$

In order to apply to GRBs,

several revisions are required.

1. Time dependence
   * Light velocity is finite for the relativistic medium.
2. Lorentz transformation
   * Relativistic beaming
   * Energy variation
3. Compton scattering
   * Photon energy is as high as electron rest mass.

1. Time dependence

* Time-dependent radiative transfer eq.

$$\frac{1}{c}\frac{\partial I\left(s,\nu,\Omega\right)}{\partial t}+\boldsymbol{n}\cdot\nabla I\left(s,\nu,\Omega\right)=\eta_{\mathrm{all}}\left(s,\nu,\Omega\right)-\alpha\left(s,\nu,\Omega\right)I\left(s,\nu,\Omega\right)$$

* First, we discretize the time-derivative term,

$$\frac{\partial I(s,\nu,\Omega)}{\partial t}=\frac{I^{n+1}(s,\nu,\Omega)-I^{n}(s,\nu,\Omega)}{\Delta t}$$

$$\tilde{\alpha}=\alpha+\frac{1}{c\Delta t}\qquad\tilde{\eta}_{\mathrm{all}}=\eta_{\mathrm{all}}+\frac{I^{n}}{c\Delta t}$$

$$\boldsymbol{n}\cdot\nabla I^{n+1}=\tilde{\eta}_{\mathrm{all}}-\tilde{\alpha}I^{n+1}$$

Baron+09, Hiller&Dessart12, Jack+12

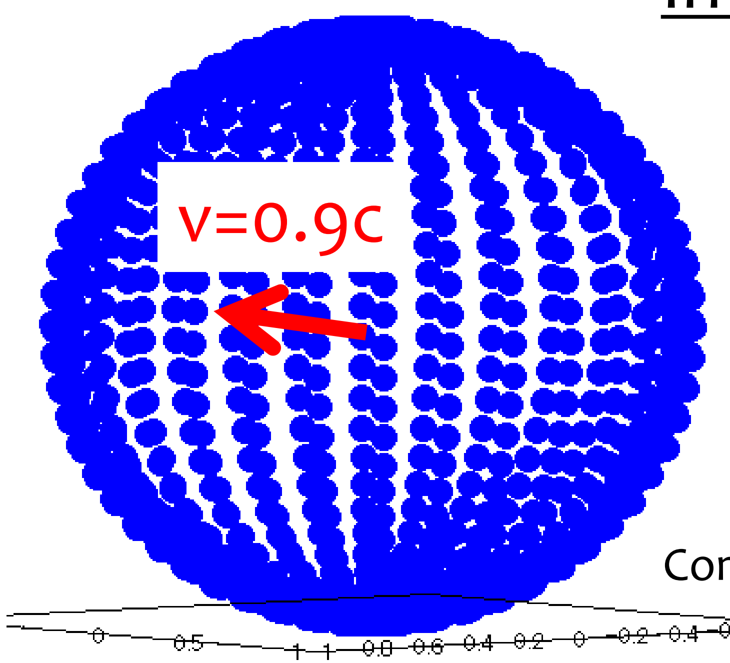
2. Lorentz transformation

* Photon energy
* Direction

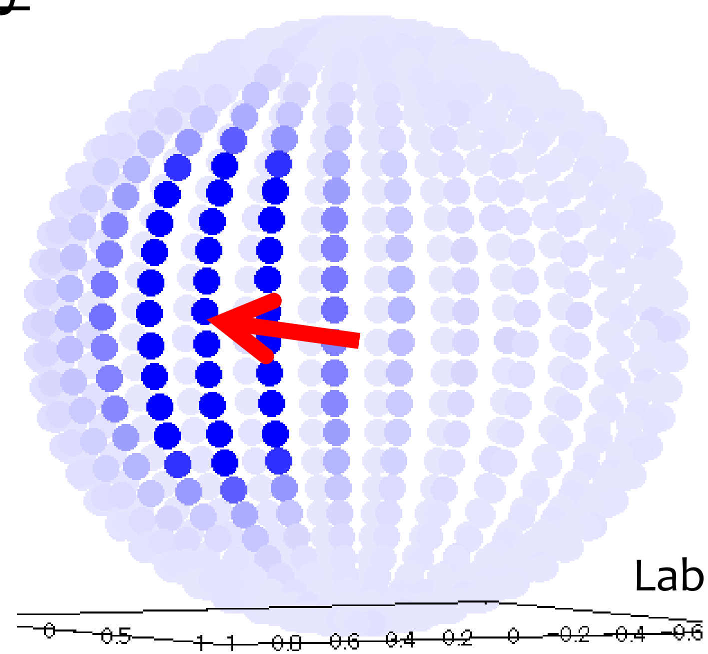
$$\nu_0 = \Gamma\nu\left(1 - \frac{\boldsymbol{n}\cdot\boldsymbol{v}}{c}\right) \qquad \boldsymbol{n}_0 = \frac{\nu}{\nu_0}\left\{\boldsymbol{n} - \Gamma\frac{\boldsymbol{v}}{c}\left[1 - \frac{\Gamma}{\Gamma+1}\frac{\boldsymbol{n}\cdot\boldsymbol{v}}{c}\right]\right\}$$

E.g., Mihalas & Weibel Miharas 84

Intensity



Comoving frame



Laboratory frame

3. Compton scattering

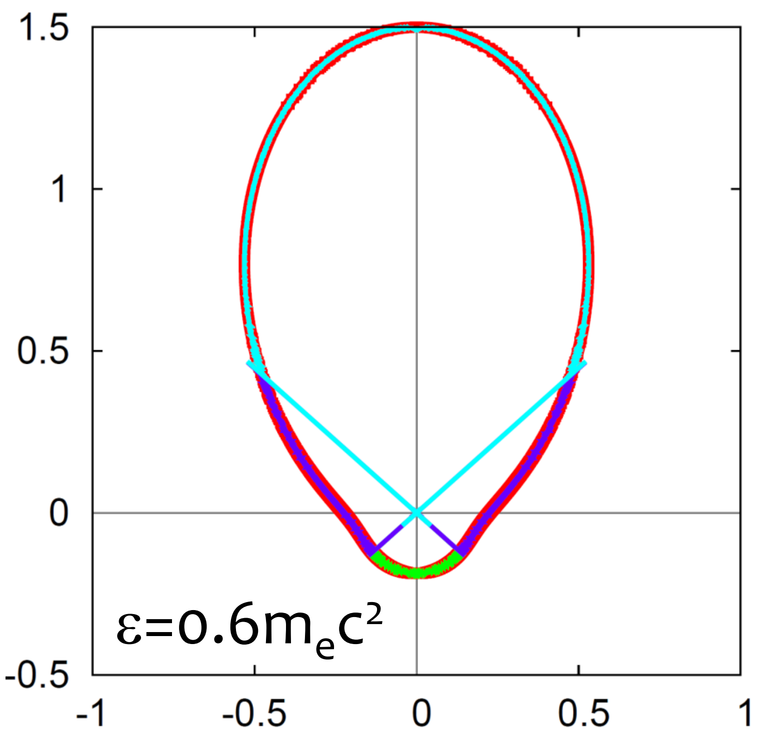
- Photon energy

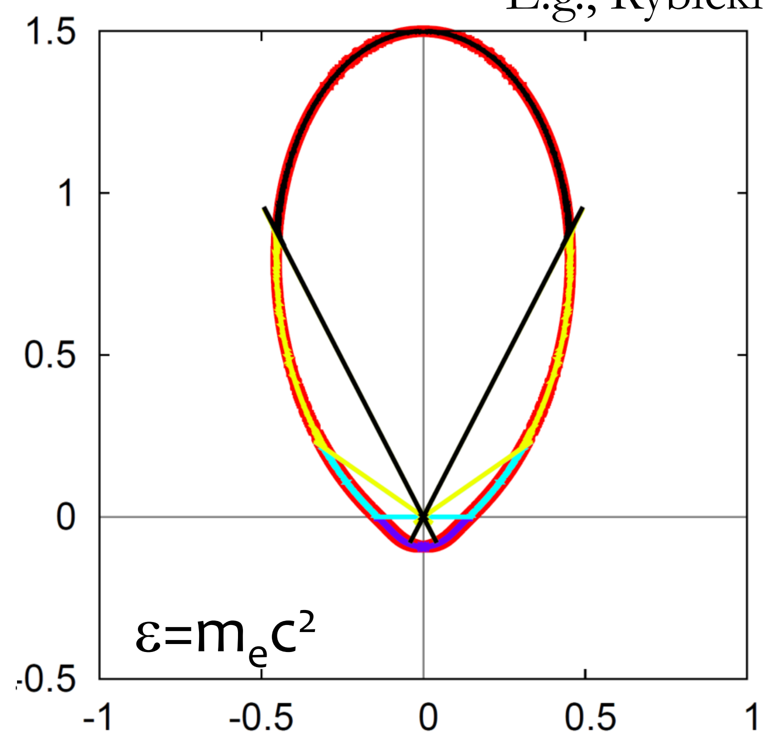
$$\varepsilon_1 = \frac{\varepsilon}{1 + \frac{\varepsilon}{mc^2}(1-\cos\theta)}$$

- Differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\frac{\varepsilon_1}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon_1} + \frac{\varepsilon_1}{\varepsilon} - \sin^2\theta\right)$$

E.g., Rybicki & Lightman 85





Intensity

$\varepsilon_1 = 0.2m_ec^2$

$0.4m_ec^2$

$0.6m_ec^2$

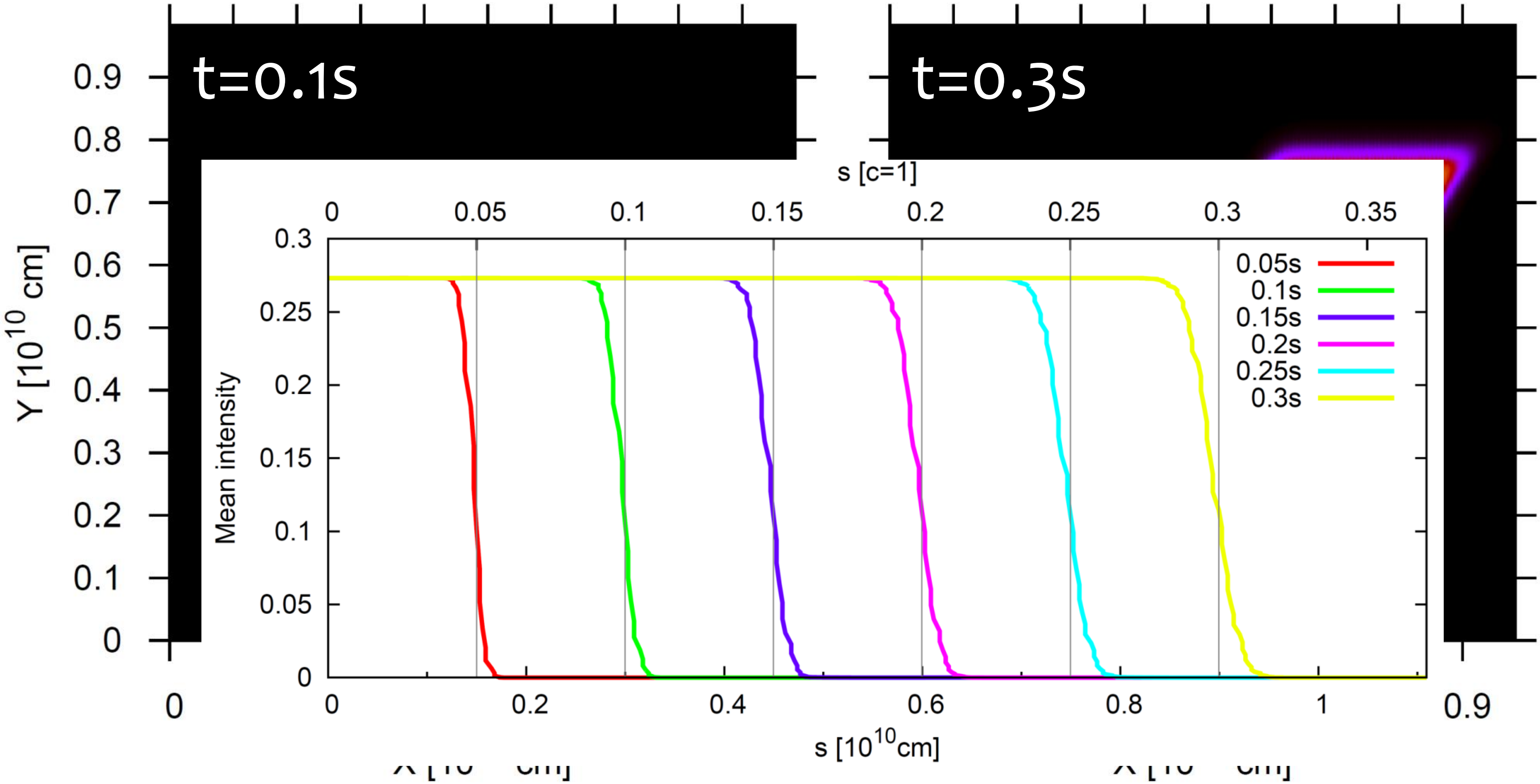
$0.8m_ec^2$

$1.0m_ec^2$

Test problems

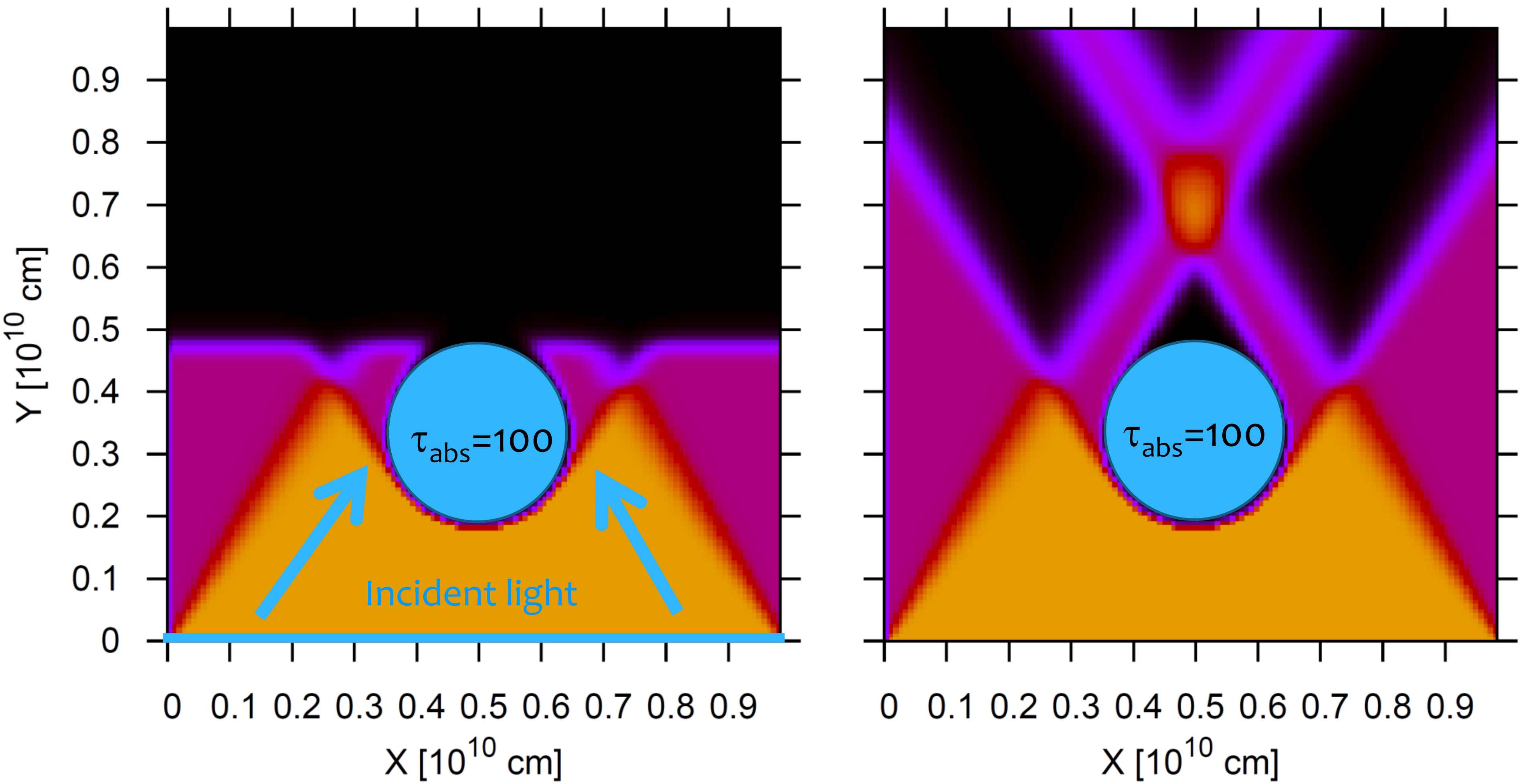
Beam test

- Optically thin medium: 512(x)x512(y)x4(θ)x8(φ)



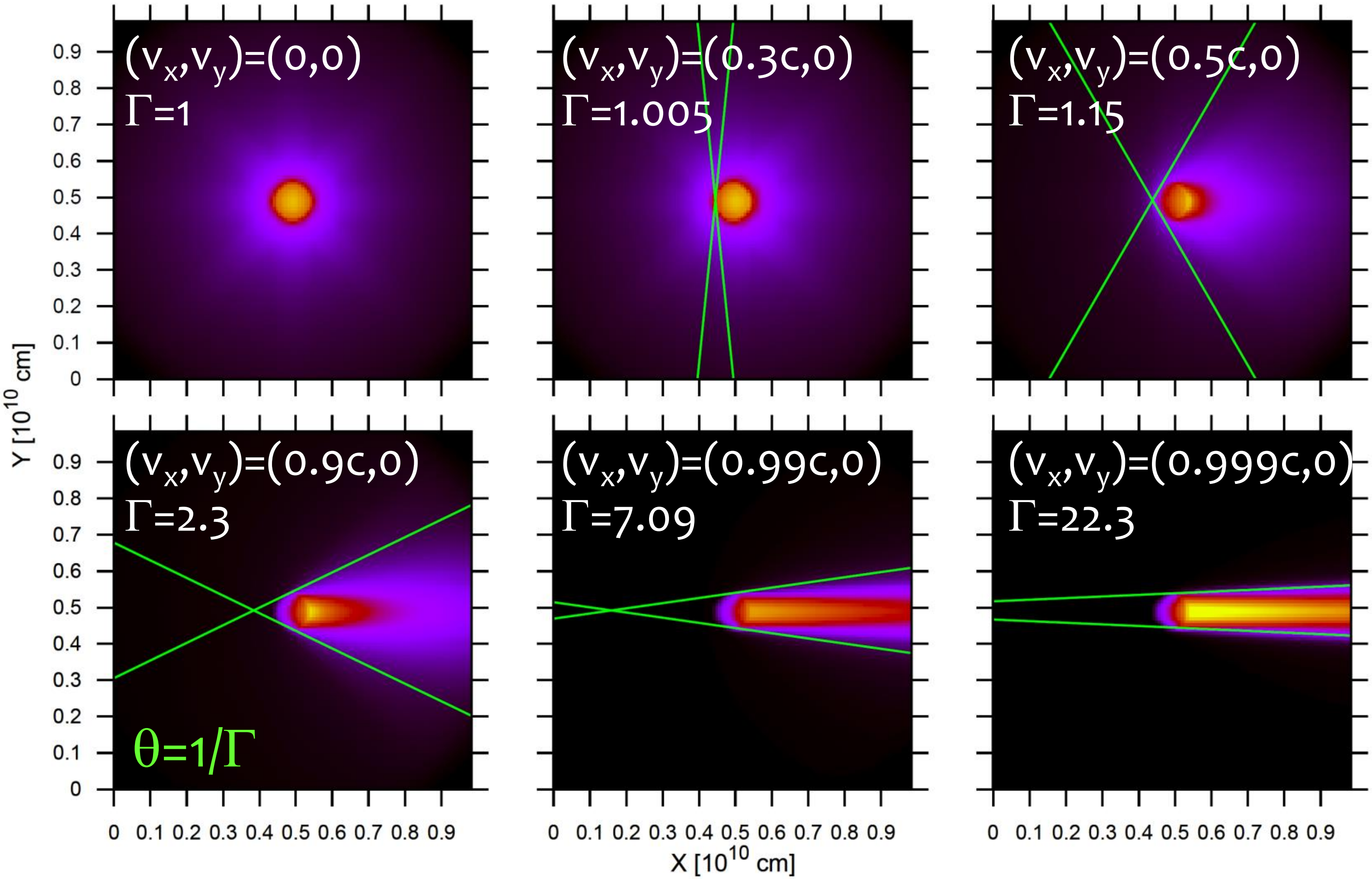
Two beam with shadow

- Optically thick cylinder:128(x)x128(y)x32(θ)x64(ϕ)



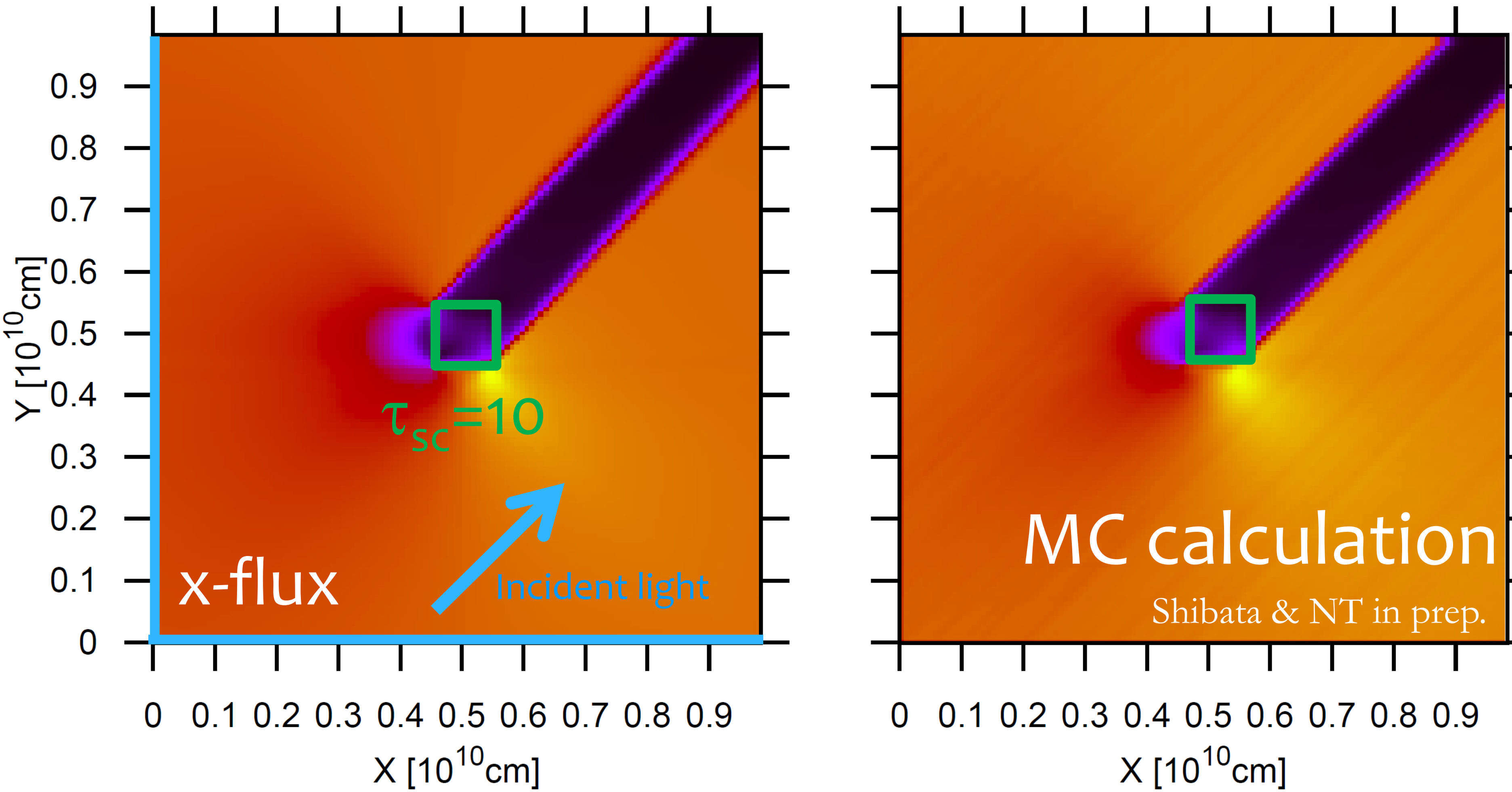
Relativistic beaming

- Thermal cylinder: 128(x)x128(y)x32(θ)x64(ϕ)



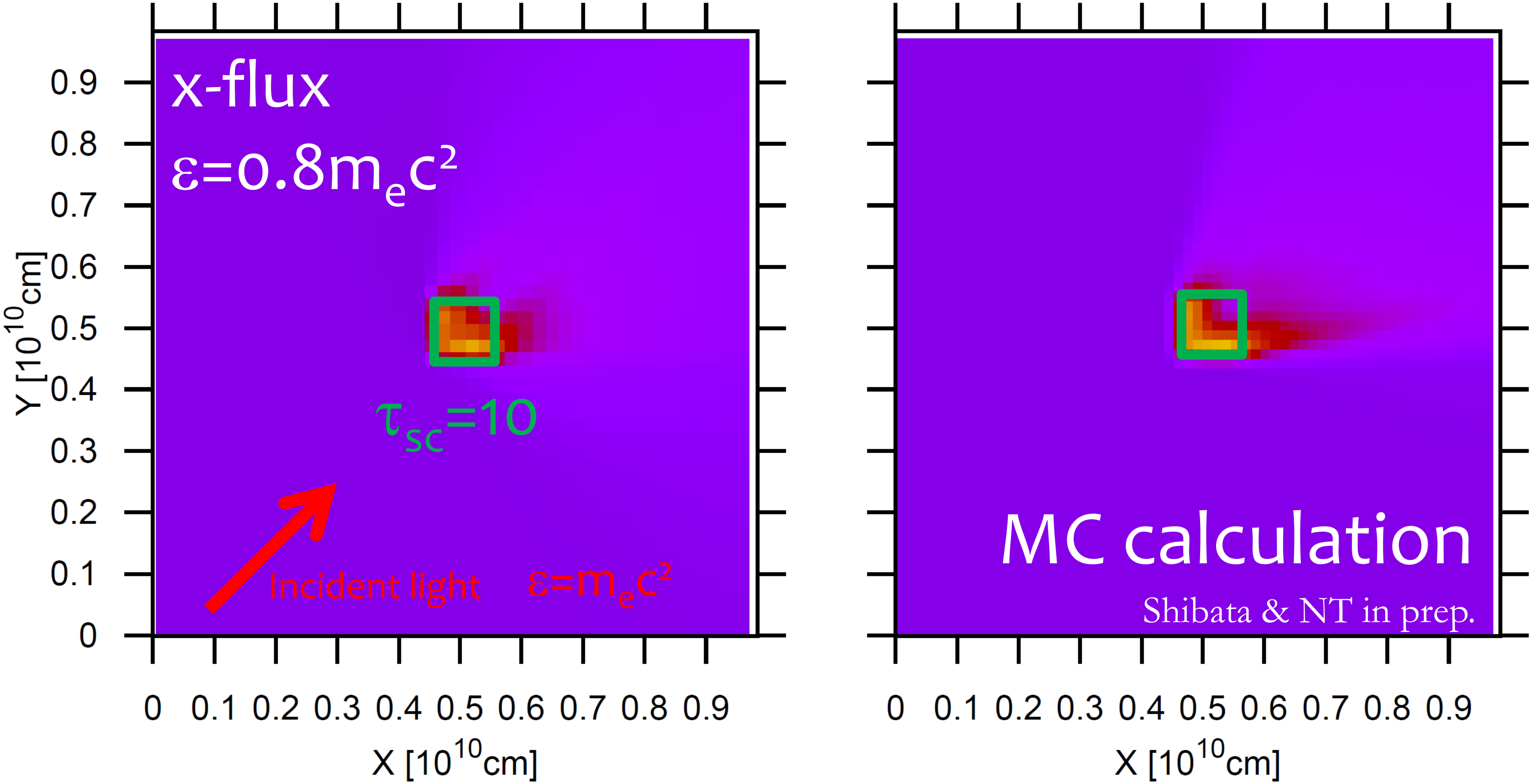
Comparison with MC cal. - Thomson scattering-

- Optically thick cuboid: 128(x)x128(y)x16(θ)x32(ϕ)



Comparison with MC cal. -Compton scattering-

- Optically thick cuboid: 64(x)x64(y)x64(θ)x128(ϕ)x10(ν)



Summary

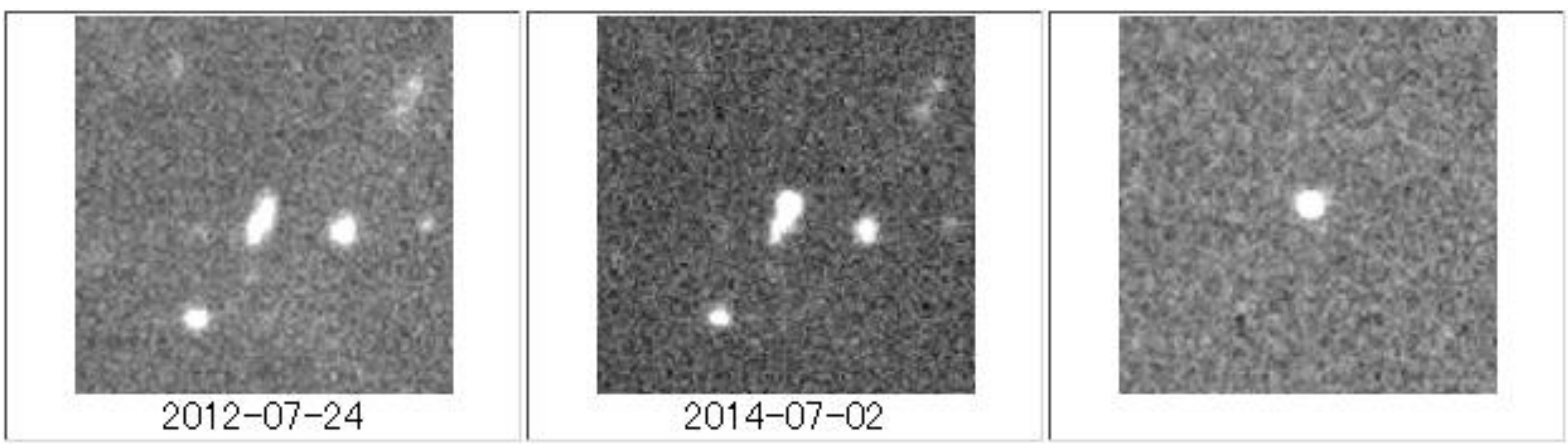
* A multidimensional time-dependent relativistic radiative transfer code is essential to numerically study the radiation from GRBs.
* We develop two codes.
* The MC method is ready for spectra synthesis. Photospheric emission may explain the Band function.
* The SHDOM is under development and passes several test problems for the relativistic radiative transfer calculation.

Appendix

The Astronomer's Telegram

First supernova candidates discovered with Subaru/Hyper Suprime-Cam

ATel #6291; *Nozomu Tominaga (Konan U./Kavli IPMU, U. Tokyo), Tomoki Morokuma (U. Tokyo), Masaomi Tanaka (NAOJ), Naoki Yasuda (Kavli IPMU, U. Tokyo), Hisanori Furusawa (NAOJ), Jian Jiang (U. Tokyo), Satoshi Miyazaki (NAOJ), Takashi J. Moriya (U. Bonn), Junichi Noumaru (NAOJ), Kiaina Schubert (NAOJ), and Tadafumi Takata (NAOJ)*
on ***4 Jul 2014; 15:51 UT***



http://tpweb2.phys.konan-u.ac.jp/~tominaga/HSC-SN/