Relativistic Magnetic Reconnection

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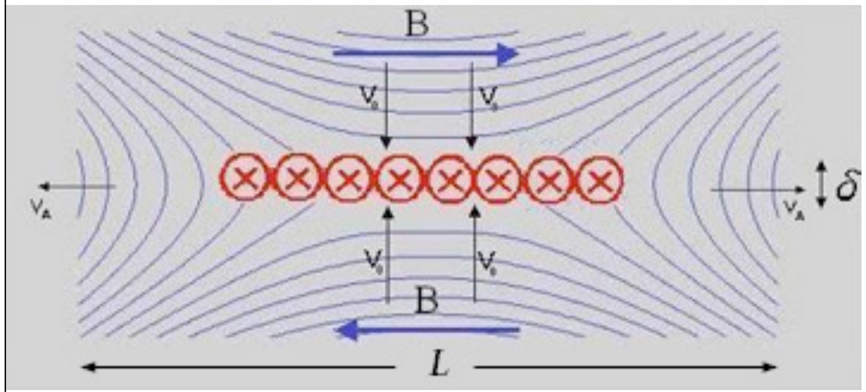
Non-relativistic MHD reconnection model

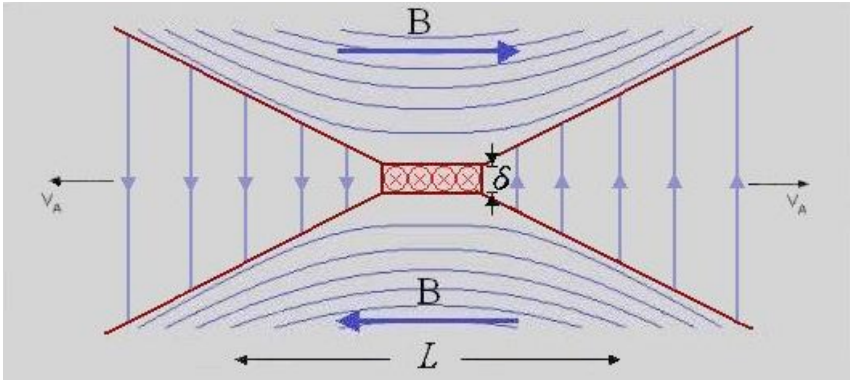
Sweet-Parker model

Petschek model

Sweet '58, Parker'57



Petschek '64



http://www.psfc.mit.edu

Magnetic energy is dissipated by Ohmic diffusion in the diffusion region.

Magnetic energy is liberated not only the diffusion region but mainly at the slow shock.

outflow speed ~ Alfvén vel.

reconnection rate $\mathcal{R} \simeq R_M^{-0.5}$

slow reconnection rate

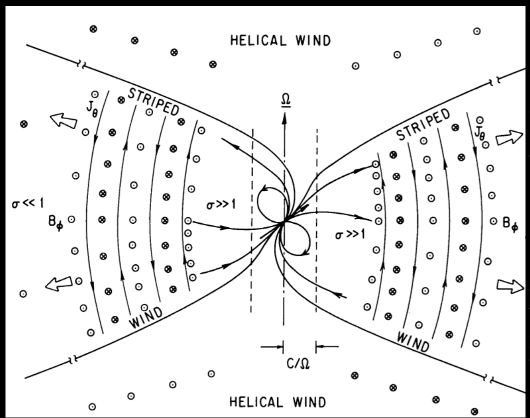
outflow speed ~ Alfvén vel.

reconnection rate $\mathcal{R} \simeq (\log R_M)^{-1}$

faster energy conversion

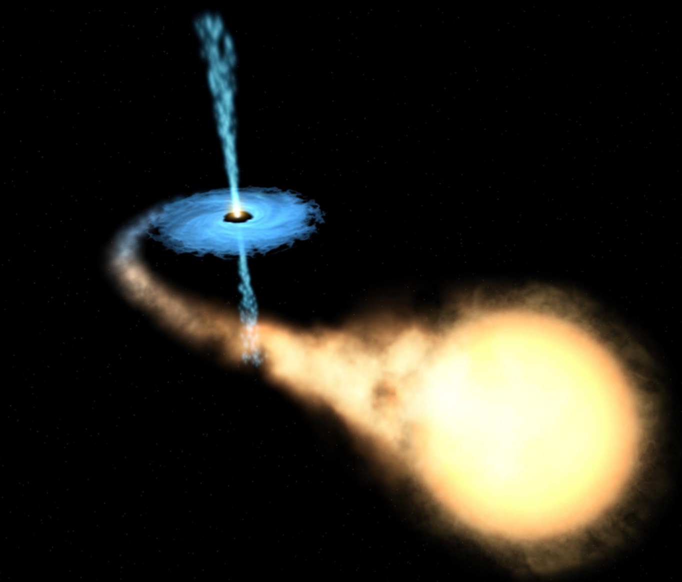
Reconnection Site in High Energy Astrophysical Phenomena

pulsar

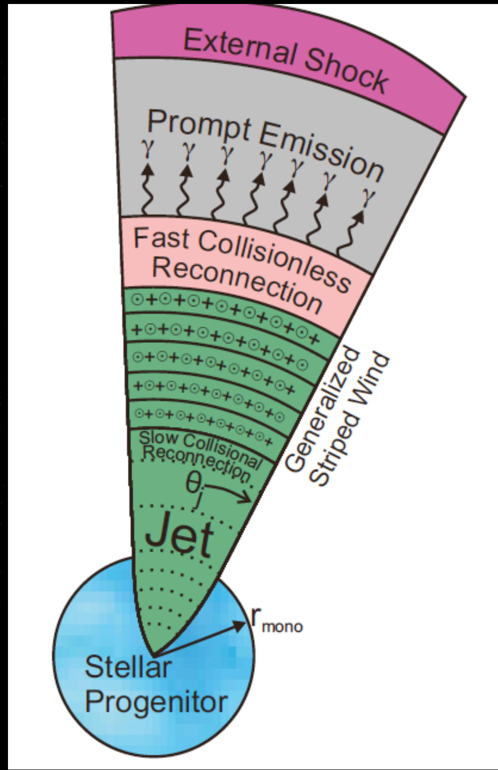


Coroniti+ ’90

BH accretion disks



GRB



McKinney & Uzdensky ’12

What we want to know about MRX

1. **What triggers the magnetic reconnection?**
   What is the origin of the electric resistivity ?

2. **Particle acceleration**
   How are the non-thermal particles generated in MRX?
   This problem would be related to the origin of hard X-ray observed in BHB.

3. **Energy conversion**
   What kind of energy is the magnetic energy finally converted into.

4. **Reconnection rate**
   How fast can MRX convert magnetic energy?

In this talk, I discuss about Energy conversion and Reconnection rate in the framework of fluid approximation.

Non Relativistic Sweet-Parker Type Magnetic Reconnection

Sweet '58, Parker '57, Priest & Forbes '98

- assume incompressibility

$$\rho = \mathrm{const}$$

- Equation of motion along x axis.

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} \right) = -\frac{j_z B_y}{c} - \frac{\partial p}{\partial x}$$

- induction equation

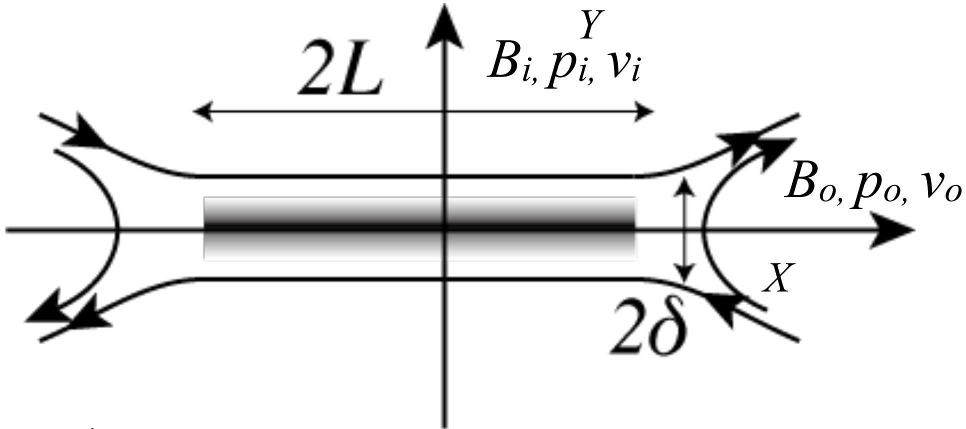
$$j_z \simeq -\frac{cB_i}{4\pi\delta}$$

- Gauss's law

$$B_i L \simeq B_y \delta$$

$$\Rightarrow \frac{\partial}{\partial x}\left(\frac{\rho v_x^2}{2}\right) = \frac{B_i^2}{4\pi L} - \frac{\partial p}{\partial x}$$

integrate with pressure balance along y.



$$v_o^2 = 2v_A^2 + \frac{2(p_i - p_o)}{\rho}$$

outflow velocity

$$v_o \simeq \frac{B_i}{\sqrt{4\pi\rho}} = v_A$$

Outflow velocity is close to the Alfvén velocity of the sheath plasma.

Outflow velocity and reconnection rate

✦mass conservation relation between the inflow and outflow:

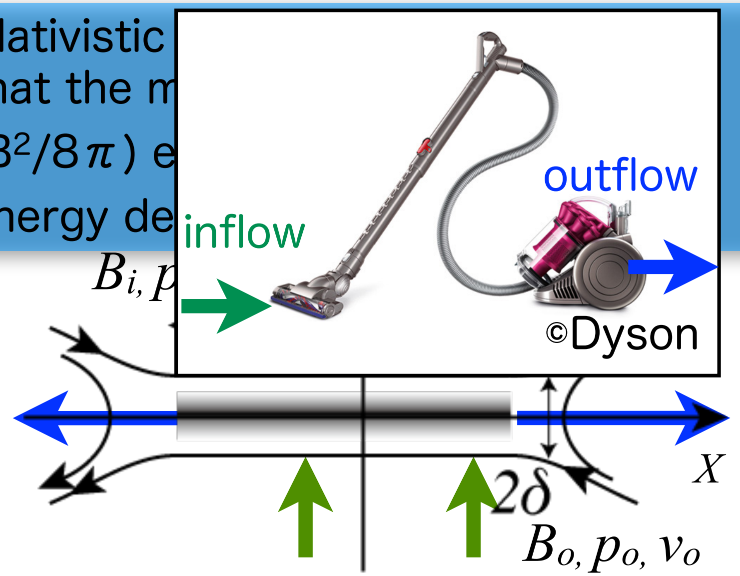
$$\rho_i v_i \gamma_i L = \rho_o v_o \gamma_o \delta$$

$$\frac{v_i}{v_o} = \frac{\rho_o}{\rho_i}\frac{\delta}{L}\frac{\gamma_o}{\gamma_i} \quad (1)$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$

=(compression ratio)
x (aspect ratio)
x (Lorentz contraction)

Relativistic
that the m
($B^2/8\pi$) e
energy de



✦**non-relativistic** ($\rho c^2 >> B^2/8\pi$)

Magnetic energy is converted into the kinetic energy:

$$\rightarrow \quad v_o \simeq V_A = \frac{B}{\sqrt{4\pi\rho}} \quad (2)$$

$$\frac{v_i}{v_o} \simeq \frac{v_i}{V_A} \simeq \begin{cases} R_M^{-0.5} & \text{Sweet-Parker} \\ (\log R_M)^{-1} & \text{Petscheck} \end{cases}$$

See, also
Blackman & Field '94
Lyutikov & Uzdensky '03
Lyubarsky '05

✦**relativistic** ($\rho c^2 << B^2/8\pi$)
→from equation 2, outflow velocity approaches to c for a larger B:
→from equation 1, rec. rate might be enhanced by factor γ_o in relativistic regime?

Relativistic Resistive MHD(R2MHD)

mass conservation equation

$$\frac{\partial \rho\gamma}{\partial t} + \frac{\partial}{\partial x^{\nu}}(\rho\gamma v^{\nu}) = 0$$

gas energy conservation

$$\frac{\partial}{\partial t}\left[E_{\rm hydro} + E_{\rm EM}\right] + \nabla\cdot\left[\boldsymbol{m}_{\rm hydro} + \boldsymbol{m}_{EM}\right] = 0$$

gas momentum equation

$$\frac{\partial}{\partial t}\left[\boldsymbol{m}_{\rm hydro} + \boldsymbol{m}_{EM}\right] + \nabla\cdot\left[\boldsymbol{P}_{\rm hydro} + \boldsymbol{P}_{\rm EM}\right] = 0$$

Maxwell equations

$$\frac{\partial \boldsymbol{B}}{\partial t} + c\nabla\times\boldsymbol{E} = 0 \qquad \nabla\cdot\boldsymbol{E} = 4\pi q$$

$$\frac{\partial \boldsymbol{E}}{\partial t} - c\nabla\times\boldsymbol{B} = -4\pi\boldsymbol{j} \qquad \nabla\cdot\boldsymbol{B} = 0$$

algebraic equations

gas : E.o.S. $p = p(\epsilon)$

E.M. : Ohm's law

$$\boldsymbol{j}' = \frac{\boldsymbol{E}'}{\eta}$$

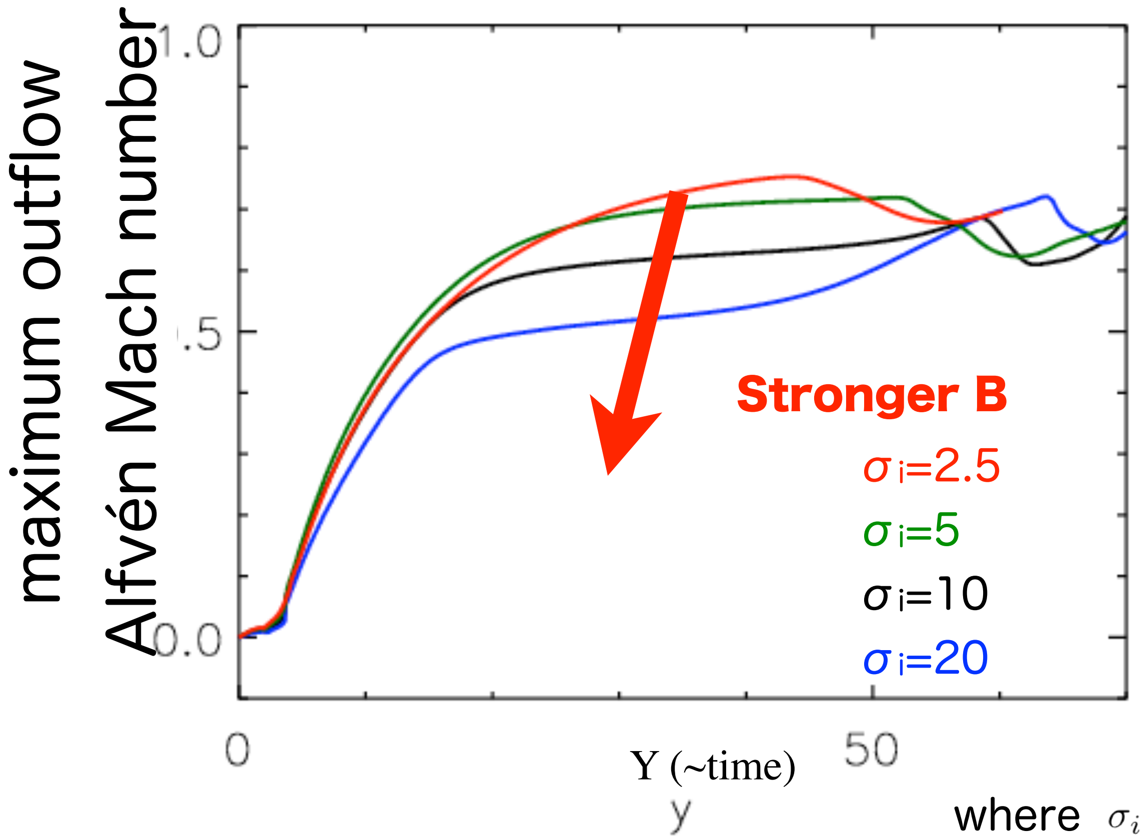
We solve 11 hyperbolic equation.

Relativistic Sweet-Parker MRX

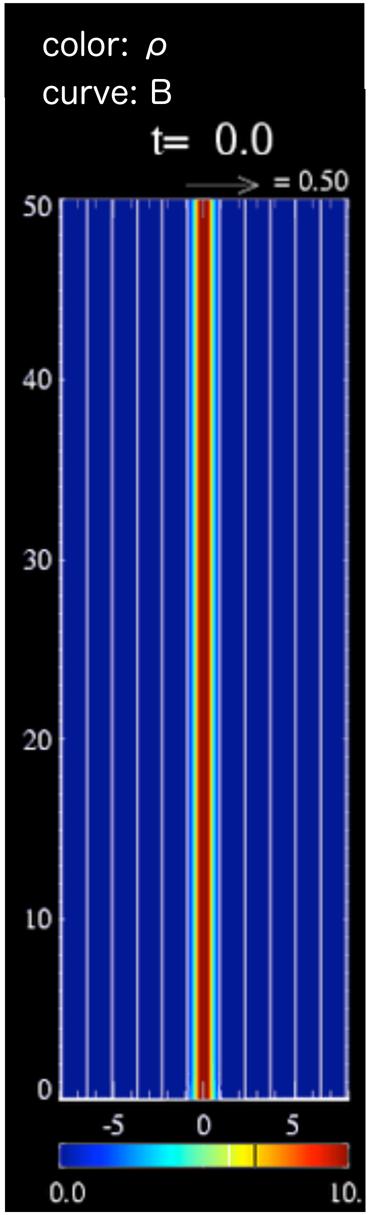
Relativistic Resistive Magnetohydrodynamic (R2MHD) simulations

Maximum outflow velocity on the plane of x=0.

Takahashi+ 11



where $\sigma_i = \frac{B_i^2}{4\pi\rho_i c^2}$



M_A increases with time and it saturates.

The saturated M_A is smaller for a larger B in the relativistic regime?

Mach number seems to decrease with increasing B

mildly relativistic outflow

Energy conversion in Relativistic MRX

Lyubarsky '05, HRT et al. '10

- Continuity equation

$$\rho_i \gamma_i v_i L = \rho_o \gamma_o v_o \delta$$

- Energy conservation

$$\left(\rho_i \xi_i \gamma_i^2 + \frac{B_i^2}{4\pi}\right) v_i L$$
$$= \left(\rho_o \xi_o \gamma_o^2 + \frac{B_o^2}{4\pi}\right) v_o \delta$$

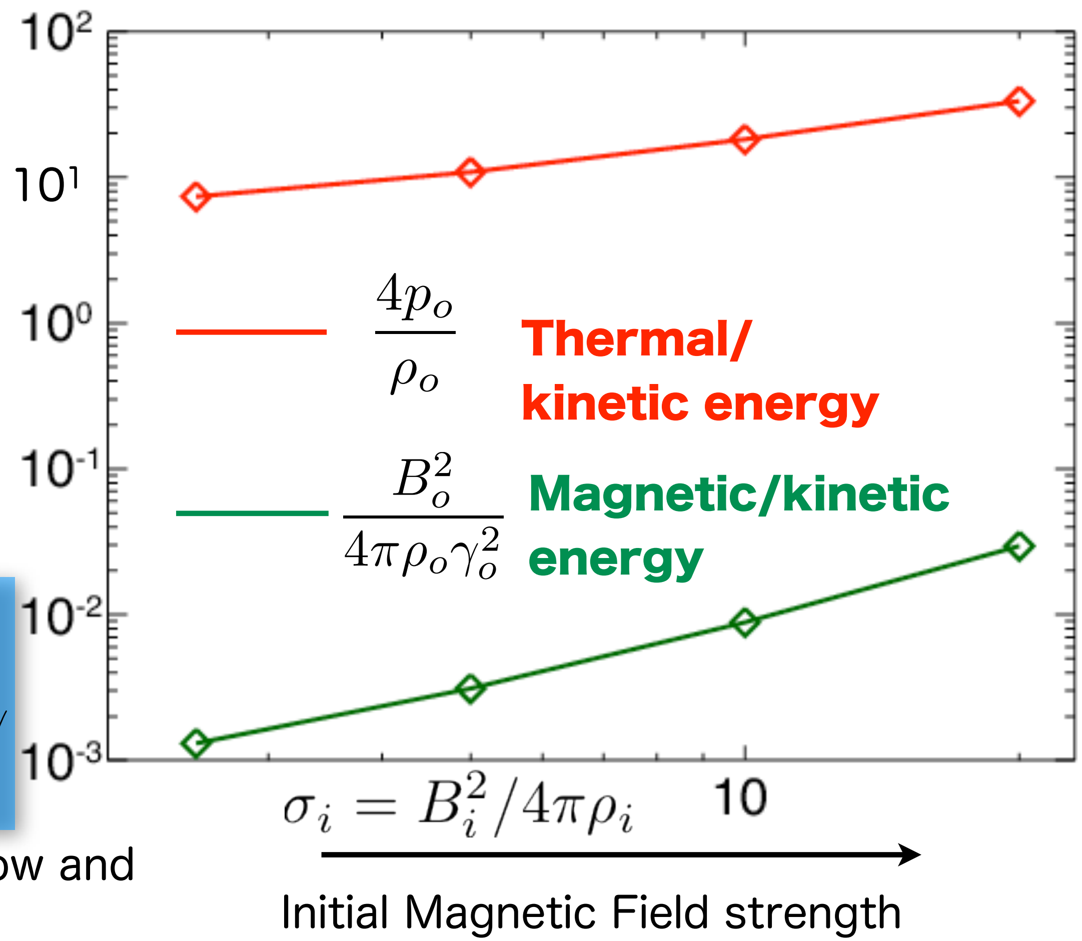
where $\xi = 1 + \frac{\Gamma}{\Gamma - 1} \frac{p}{\rho}$

Bernoulli constant

$$h = \left(1 + \frac{4p}{\rho} + \frac{B^2}{4\pi\rho\gamma^2}\right)\gamma$$

h is almost constant between inflow and outflow (assuming steady state).

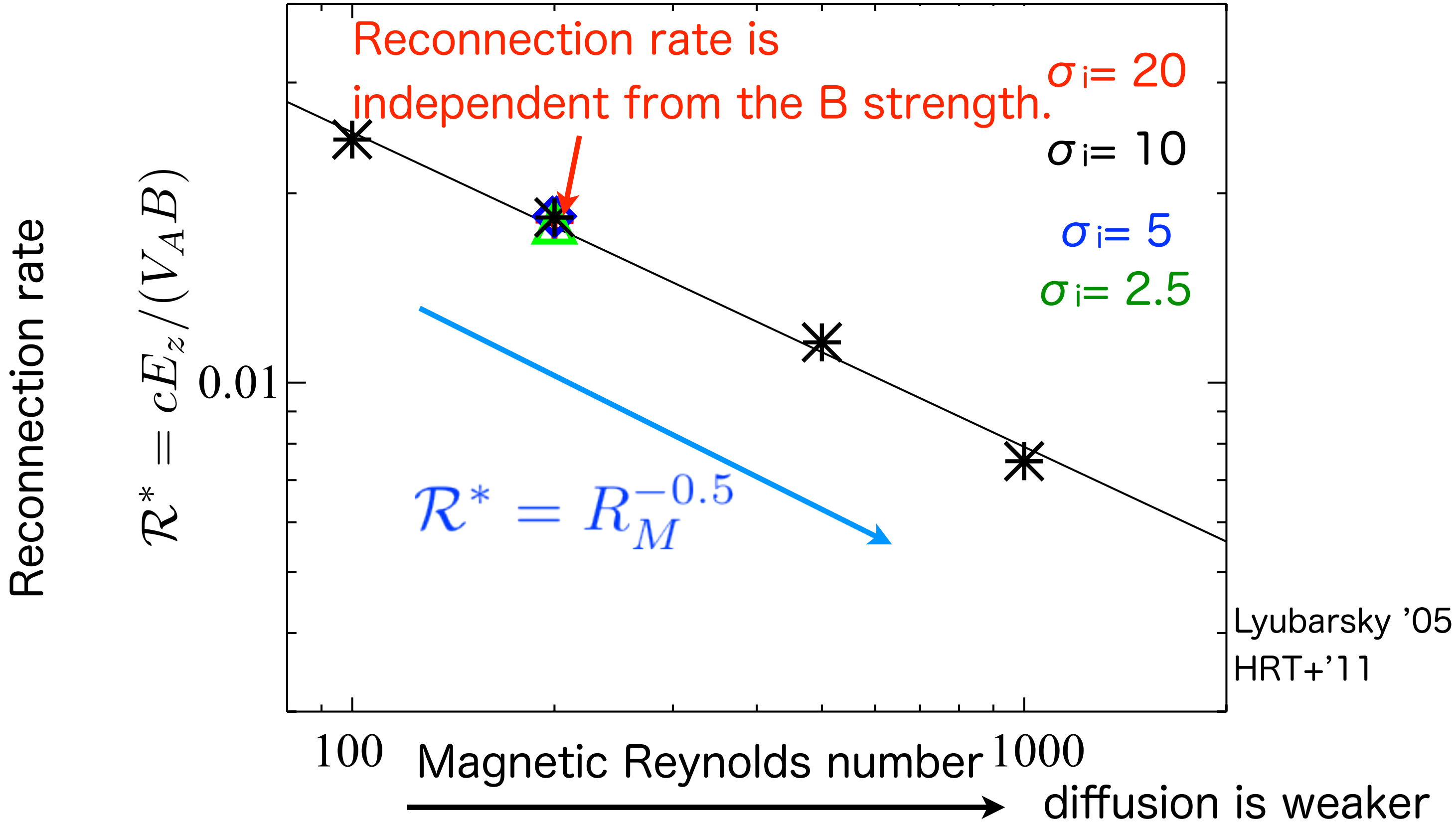
Energy composition of outflow



Most of magnetic energy is converted into the thermal energy.
Outflow is overpressured.

Takahashi+ 11

Reconnection Rate



Most of magnetic energy is converted to the thermal energy

-> The thermal energy contributes to the plasma inertia.

-> The plasma is hard to be accelerated and the bulk Lorentz factor is of order 1.

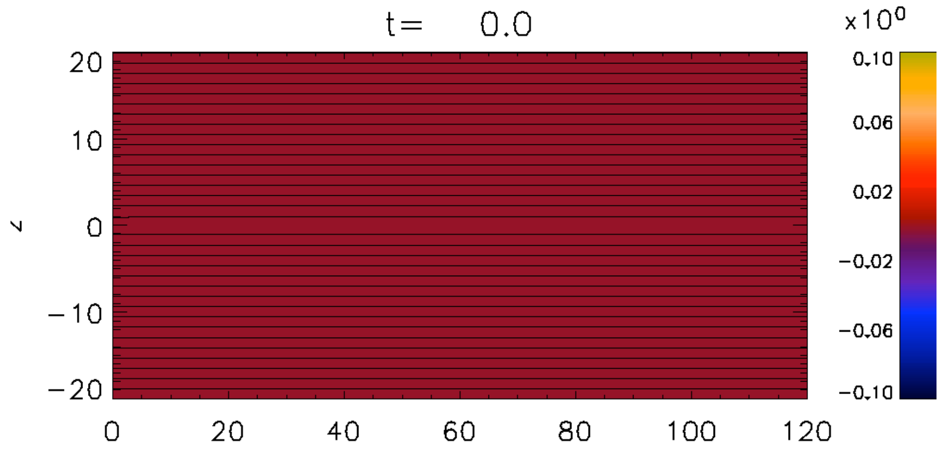
-> The effect of Lorentz contraction does not work efficiently.

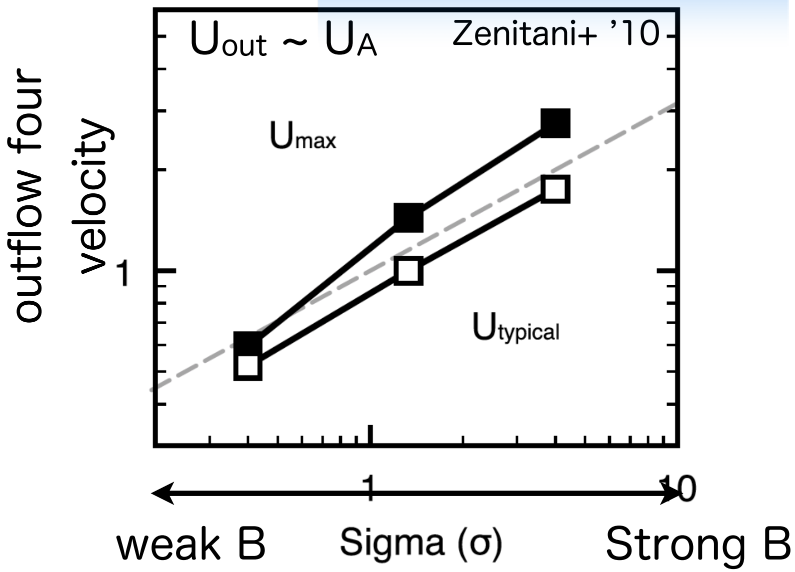
-> Sweet-Parker type magnetic reconnection is a slow process.

Relativistic Petschek type MRX

see,
Watanabe & Yokoyama '06,
Zenitani et al. '10,
Zanotti & Dumbser '11

Numerical simulations of the Relativistic Petschek type magnetic reconnection with spatially localized resistivity.



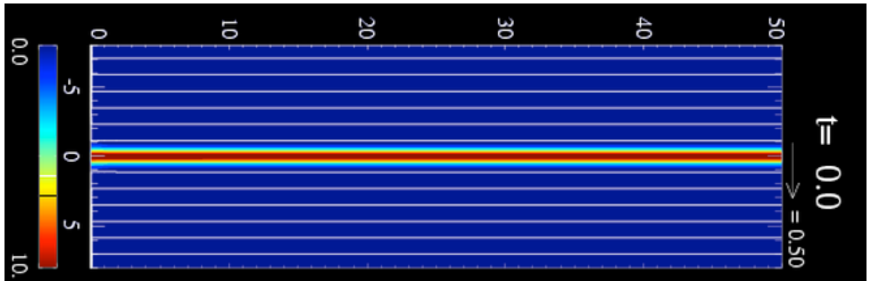


Relativistic Petschek type Magnetic Reconnection.

- Outflow is accelerated up to the Alfvén velocity by the magnetic tension force.
- Reconnection rate is enhanced in the relativistic regime (Watanabe & Yokoyama '06).
- The thermal energy is comparable to the kinetic energy in the outflow (Zenitani '09)

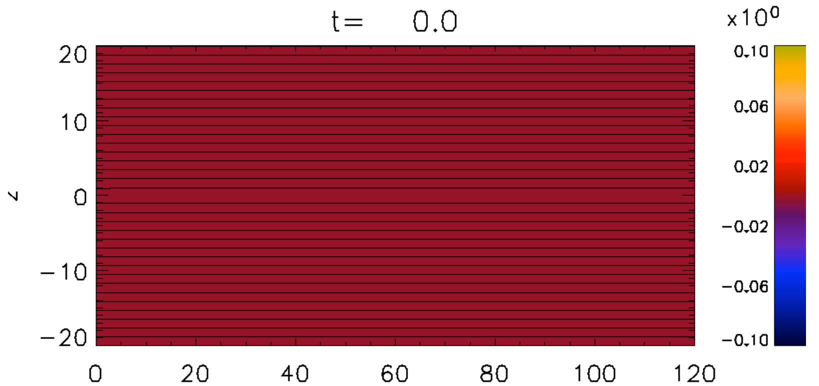
Summary of Relativistic Magnetic Reconnection

Sweet-Parker model



Takahashi+ '11

Petschek model



Watanabe & Yokoyama '06, Zenitani+ '10, Zanotti & Dumbser '11

Magnetic energy is liberated by Ohmic dissipation in the diffusion region.

mildly relativistic outflow

reconnection rate $\mathcal{R} \simeq R_M^{-0.5}$

slow reconnection rate

Magnetic energy is liberated not only the diffusion region but mainly at the slow shock

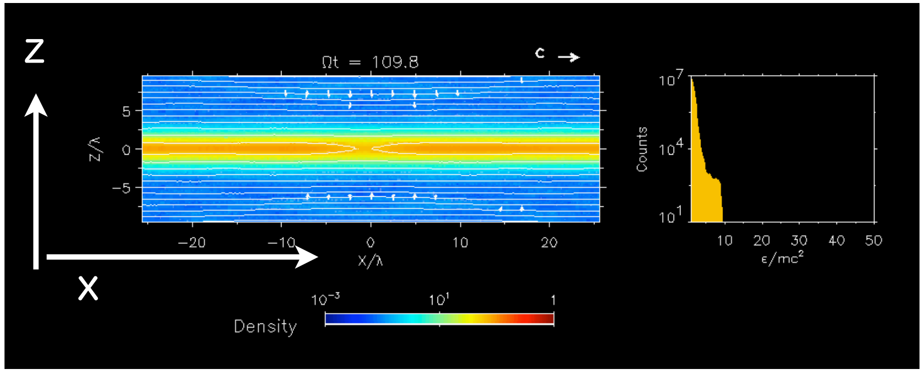
outflow speed ~ Alfvén velocity~relativistic?

reconnection rate $\mathcal{R} \simeq (\log R_M)^{-1}$

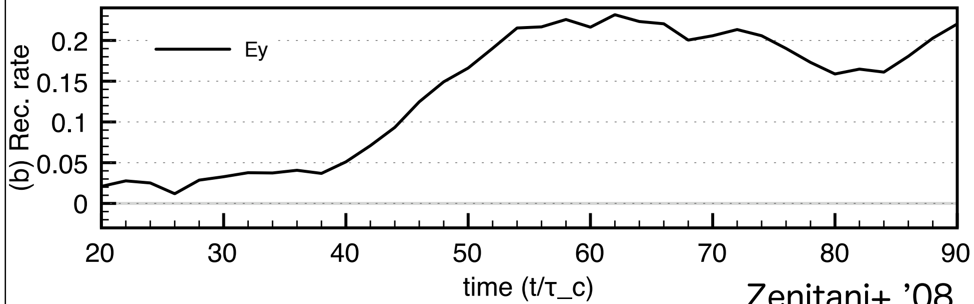
faster energy conversion

short comments: collisionless MRX

Reconnection rate is large.



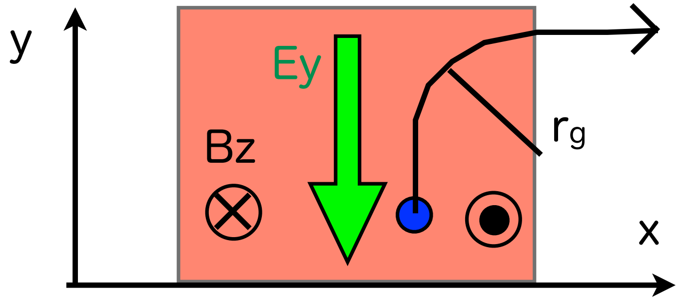
Zenitani+ '01

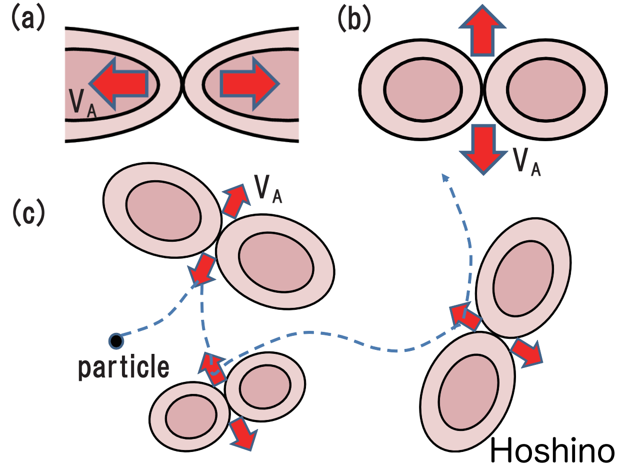


Zenitani+ '08

In the collisionless plasma, phenomenological resistive parameter is not introduced.
Reconnection proceeds
very fast (reconnection rate is of order 0.1).

Particles are efficiently accelerated.



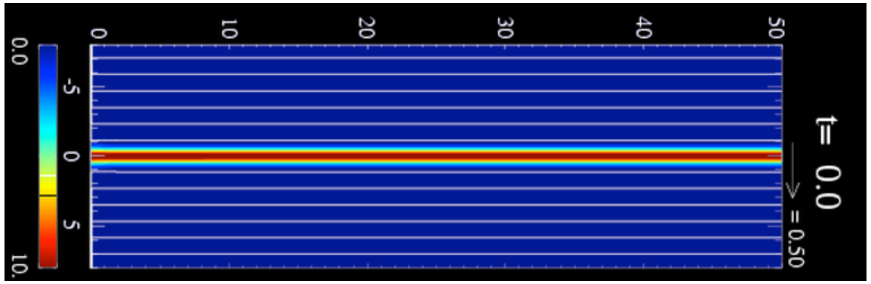


Hoshino '12

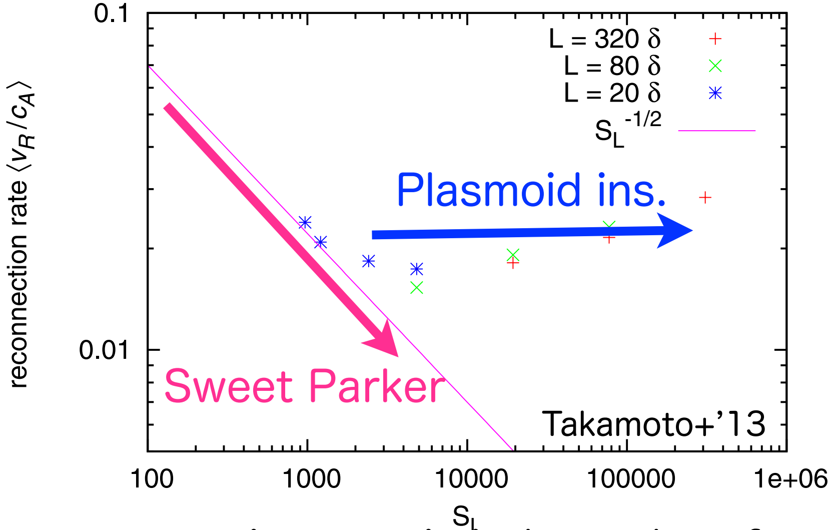
Particles are efficiently accelerated
by the reconnection E field.
Also multiple acceleration does work in multiple MRX (Matsumoto san).

Turbulence

Sweet-Parker type current sheet is not stable to the tearing mode instability with high Magnetic Reynolds number. Shibata & Tanuma ‘01

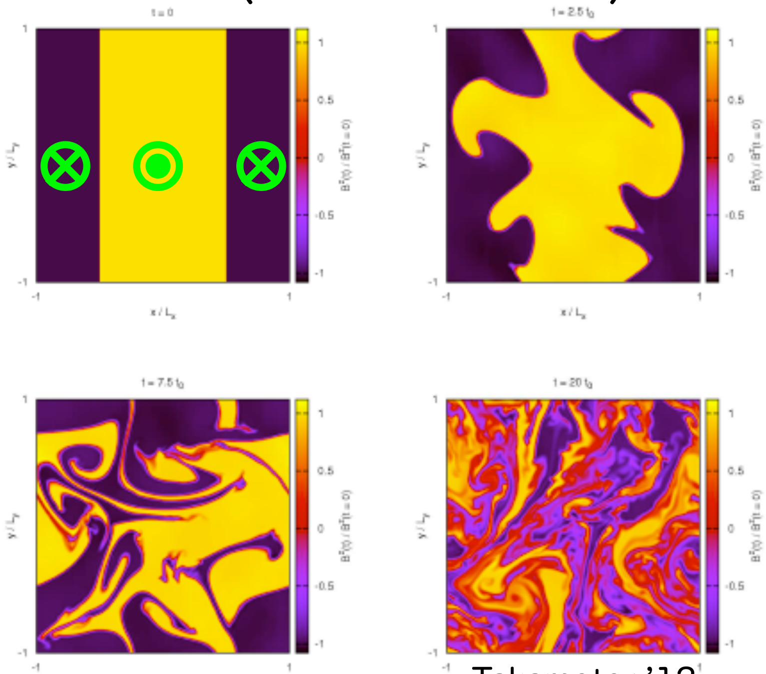


HRT+’11



The reconnection rate is independent from the magnetic Reynolds number also in the relativistic plasma.

Magnetic Energy dissipation in turbulence (not reconnection).



Takamoto+’12

The energy dissipation rate is independent from the resistivity.

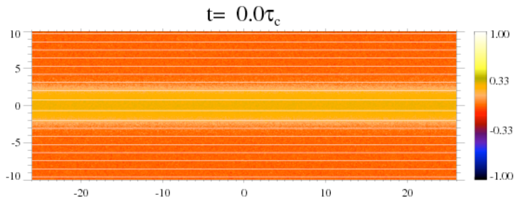
The Relativistic GEM Challenge

[Zenitani, Takahashi, Takamoto, & Bessho 2014]

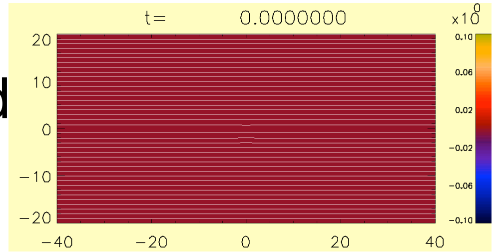
- What determines the energy efficiency in Relativistic MRX?
- Is it possible to construct resistivity model in relativistic plasma?

Geospace Environmental Modeling (GEM) Challenge
project has an aim of understanding the magnetic reconnection occurred

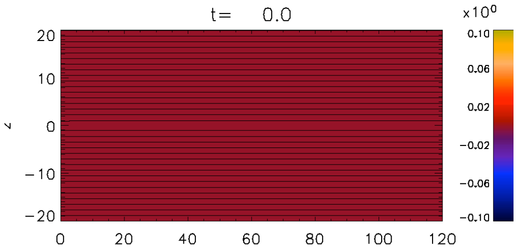
Particle



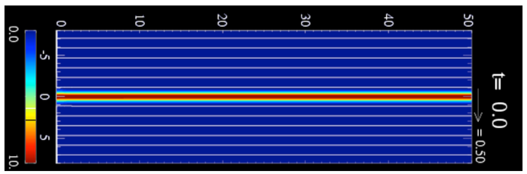
Two fluid
(pair)

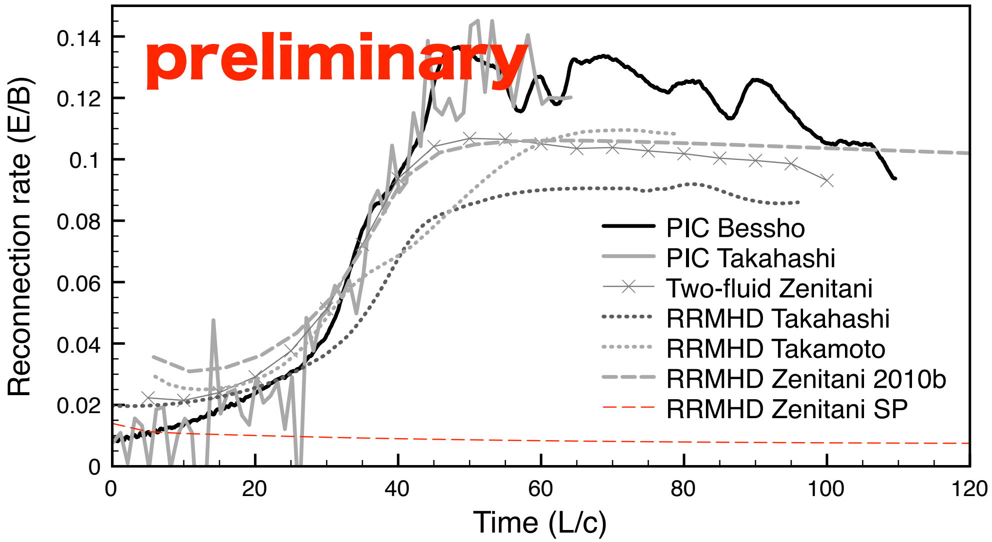


MHD
(PC)



MHD
(SP)





Reconnection Rate ($\sigma_E = 4$)

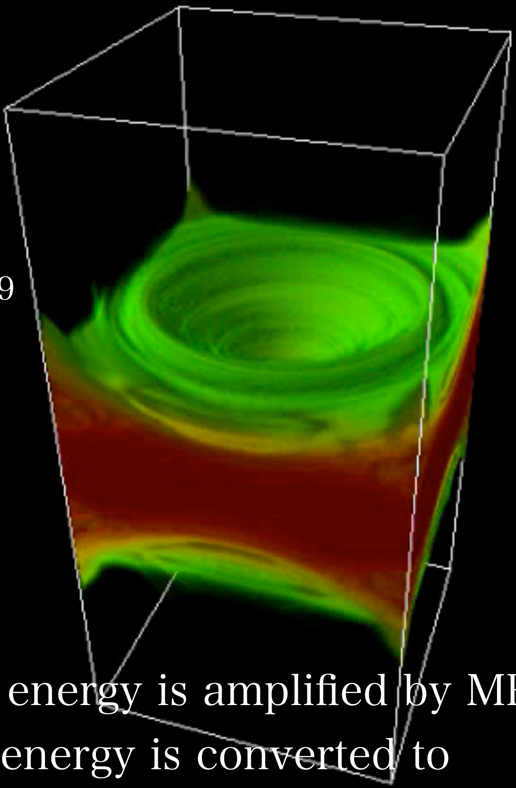
- A milestone in relativistic reconnection research!

High energy astronomical environment…,

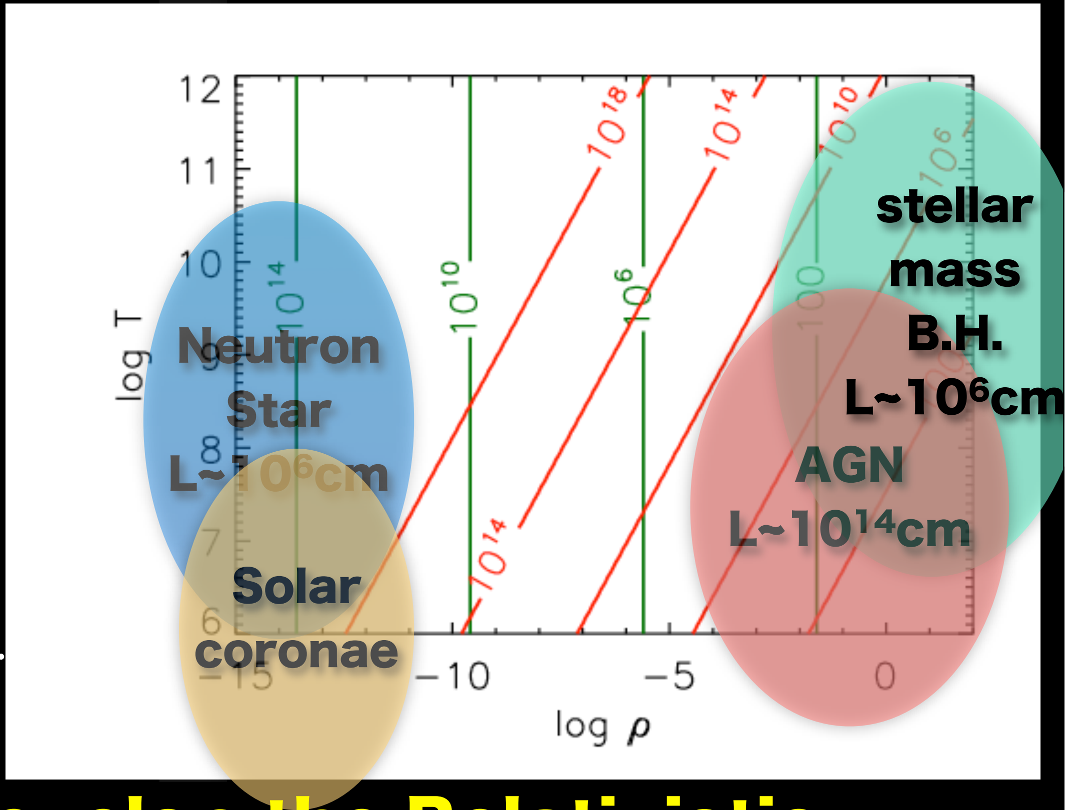
Radiation effect cannot be ignored

non-relativistic radiation MHD simulation of super critical accretion flow onto the stellar mss BH.

Ohsuga '09



mean free path for
electron scattering
free-free emission



Magnetic energy is amplified by MRI.
Part of B energy is converted to thermal energy through MRX in accretion disks.

We develop the Relativistic Resistive Radiation MHD (R3MHD) code.

Relativistic Resistive Radiation MHD(R3MHD)

mass conservation equation

$$\frac{\partial \rho\gamma}{\partial t} + \frac{\partial}{\partial x^\nu}(\rho\gamma v^\nu) = 0$$

gas energy conservation

$$\frac{\partial}{\partial t}\left[E_{\rm hydro} + E_{\rm EM}\right] + \nabla\cdot\left[\boldsymbol{m}_{\rm hydro} + \boldsymbol{m}_{\rm MHD}\right] = G^0$$

gas momentum equation

$$\frac{1}{c^2}\frac{\partial}{\partial t}\left[\boldsymbol{m}_{\rm hydro} + \boldsymbol{m}_{\rm EM}\right] + \nabla\cdot\left[\boldsymbol{P}_{\rm hydro} + \boldsymbol{P}_{\rm MHD}\right] = \boldsymbol{G}$$

Maxwell equations

$$\frac{\partial \boldsymbol{B}}{\partial t} + c\nabla\times\boldsymbol{E} = 0 \qquad \nabla\cdot\boldsymbol{E} = 4\pi q$$

$$\frac{\partial \boldsymbol{E}}{\partial t} - c\nabla\times\boldsymbol{B} = -4\pi\boldsymbol{j} \qquad \nabla\cdot\boldsymbol{B} = 0$$

15 hyperbolic equation

Radiation moment equation

$$\frac{\partial E_r}{\partial t} + \nabla\cdot\boldsymbol{F}_r = -G^0$$

$$\frac{1}{c^2}\frac{\partial \boldsymbol{F}_r}{\partial t} + \nabla\cdot\boldsymbol{P}_r = -\boldsymbol{G}$$

algebraic equations

gas : E.o.S.

rad. : M-1 closure

E.M. : Ohm’s law

Petschek Type Reconnection in Uniformly Distributed Radiation Field

parameter:

density $1.0\text{x}10^{-2}$ g/cm^3,

T_{gas} $1\text{x}10^8$ K,

T_{rad} $1\text{x}10^8$ K,

B=$1\text{x}10^{10}$ Gauss

V_A=0.69c

β =$4.1\text{x}10^{-5}$

σ =0.89

radiation process

abs.: free-free absorption (m.f.p.=$1.6\text{x}10^4$km)

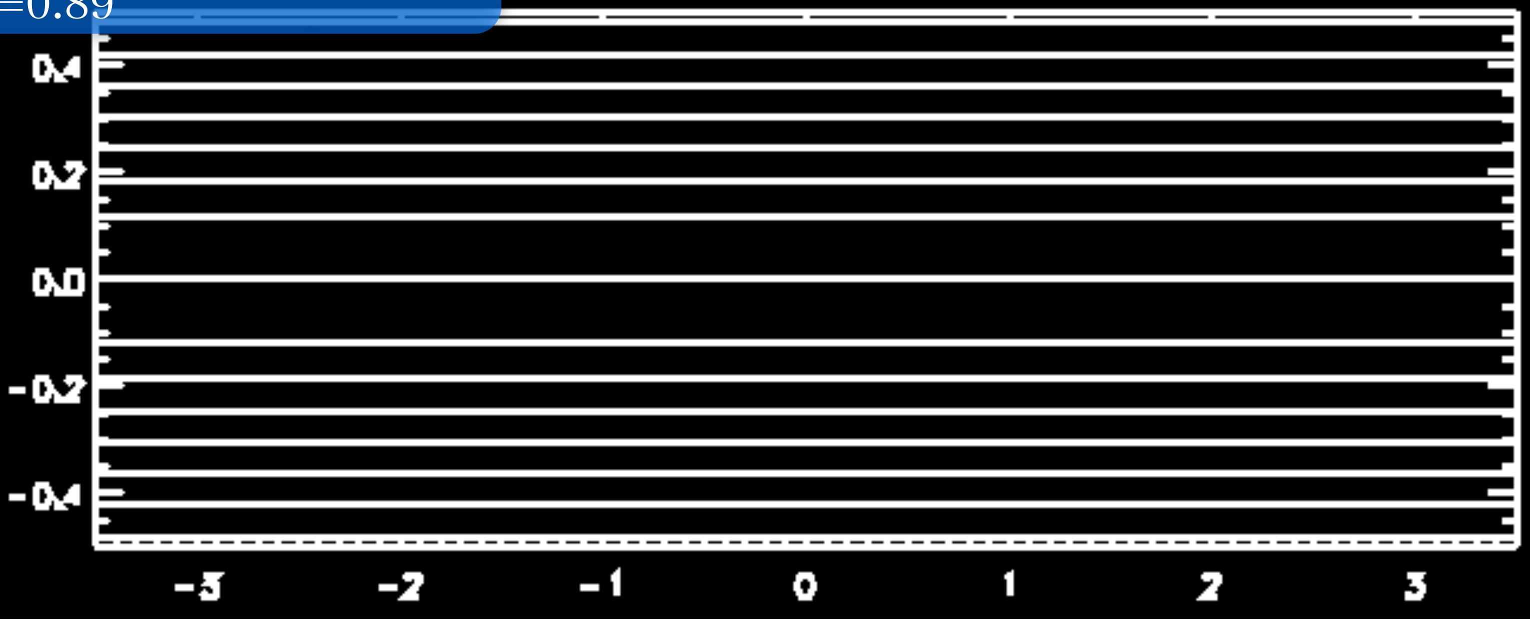
scat.: electron scattering (m.f.p.=$2.5\text{x}10^{-3}$km)

model

force-free Harris sheet

localized resistivity

radiation energy density



Petschek Type Reconnection in Uniformly Distributed Radiation Field

parameter:

density 1.0×10^{-2} g/cm^3,
T_{gas} 1×10^8 K,
T_{rad} 1×10^8 K,
$B = 1 \times 10^{10}$ Gauss
$V_A = 0.69c$
$\beta = 4.1 \times 10^{-5}$
$\sigma = 0.89$

radiation process

abs.: free-free absorption (m.f.p.=1.6×10^4km)
scat.: electron scattering (m.f.p.=2.5×10^{-3}km)

model

force-free Harris sheet
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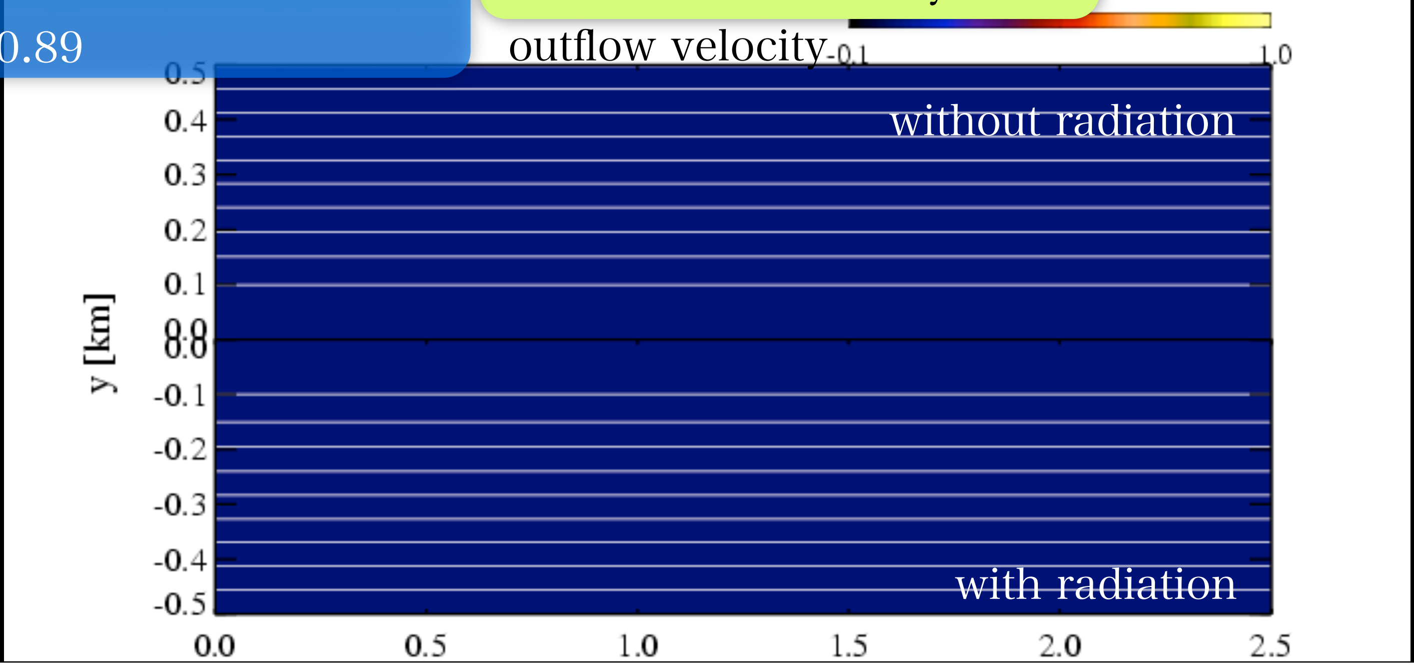
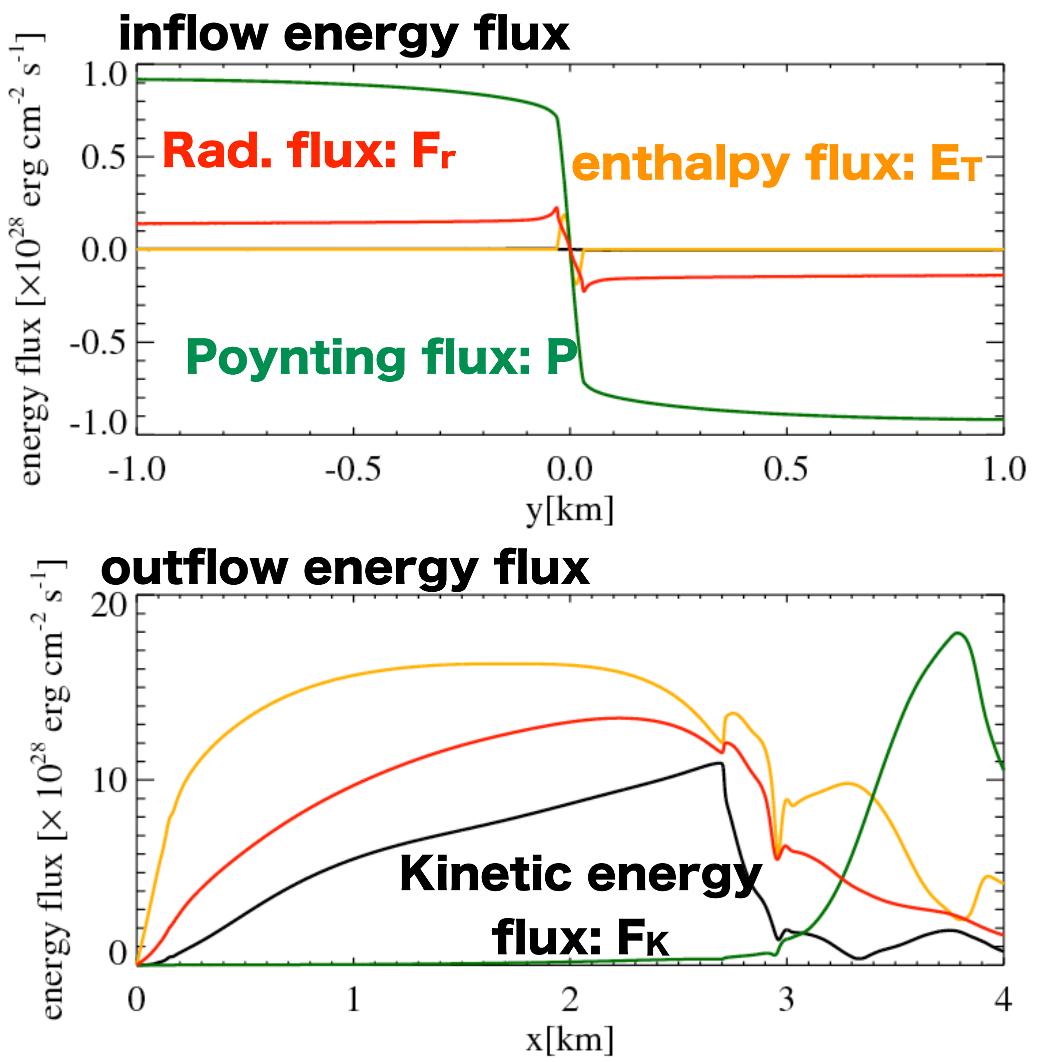


energy composition

Energy Flux $T^{0i} - \rho u^i = F_K^i + P^i + F_T^i + F_r^i$



Kinetic energy flux

$$F_K = \rho(\gamma - 1)u$$

Poynting flux

$$P = \frac{cE \times B}{4\pi}$$

Enthalpy flux

$$F_T = \frac{\Gamma}{\Gamma - 1}\gamma p_g u$$

Radiation flux

$$F_R$$

Magnetic energy ->
Thermal and kinetic energy
The radiation energy is comparable to those energies.

Radiation Dragging Force

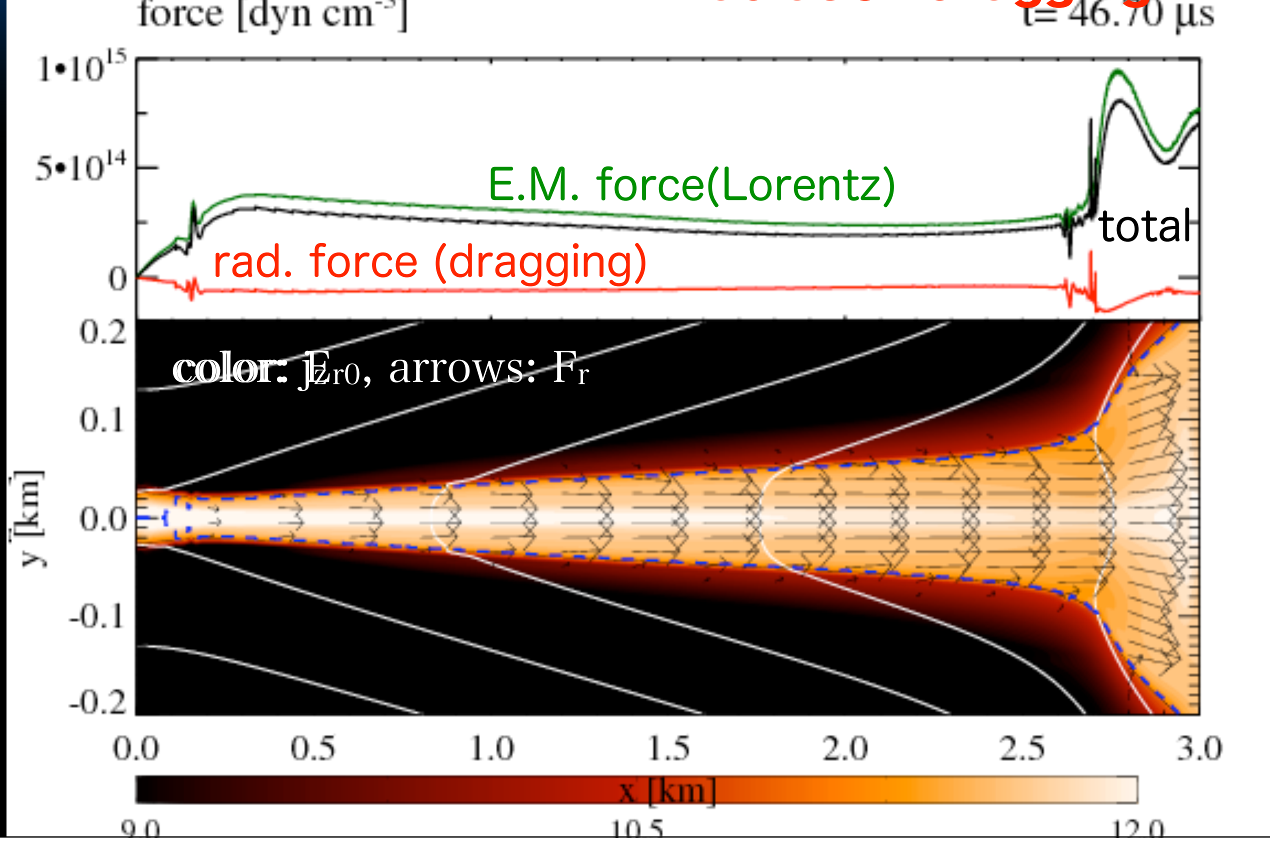
EOM for R3MHD

$$\rho h \gamma^2 \frac{\mathrm{D}v^i}{\mathrm{D}t} = -\partial^i p_g - \frac{\beta^i}{c}\frac{\partial p_g}{\partial t}$$ **gas pressure force**

$$+ \rho_e E^i + \frac{(\boldsymbol{j} \times \boldsymbol{B})^i}{c} - \frac{v^i}{c^2}(j_k E^k)$$ **EM force**

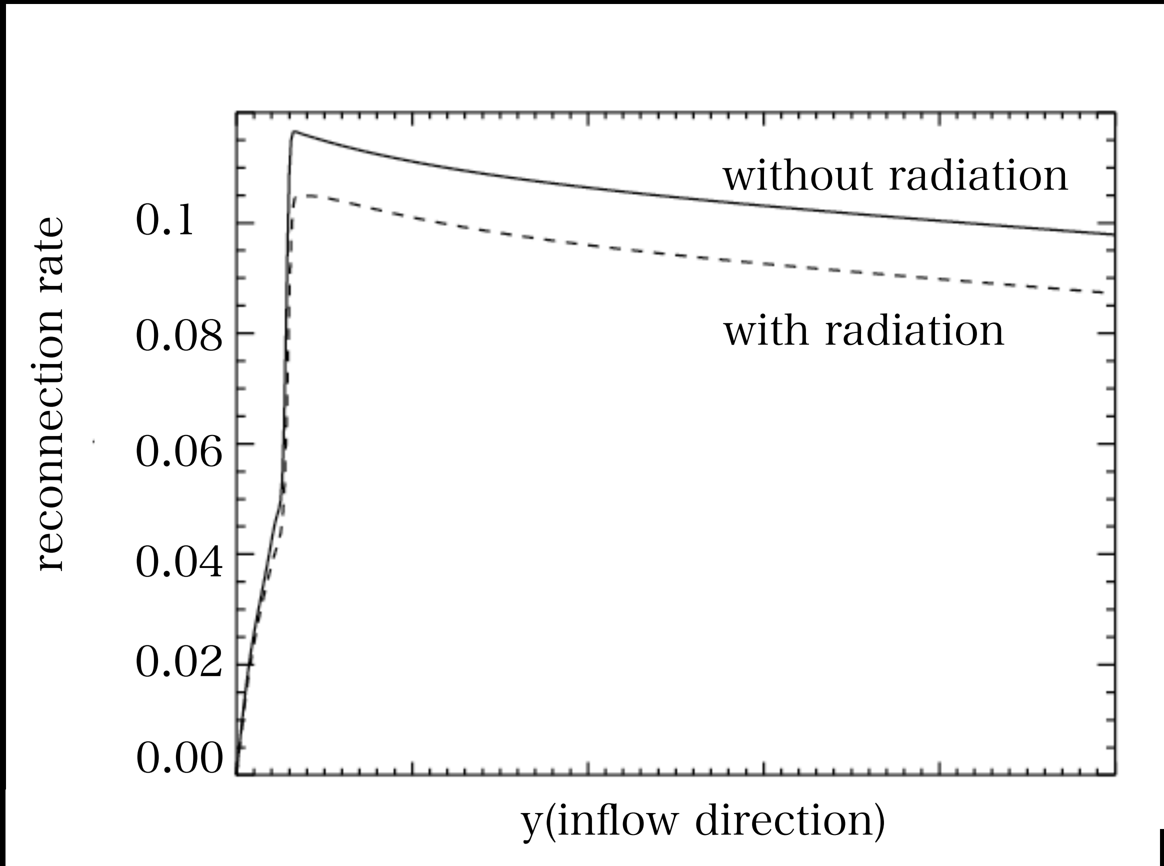
$$+ \rho(\kappa_0 + \sigma_0)\left[\gamma \frac{F_r^i}{c} - \frac{E_r u^i + u_k P_r^{ik}}{c} + \frac{u^i u_k F_r^k}{\gamma c^3}\right]$$ **rad. force**

radiation dragging



reconnection rate

=inflow velocity / Alfvén velocity

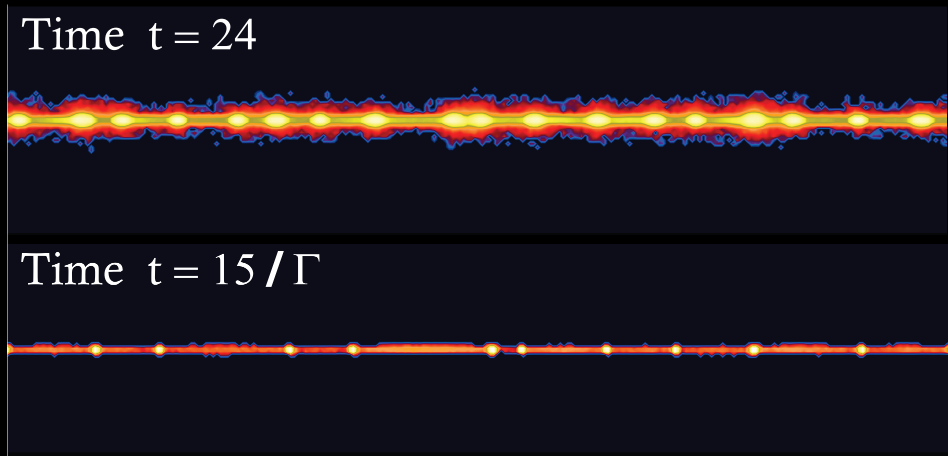


The outflow velocity decreases due to the radiation dragging force.

-> The inflow velocity slightly decreases to balance the mass conservation between the inflow and outflow:

Radiative Cooling

Particle-In-Cell simulation of the Relativistic Current Sheet with radiative cooling

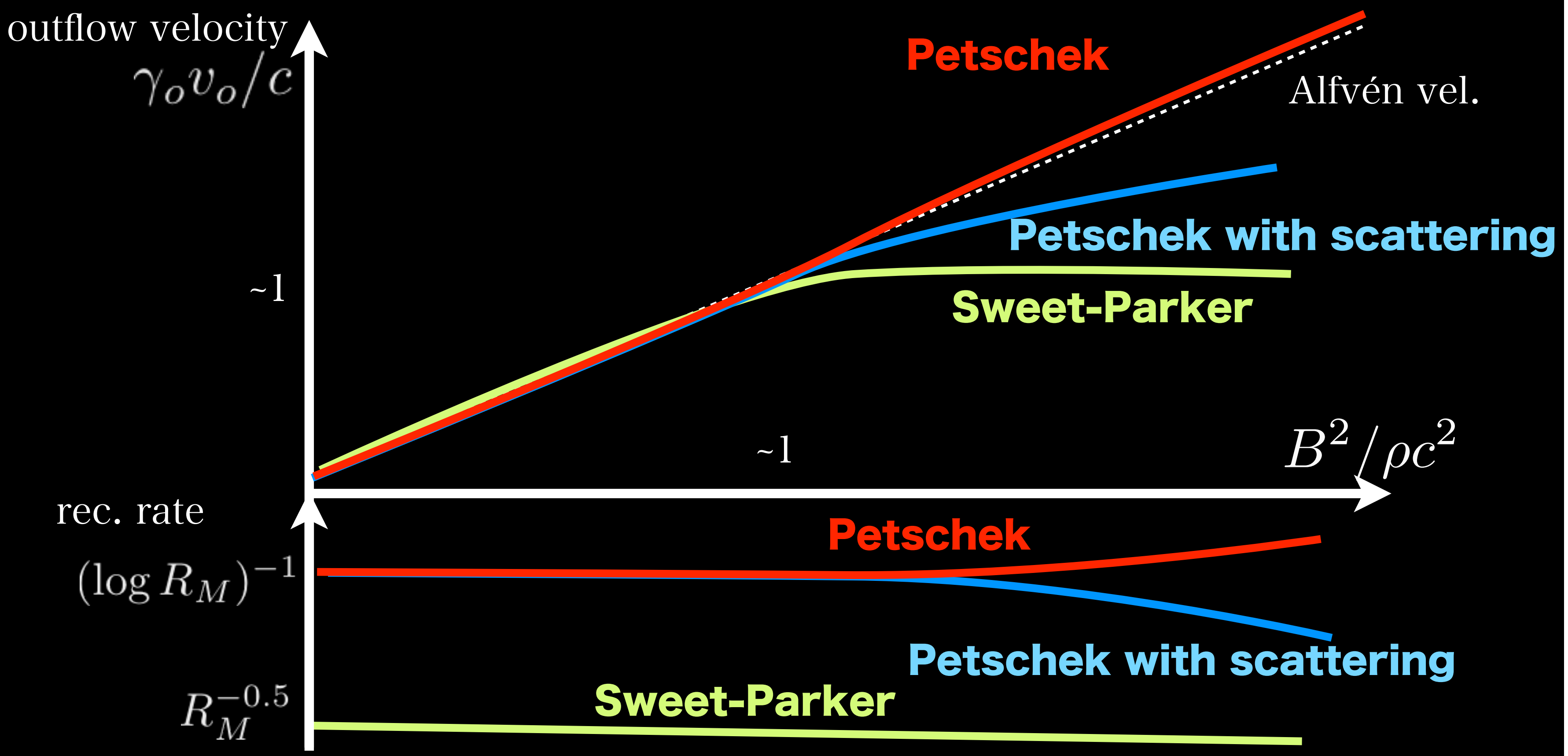


Jaroschek & Hoshino '09

The current sheet gets thinner due to the radiative cooling.
The thin current sheet becomes unstable to the tearing instability and the reconnection rate is enhanced (similar to plasmoid instability).

Also the current sheet is essentially thermally unstable?
(Uzdensky & McKinney '13).

Schematic Summary of MHD reconnection



In the relativistic reconnection,

- outflow velocity decreases due to the radiative dragging force.
- reconnection rate decreases to balance the mass conservation.

Summary

Simple MHD Model

Sweet-Parker type

- The outflow velocity is mildly relativistic ($\gamma \sim 1$).
- The reconnection rate is small: $\mathcal{R} = R_M^{-0.5}$

Petschek type

- The outflow velocity is relativistic ($\gamma = \sqrt{(1+\sigma)}$).
- The reconnection rate is large $\mathcal{R} \simeq (\log R_M)^{-1}$

Recent Progress

- Turbulence: would enhance the reconnection rate whether the turbulence is self-generated (plasmoid) or externally driven.
- Radiation:
  - Scattering effect slows down reconnection due to the dragging effects.
  - Cooling effects increases reconnection rate by accompanying tearing mode or thermal instability.
- Collisionless reconnection: is a fast process for energy conversion. Nonthermal particles are generated.
- Connection between MHD and collisionless scale is not well understood.