Radiative Transfer Analysis for Coupled Computation with Relativistic Hydrodynamics

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Numerical works of GRB

- Some radiative transfer simulations on the steady background have been performed
- Jet structure can affect the observed spectrum (Mizuta 2006, Lazzati 2009, Nagakura 2011)
- Radiative transfer computation should be implemented on inhomogeneous background

Radiative transfer computation on the unsteady fluid background is necessary

Coupled computation of radiative transfer with relativistic hydrodynamics
Preparation for coupled computation

Past study

- Relativistic hydrodynamics simulation of jets

- Radiative transfer computation (Monte Carlo, photospheric emission)
  in a simple model
  (Pe’er 2011, Ito 2013, Ito 2014, Shibata 2014)

Flow velocity of jet $\rightarrow$ almost speed of light
  (Lorentz factor $\Gamma \gtrsim 100$)

- Coupled computation with time-dependent ultrarelativistic flow
  has not been performed yet (Radiation hydrodynamics)

- Radiative transfer method on the ultrarelativistic background
  should be validated for a reliable computation
Objectives

Goal

GRBs originated with relativistic jets by coupled computation

Preparation of coupled computation

Are simulation results in different inertial frames equivalent each other in computing radiative transfer on the ultrarelativistic background?

- Implementing radiative transfer simulation in the shock rest frame and the shock moving frame
- Comparing results in the same frame
- Performing photon transport with the shock moving on the computational grids
Numerical method

radiative transfer equation including scatterings

\[
\left( \frac{1}{c} \frac{\partial}{\partial t} + \Omega \cdot \nabla \right) I = j + \frac{\rho}{4\pi} \int \int \sigma I \phi d\nu' d\Omega' - [\kappa + \sigma] \rho I
\]

\( c \): speed of light  \( t \): time  \( \Omega' \): incident direction  \( \Omega \): scattered direction

\( I \): specific intensity  \( j \): emissivity  \( \nu' \): incident frequency  \( \nu \): scattered frequency

\( \sigma \): scattering cross-section  \( \phi \): scattering kernel  \( \kappa \): absorption cross-section

computed in comoving frame

Monte Carlo method
no absorption
including Thomson scattering

emission

Lorentz transformation

CMF

OBF

electron

recomputing cross-section, free path, angle

ignoring thermal motion of electron

CMF : comoving frame
OBF : observer frame
Computing in the different inertial frames
Simulation condition

- Setting of shock wave → relativistic Rankine-Hugoniot relations
- In the comoving frame, putting every photons at the single point initially (isotropic emission)
- Computing until all photons reach the boundary
- Simulating in shock rest frame and shock moving frames ($\Gamma = 1, 10, 100$)
Directional distribution of the escaped photons

- In the shock rest frame, photons are deflected backward because of flow velocity to the negative $z$-direction.
- In the shock moving frames, photons are deflected forward in contrast.
- After transformation, the profiles are identical in all frames.
Difference due to time duration

- Computation with limited $\Delta t$ should be performed for convergent result
- Shock speed (= boundary speed) is almost speed of light

$\Gamma = 10$

$\Gamma = 100$

$L$ : width of computational cell

\begin{align*}
\Delta N(\theta) / N \times 10^3 &
\end{align*}

\begin{align*}
direction angle, \theta [\text{rad}] / \pi
\end{align*}

\begin{align*}
c\Delta t / L &\sim 10^{-5} \\
c\Delta t / L &\sim 10^{-4} \\
c\Delta t / L &\sim 10^{-3}
\end{align*}

\begin{align*}
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c\Delta t / L &\sim 10^{-5} \\
c\Delta t / L &\sim 10^{-4} \\
c\Delta t / L &\sim 10^{-3}
\end{align*}
**Constraint for time duration**

Relation of mean free path with $\Delta t$

$$\alpha = \frac{c\Delta t}{s_{min}}$$

- $\alpha$ is almost 0.2
- We should adopt $\Delta t$ that resolves the mean free path to five steps

$\Gamma = 10$

$$\frac{c\Delta t}{s_{min}} \sim 0.1931$$

$\Gamma = 100$

$$\frac{c\Delta t}{s_{min}} \sim 0.1935$$

Transformation of free path

$$s_{min} = \frac{\sqrt{1-(v/c)^2}}{1+v/c} s_{cmf}$$

- $s_{min}$: mean free path of photon traveling opposite to flow velocity
- $c$: speed of light
- $\Delta t$: time duration

Flow velocity

- $\alpha$ is almost 0.2
- We should adopt $\Delta t$ that resolves the mean free path to five steps
Energy spectra of the escaped photons

- In each frame, the peak energy is shifted due to difference of flow velocity
- After transforming to the same frame, the profiles are identical each other
- Double peaks are found → bulk-Compton scattering
Summary

Radiative transfer computation in ultrarelativistic fluid background has been validated

- Simulation results in the differential frames were identical in the same frame
- Double peaks of energy spectrum were found due to bulk-Compton scattering
- In the Eulerian fluid background, no photon can catch up the shock front of $\Gamma \sim 220$
- Validation in more realistic situation should be performed
Future works

Goal

Reproducing observed high energy photons by coupled computation of radiative transfer with relativistic hydrodynamics

- Introducing electron energy distribution
- Selecting proper emission position
- Performing coupled computation with one-dimensional relativistic hydrodynamics

Thank you for your attention!
Bulk-Compton scattering after transformation to the shock rest frame

- Double peaks are found
- Bulk-Compton scattering occurs across the shock

Trajectory of high-energy photon

Shock rest frame
Lorentz factor 10
Lorentz factor 100

\[ \log E \text{ [keV]} \]

\[ \Delta \frac{N(E)}{N} \]

\[ \text{photons energy, } E \text{ [keV]} \]

\[ x \times 10^8 \text{ cm} \]

\[ z \times 10^{10} \text{ cm} \]

- 0.66667 c
\( (\Gamma \sim 1.34) \)

- 0.99999 c
\( (\Gamma \sim 220) \)