

Relativistic HD/MHD Flow for GRB Jets

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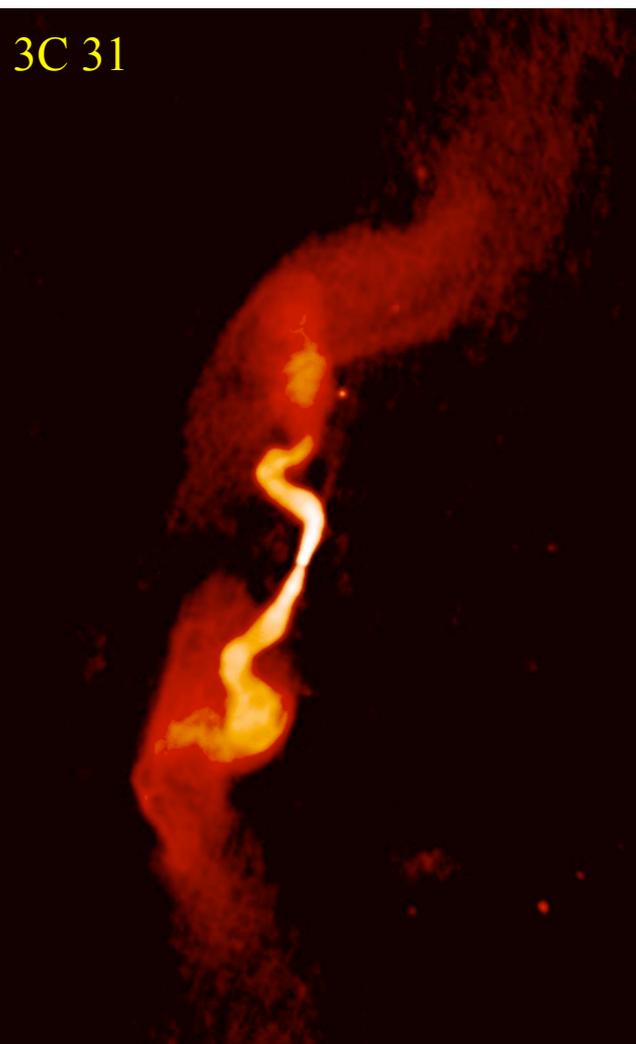
What a relativistic jet?

collimated bipolar outflow from gravitationally bounded object

- active galactic nuclei (AGN) jet: $\gamma \sim 10$
- microquasar jet: $v \sim 0.9c$
- Gamma-ray burst: $\gamma > 100$

Lorentz factor

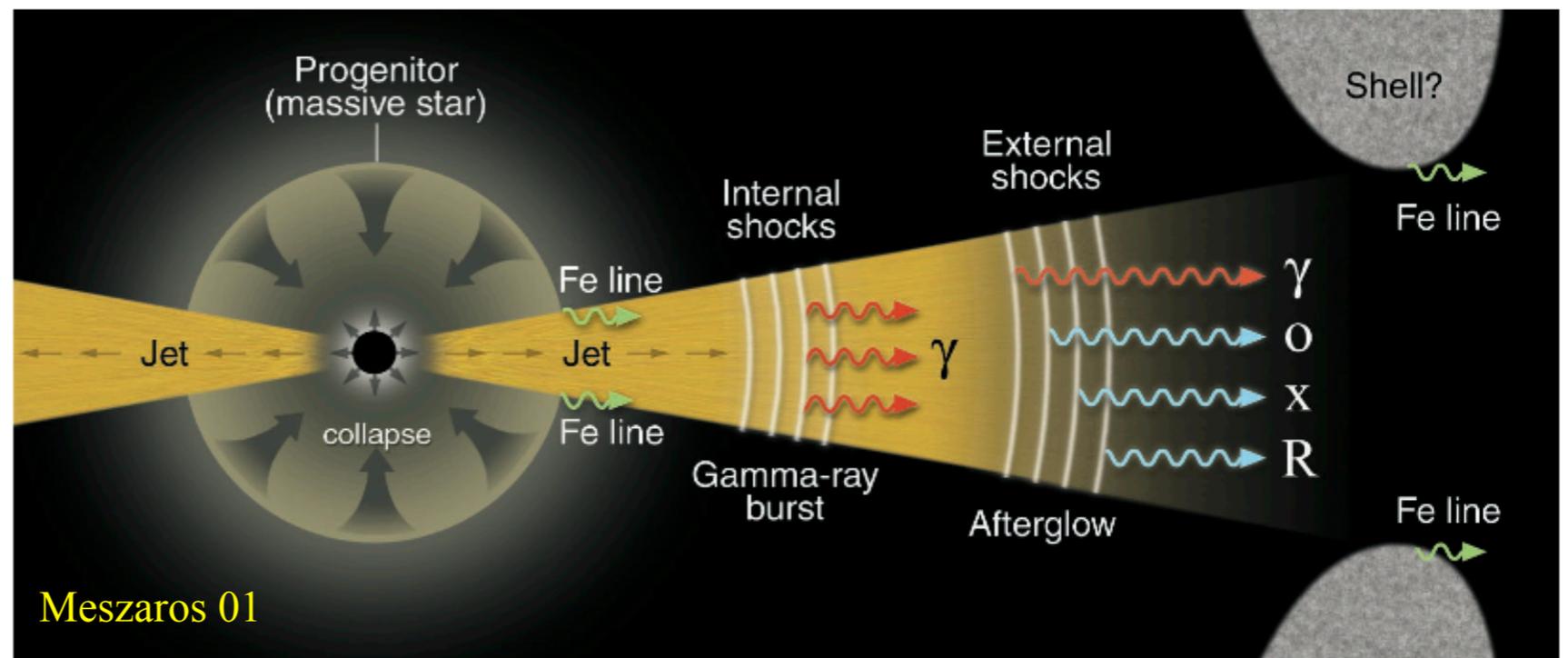
$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$



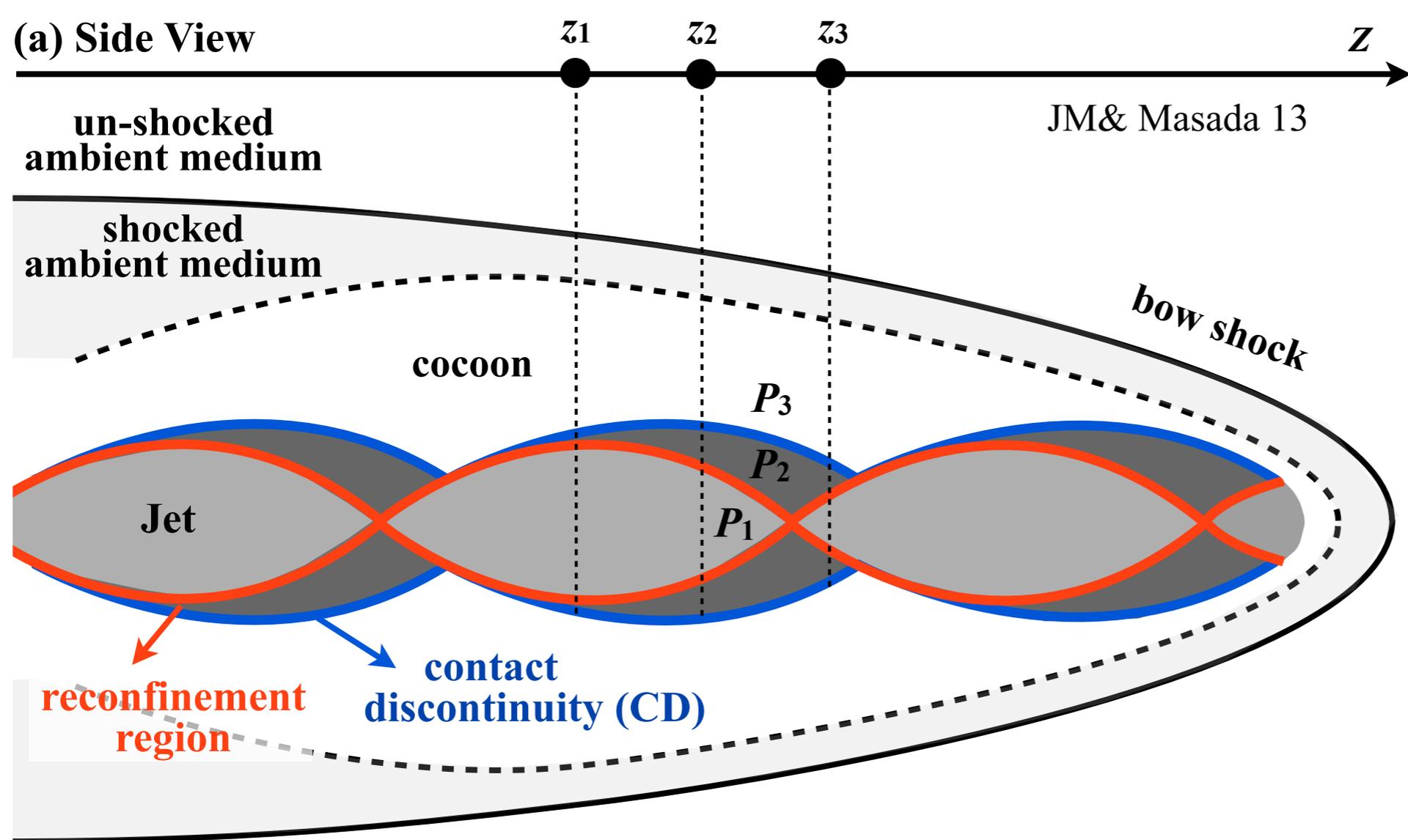
3C 31

AGN jet

schematic picture of the GRB jet



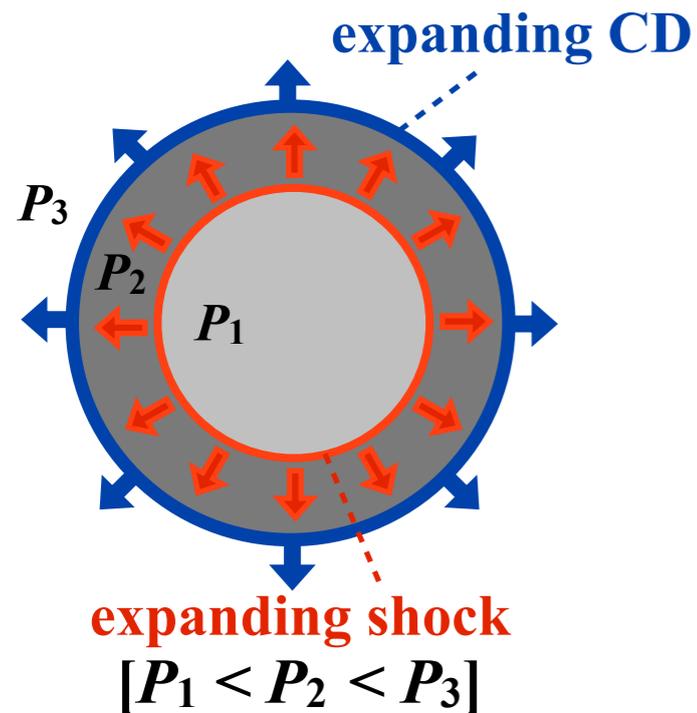
A relativistic jet is considered to be launched from the central engine and propagates the progenitor star.



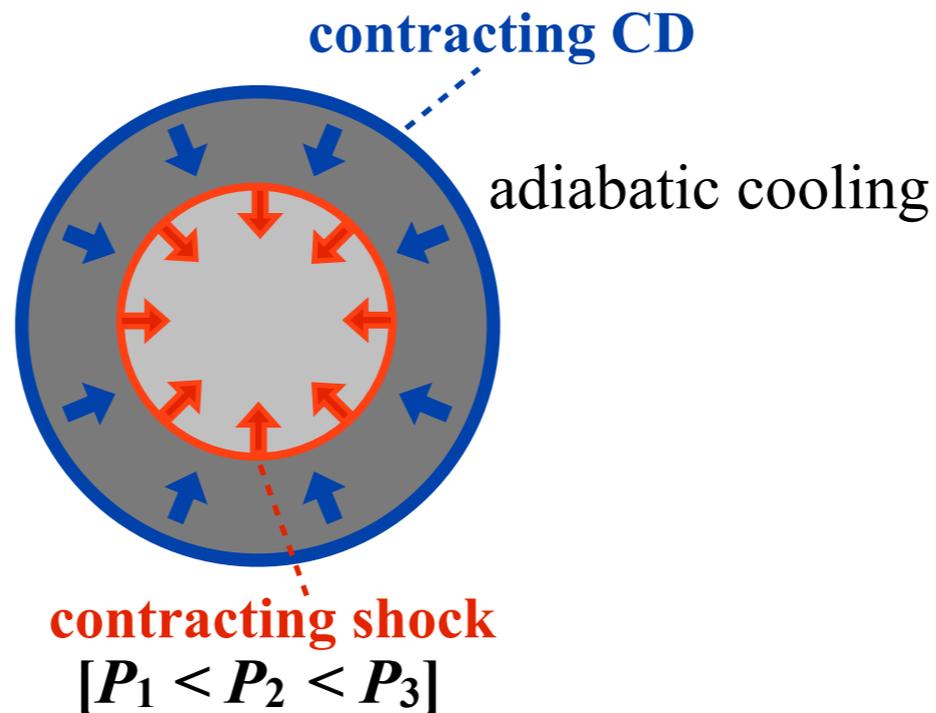
- many numerical works in order to investigate the propagation dynamics of the relativistic jet (e.g., Marti+ 97, Aloy+ 00, Zhang+ 03,04, Mizuta+ 06, Perucho+ 08, Morsony+07, Lazzati+ 09, Lopez-Camara+ 13)
- reconfinement shock in the collimated jet (Norman et al. 1982; Sanders 1983)
- **radial oscillating motion** and repeated excitation of the reconfinement region (e.g., Gomez+ 97, JM+ 12)

(b) Top View

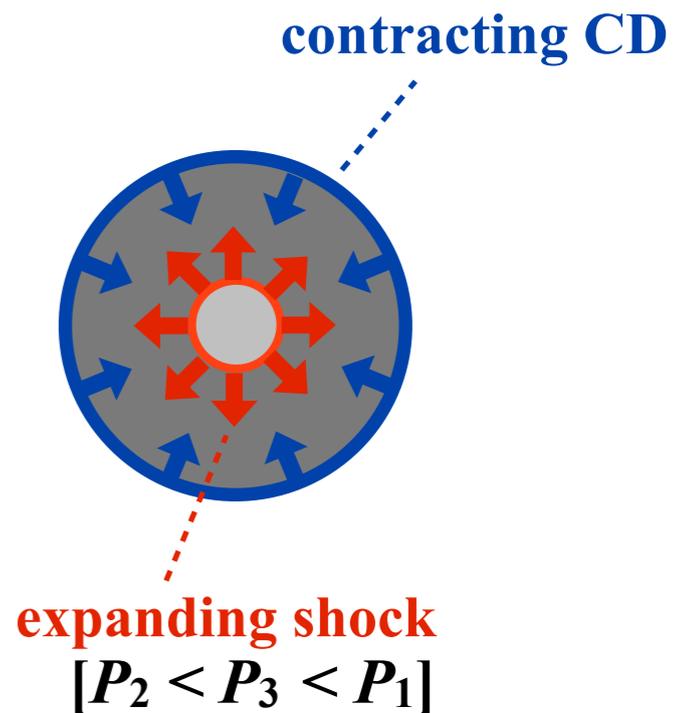
(b1) Expansion Phase [$z=z_1$]



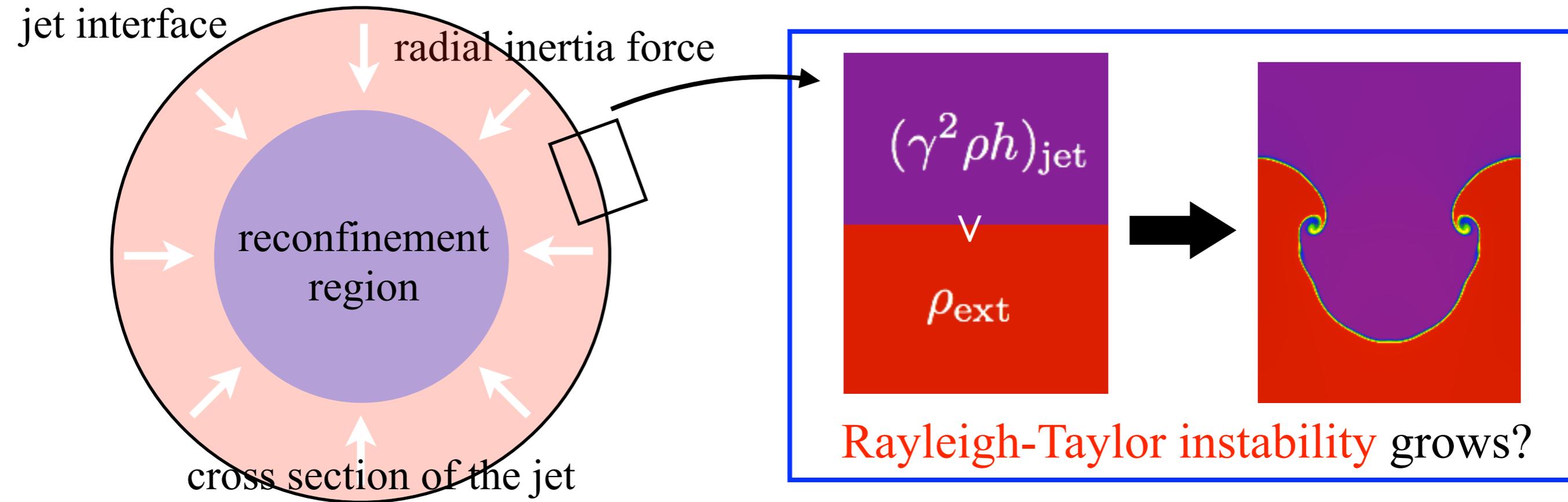
(b2) Contraction Phase (I) [$z=z_2$]



(b3) Contraction Phase (II) [$z=z_3$]



Motivation of Our Study



To investigate the propagation dynamics and stability of the relativistic jet

- using relativistic hydrodynamic simulations

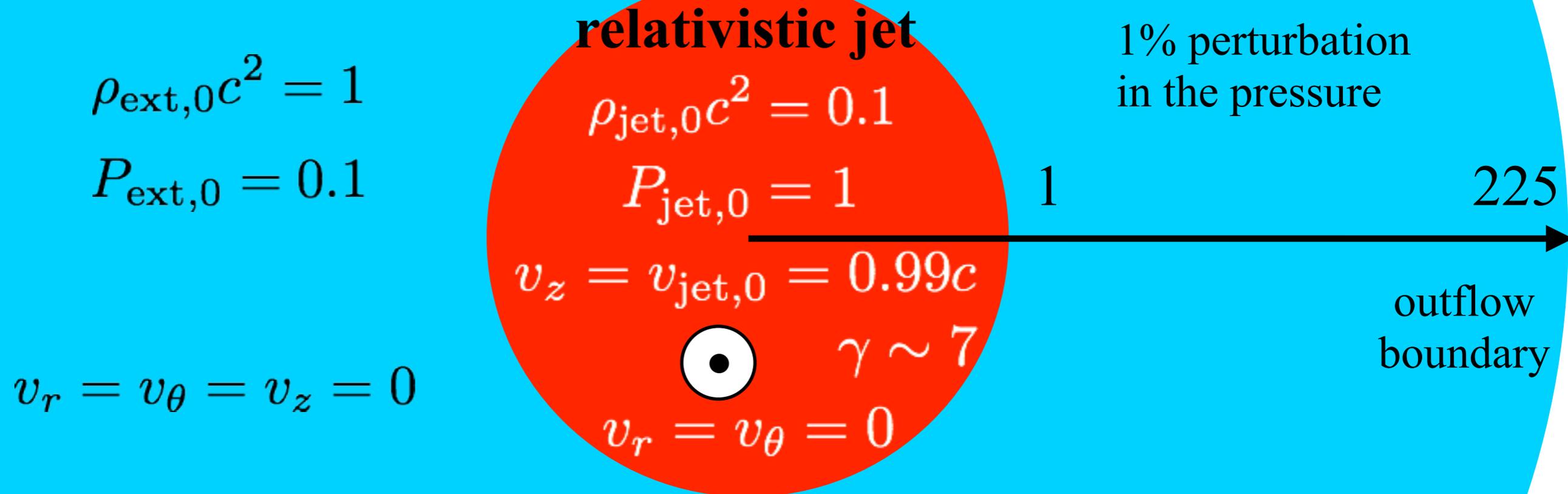
focus on the transverse structure of the jet

- 2D simulations: evolution of the cross section of the relativistic jet
- 3D simulation: propagation of the relativistic jet

2D simulations:
evolution of the cross section of the relativistic jet

JM & Masada, ApJL, **772**, L1 (2013)

Numerical Setting: 2D Toy Model



- cylindrical coordinate: ($r - \theta$ plane)
- relativistically hot jet (z -direction)
- ideal gas
- numerical scheme: HLLC (Mignone & Bodo 05)
- $\Delta r = 20/640$ ($0 < r < 20$), $\Delta \log r = \text{const.}$ ($20 < r < 225$)
- $\Delta \theta = 2\pi/200$

Basic Equations

mass conservation		$\frac{\partial}{\partial t}(\gamma\rho) + \frac{1}{r} \frac{\partial}{\partial r}(r\gamma\rho v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\gamma\rho v_\theta) = 0$	$\frac{\partial}{\partial z}$
momentum conservation	: r	$\frac{\partial}{\partial t}(\gamma^2 \rho h v_r) + \frac{1}{r} \frac{\partial}{\partial r}(r(\gamma^2 \rho h v_r^2 + P)) + \frac{1}{r} \frac{\partial}{\partial \theta}(\gamma^2 \rho h v_r v_\theta) = \frac{P}{r}$	
	: θ	$\frac{\partial}{\partial t}(\gamma^2 \rho h v_\theta) + \frac{1}{r} \frac{\partial}{\partial r}(r(\gamma^2 \rho h v_\theta v_r)) + \frac{1}{r} \frac{\partial}{\partial \theta}(\gamma^2 \rho h v_\theta^2 + P) = -\frac{\gamma^2 \rho h v_r v_\theta}{r}$	
	: z	$\frac{\partial}{\partial t}(\gamma^2 \rho h v_z) + \frac{1}{r} \frac{\partial}{\partial r}(r(\gamma^2 \rho h v_z v_r)) + \frac{1}{r} \frac{\partial}{\partial \theta}(\gamma^2 \rho h v_z v_\theta) = 0$	
energy conservation		$\frac{\partial}{\partial t}(\gamma^2 \rho h - P) + \frac{1}{r} \frac{\partial}{\partial r}(r(\gamma^2 \rho h v_r)) + \frac{1}{r} \frac{\partial}{\partial \theta}(\gamma^2 \rho h v_\theta) = 0$	

specific enthalpy

$$\frac{h}{c^2} = 1 + \frac{\Gamma}{\Gamma - 1} \frac{P}{\rho c^2}$$

ratio of specific heats

$$\Gamma = \frac{4}{3}$$

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - (v_r^2 + v_\theta^2 + v_z^2)}}$$

Time Evolution of Jet Cross Section

The effective inertia is important.

relativistically hot plasma:

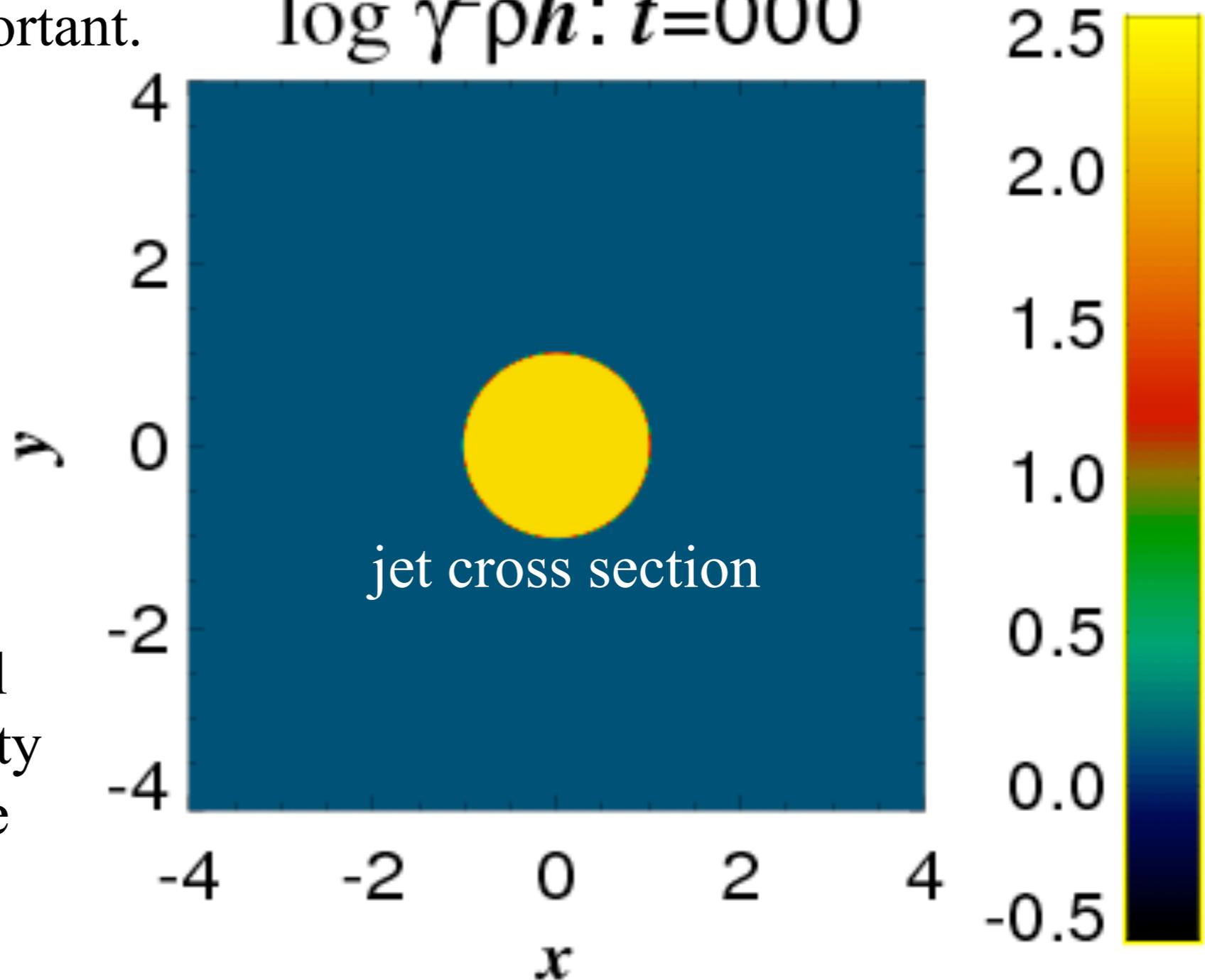
$$\rho_{\text{jet}} c^2 \leq P_{\text{jet}}$$

effective inertia:

$$\gamma^2 \rho h = \gamma^2 (\rho c^2 + 4P)$$

The effective inertia of the jet is larger than the external medium although the density of the jet is smaller than the external medium in our setting.

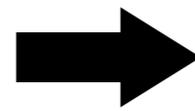
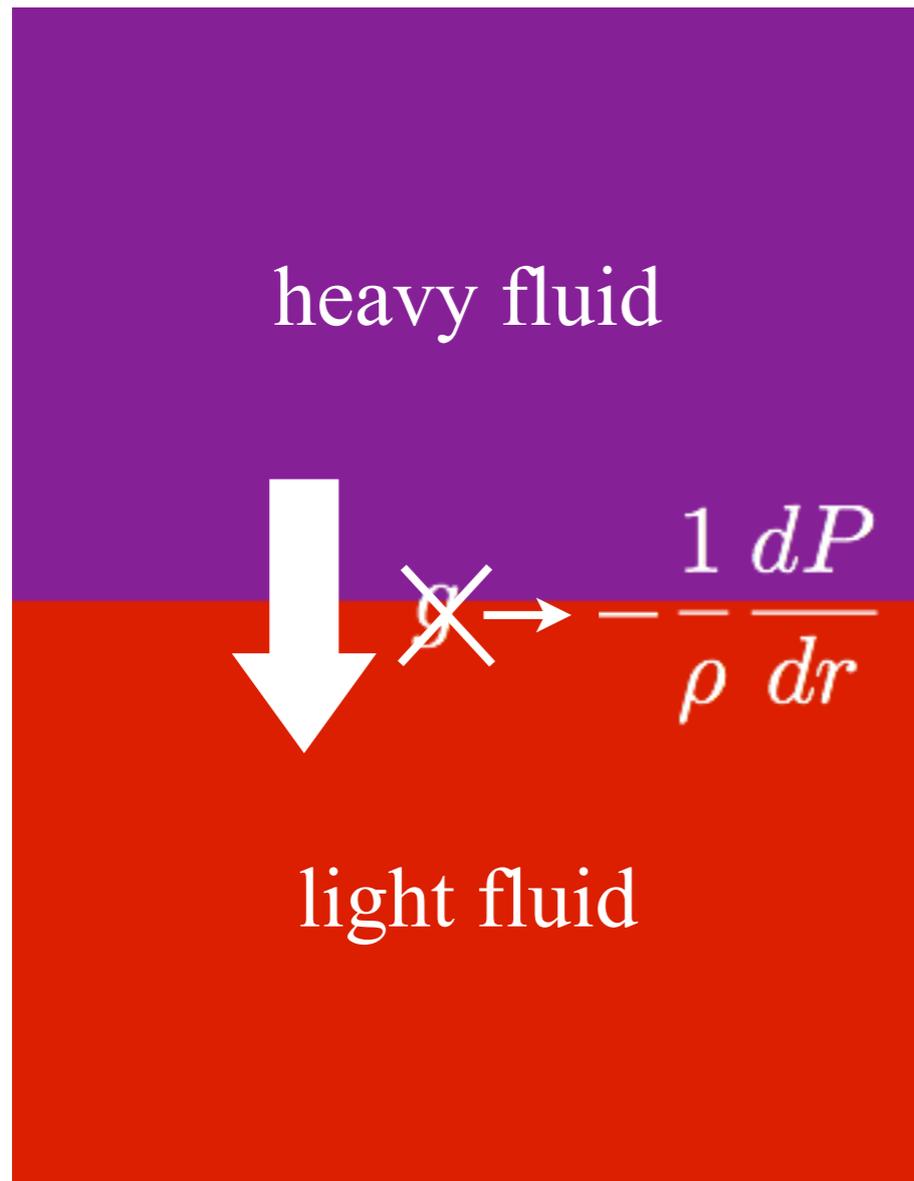
$\log \gamma^2 \rho h: t=000$



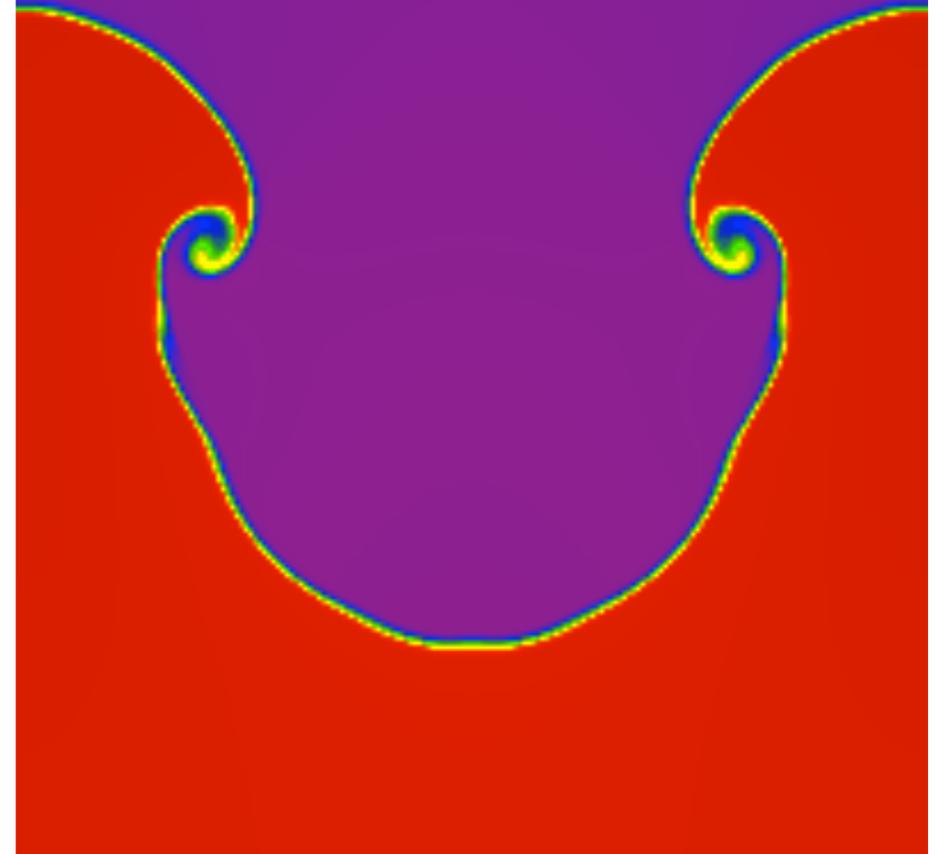
The amplitude of the corrugated jet interface grows as time passes.

A finger-like structure is a typical outcome of the Rayleigh-Taylor instability.

Growth of RTI in SN Explosion



The condition for the RTI

$$\frac{dP}{dr} \frac{d\rho}{dr} < 0, \quad (\text{Chevalier 76})$$


It is well known that a contact discontinuity, formed where a heavy fluid is supported above a light fluid against gravity, is unstable to perturbations to the boundary between the two fluids.

Although the gravity is negligible in SN explosions, the acceleration of the gas works as a gravity.

The mechanism of the growth of RTI

The effective inertia is important.

relativistically hot plasma:

$$\rho_{\text{jet}} c^2 \leq P_{\text{jet}}$$

effective inertia:

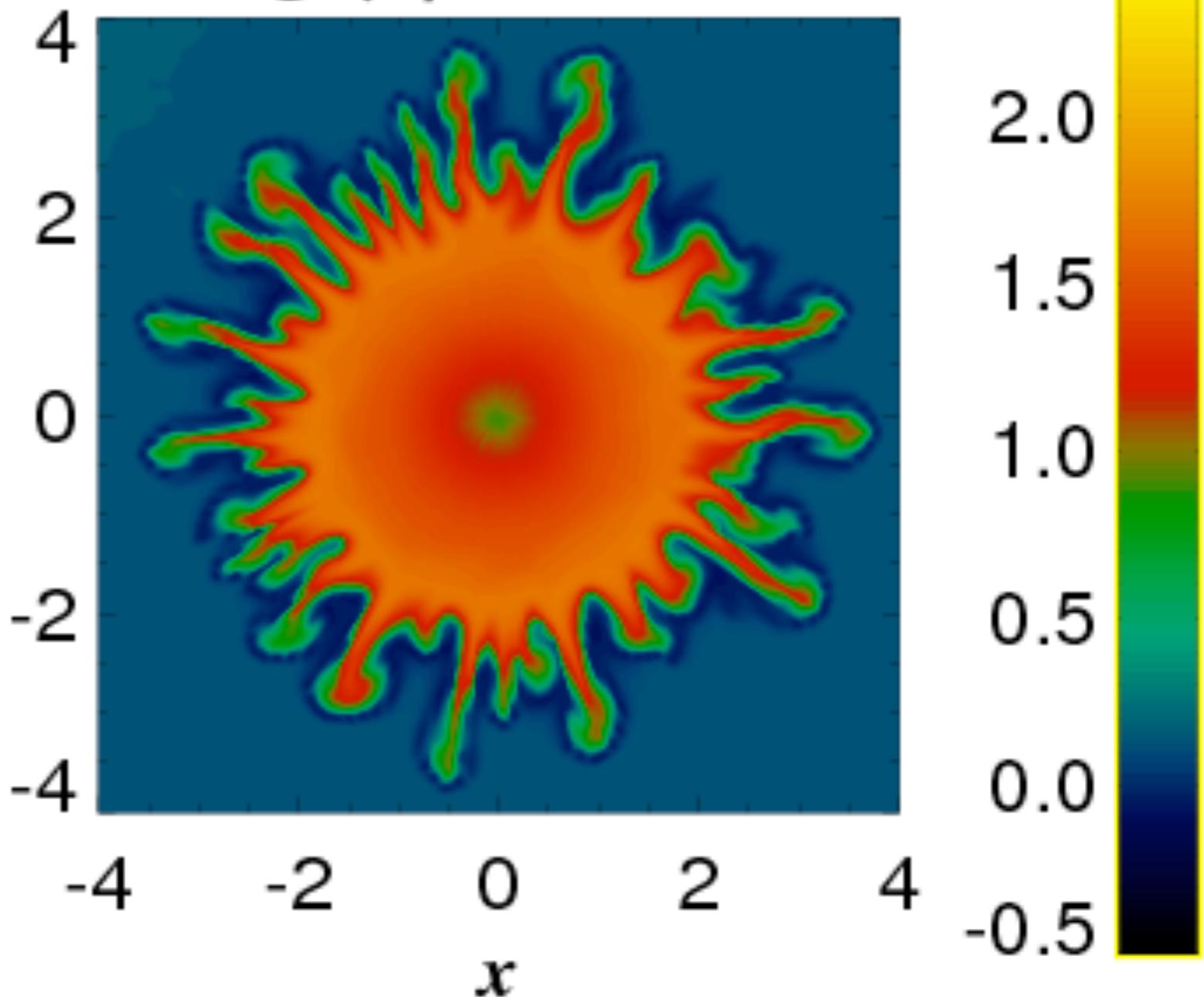
$$\gamma^2 \rho h = \gamma^2 (\rho c^2 + 4P)$$

$$\frac{d\rho}{dr} > 0, \quad \frac{dP}{dr} > 0, \quad \frac{d(\gamma^2 \rho h)}{dr} < 0$$

$$\frac{dP}{dr} \frac{d\rho}{dr} > 0$$

$$\frac{dP}{dr} \frac{d(\gamma^2 \rho h)}{dr} < 0$$

$\log \gamma^2 \rho h: t=120$



In addition to the growth of the Rayleigh-Taylor instability, the growth of the Richtmyer-Meshkov instability is also contributed to the finger like structures.

Richtmyer-Meshkov Instability



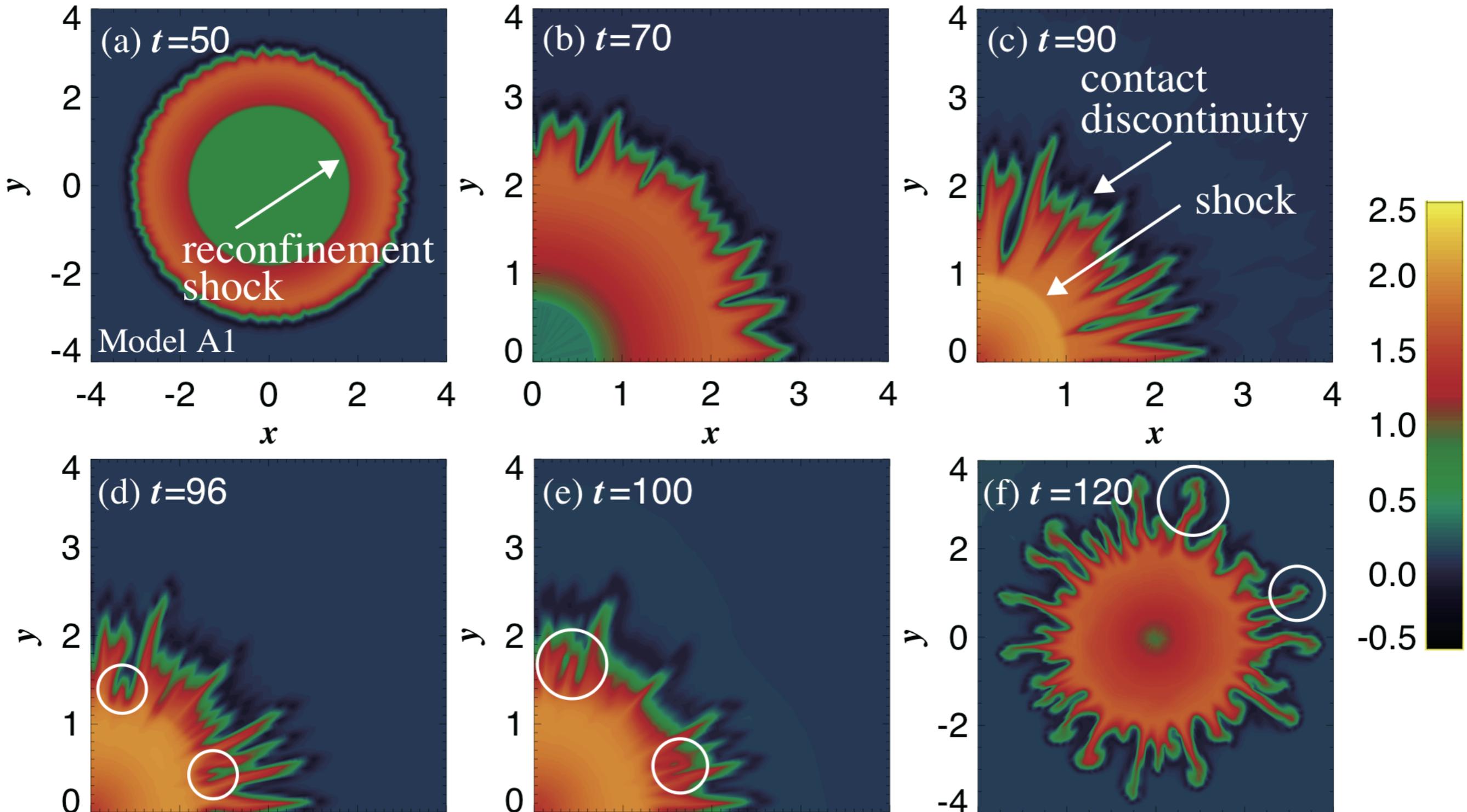
contact discontinuity

- The Richtmyer-Meshkov instability is induced by impulsive acceleration due to shock passage.
- The perturbation amplitude grows linearly in time (Richtmyer 1960)

$$\frac{\partial \delta}{\partial t} = k \delta_0^* A^* v^* , \quad A^* = \frac{\rho_1^* - \rho_2^*}{\rho_1^* + \rho_2^*}$$

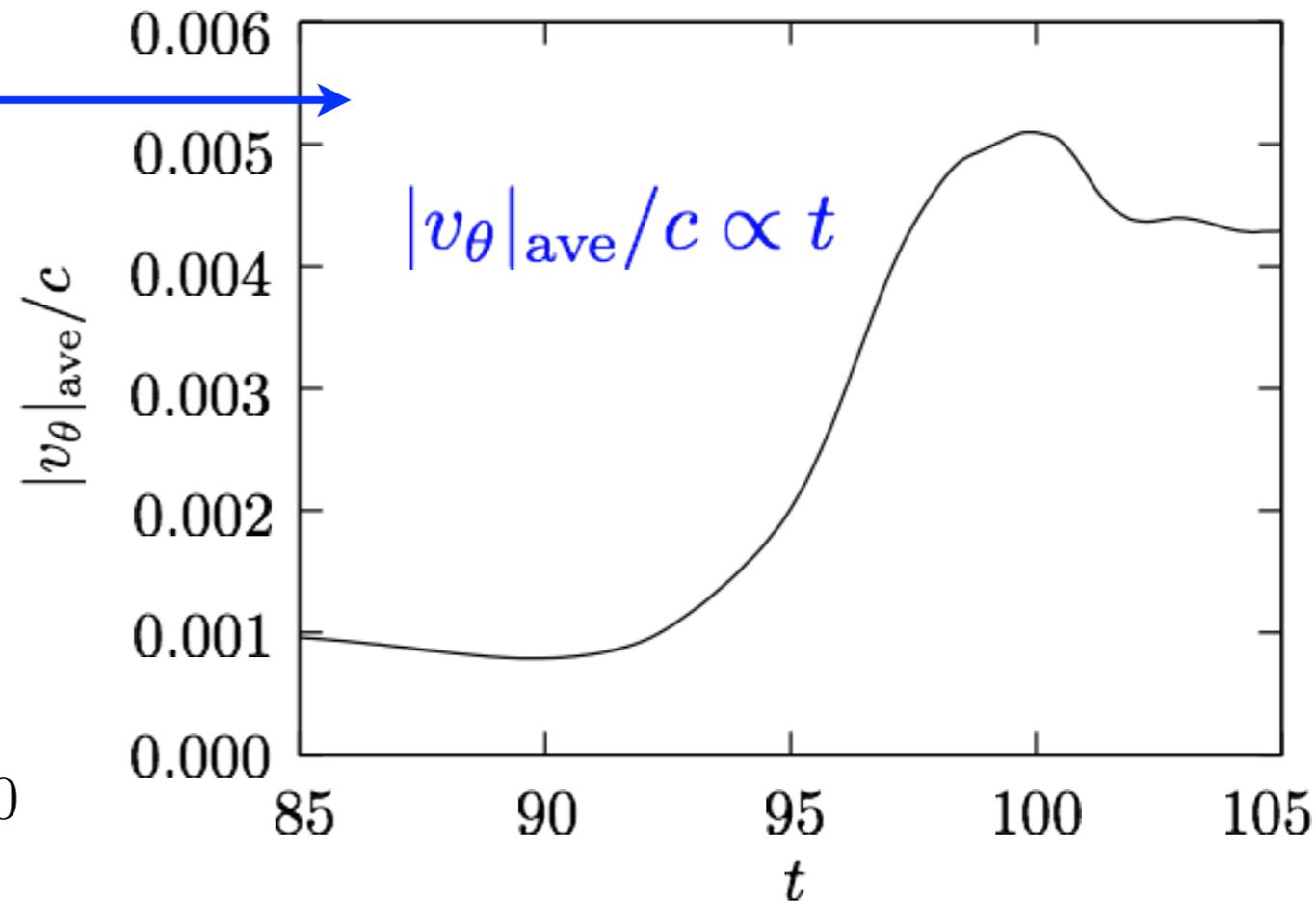
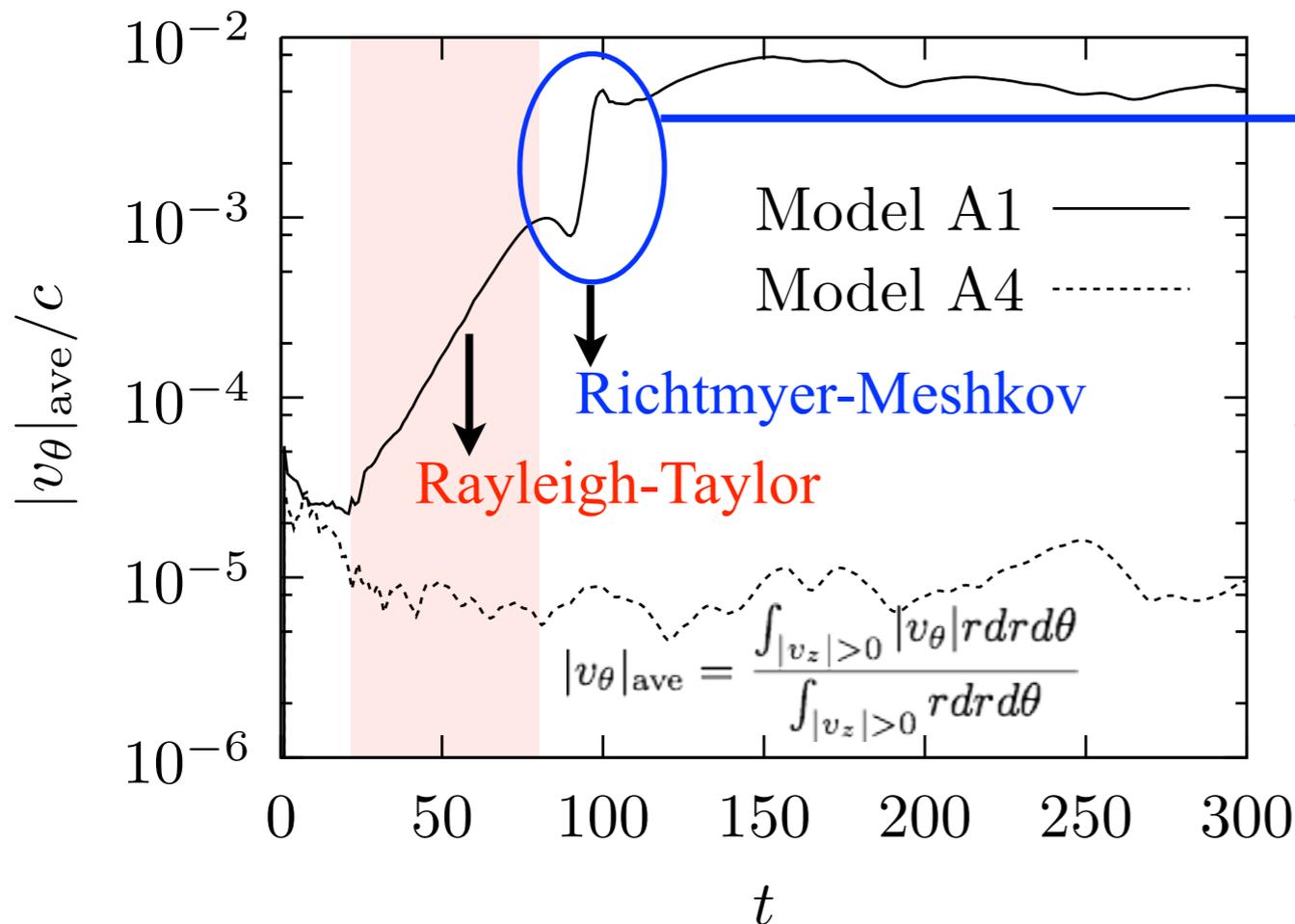
Time Evolution of Jet Cross Section

Effective inertia: $\log \gamma^2 \rho h$



Richtmyer-Meshkov instability is secondary excited between the RTI fingers.
Almost all finger-like structures in panel (f) have their origin in the RMI.

Synergetic Growth of Rayleigh-Taylor and Richtmyer-Meshkov Instabilities



development of the **Rayleigh-Taylor instability** at the jet interface

$|v_\theta|_{\text{ave}}$ increases exponentially.

excitation of the **Richtmyer-Meshkov instability** at the jet interface

$|v_\theta|_{\text{ave}}$ grows linearly with time.

The transverse structure of the jet is dramatically deformed by a synergetic growth of the Rayleigh-Taylor and Richtmyer-Meshkov instabilities once the jet-external medium interface is corrugated in the case with the pressure-mismatched jet.

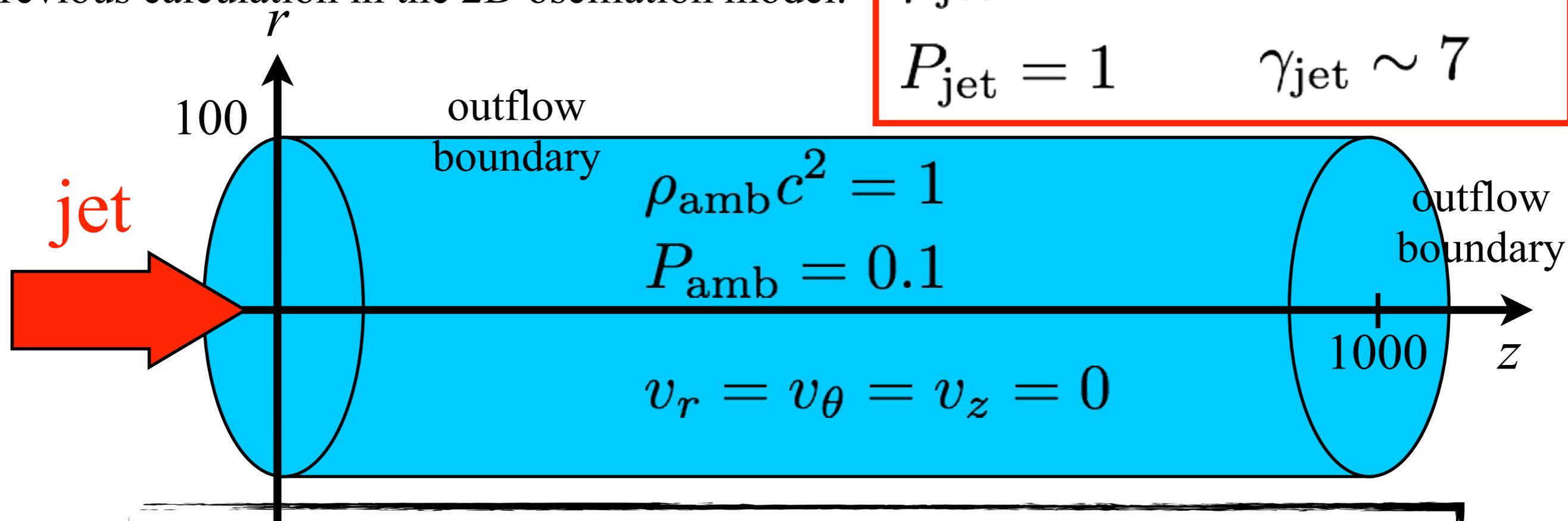
3D simulation: propagation of the relativistic jet

focusing on the impact of the oscillation-induced
Rayleigh-Taylor and Richtmyer-Meshkov instabilities on
the 3D jet propagation.

Numerical Setting: 3D Toy Model

Hydrodynamic variables are the same as the previous calculation in the 2D oscillation model.

$$\rho_{\text{jet}} c^2 = 0.1 \quad v_z = 0.99c$$
$$P_{\text{jet}} = 1 \quad \gamma_{\text{jet}} \sim 7$$



- cylindrical coordinate
- relativistic jet (z -direction)
- ideal gas
- numerical scheme: HLLC (Mignone & Bodo 05)
- uniform grid: $\Delta r = 0.0666$, $\Delta \theta = 2\pi/160$, $\Delta z = 1$

Basic Equations

mass
conservation

$$\frac{\partial}{\partial t}(\gamma\rho) + \nabla \cdot (\gamma\rho\mathbf{v}) = 0$$

momentum
conservation

$$\frac{\partial}{\partial t}(\gamma^2\rho h\mathbf{v}) + \nabla \cdot (\gamma^2\rho h\mathbf{v}\mathbf{v} + Pc^2\mathbf{I}) = 0$$

energy
conservation

$$\frac{\partial}{\partial t}(\gamma^2\rho h - P) + \nabla \cdot (\gamma^2\rho h\mathbf{v}) = 0$$

specific enthalpy

$$\frac{h}{c^2} = 1 + \frac{\Gamma}{\Gamma - 1} \frac{P}{\rho c^2}$$

ratio of specific heats

$$\Gamma = \frac{4}{3}$$

Lorentz factor

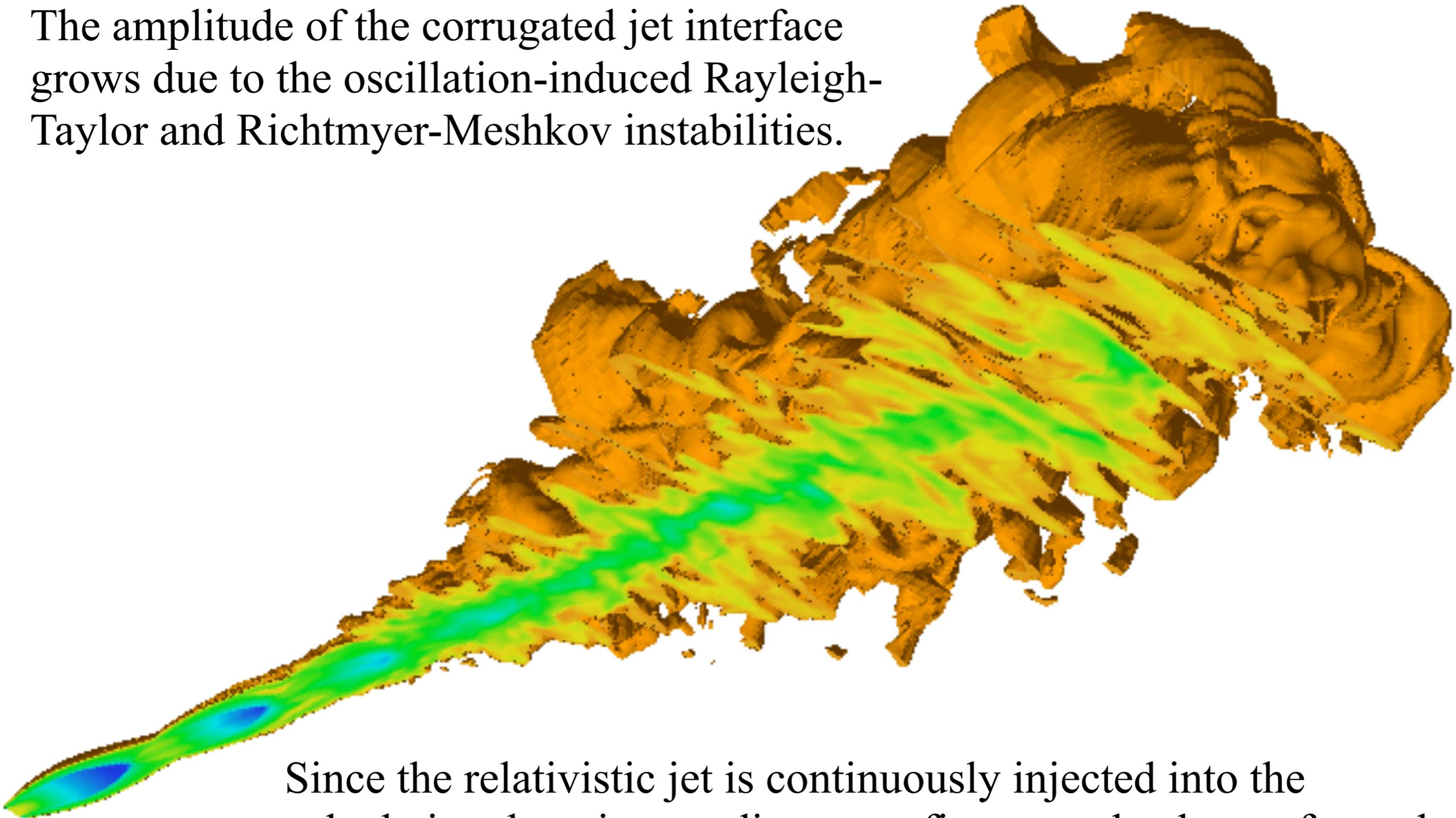
$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

In 3D case, we calculated the numerical flux to all directions.

Result: Density

Only the jet component is shown.

The amplitude of the corrugated jet interface grows due to the oscillation-induced Rayleigh-Taylor and Richtmyer-Meshkov instabilities.

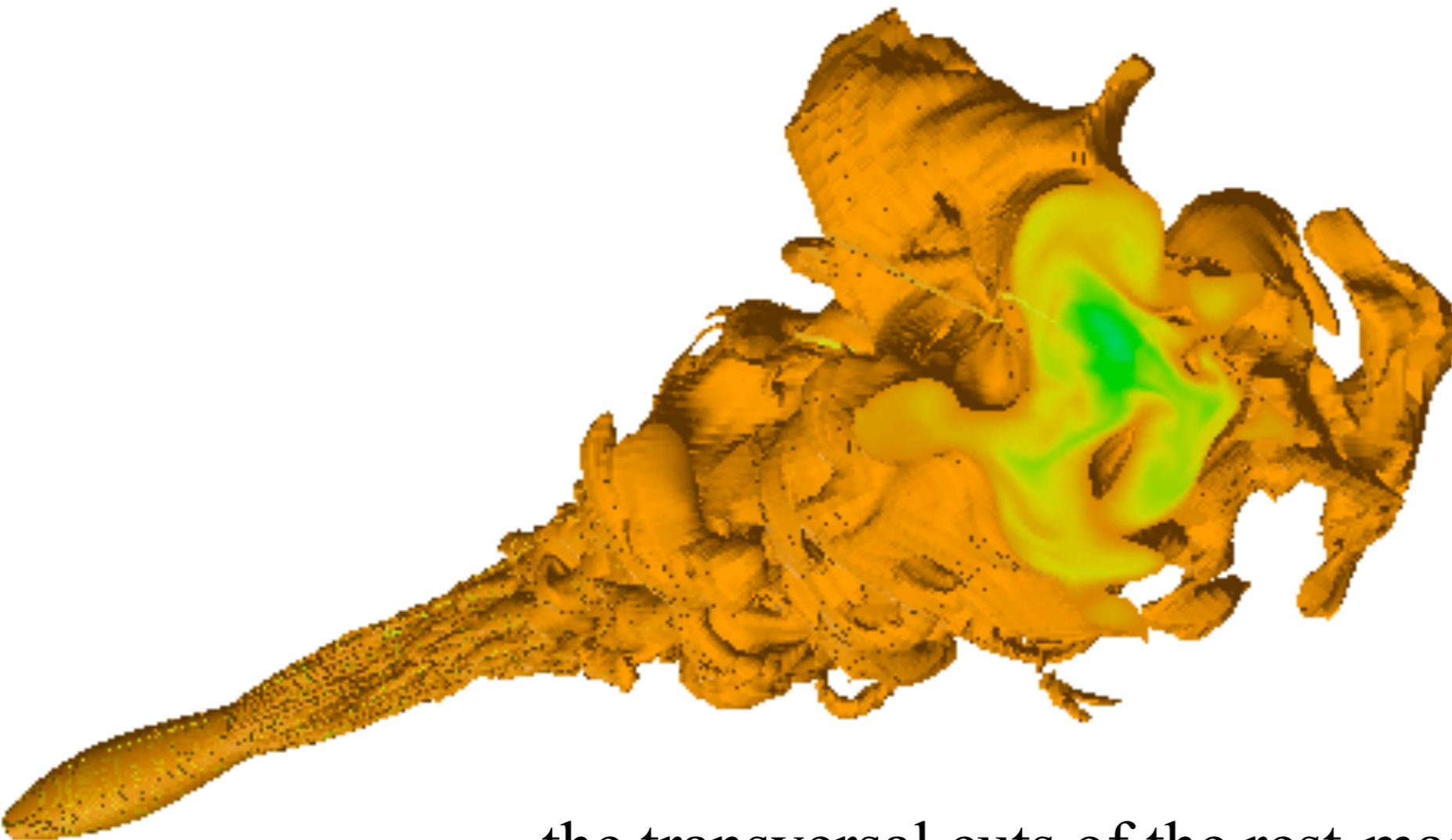


Since the relativistic jet is continuously injected into the calculation domain, standing reconfinement shocks are formed.

Result: Density

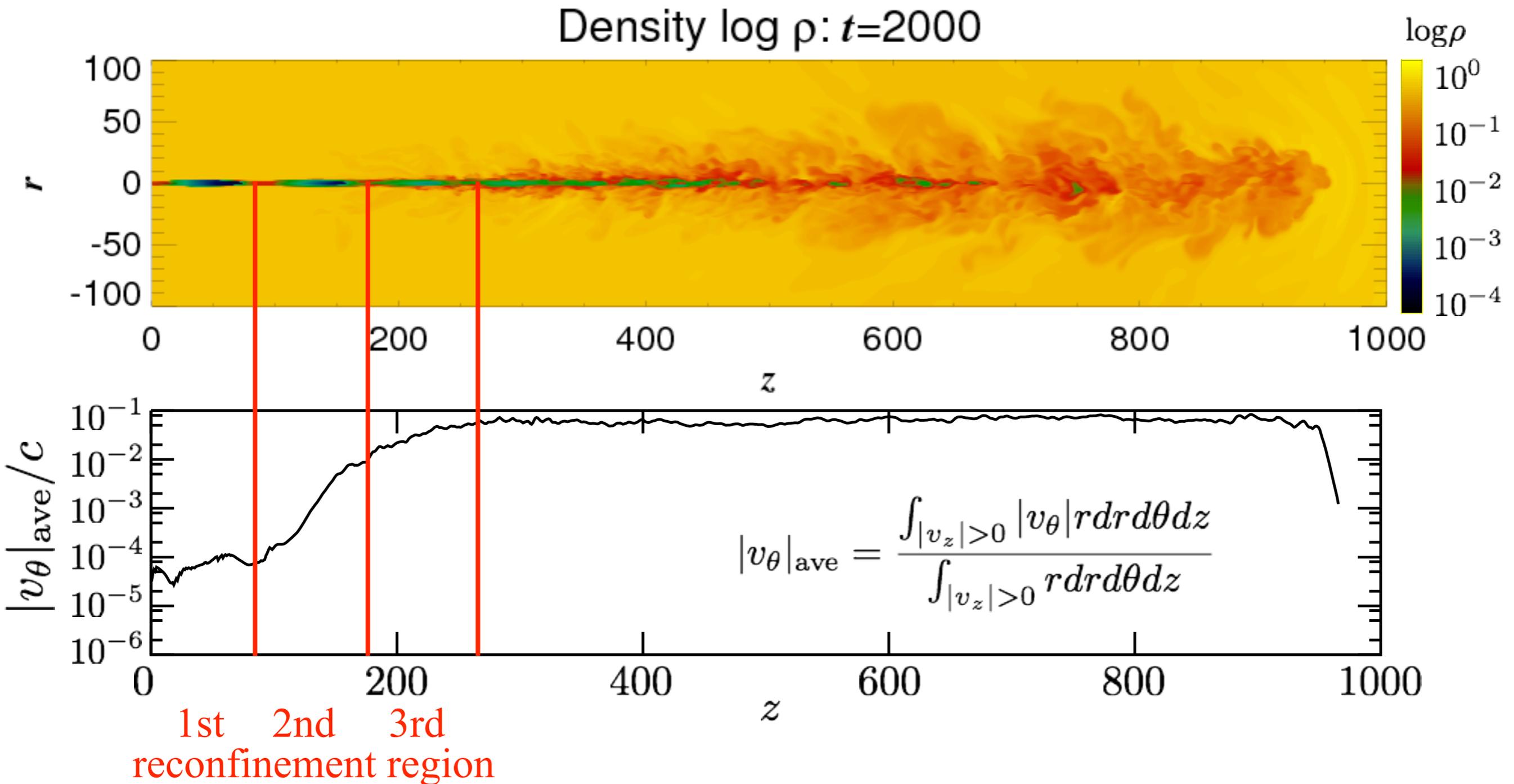
Only the jet component is shown.

We confirmed that the Rayleigh-Taylor and Richtmyer-Meshkov instabilities grows at the interface of the jet. The material mixing due to the Rayleigh-Taylor and Richtmyer-Meshkov instabilities between the jet and surrounding medium leads to the deformation of the jet.



the transversal cuts of the rest-mass density

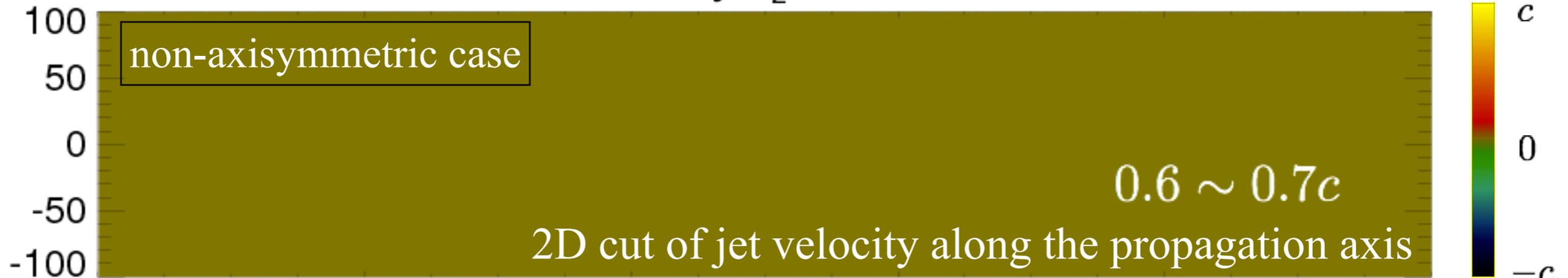
Spatial Growth Rate of Azimuthal Velocity



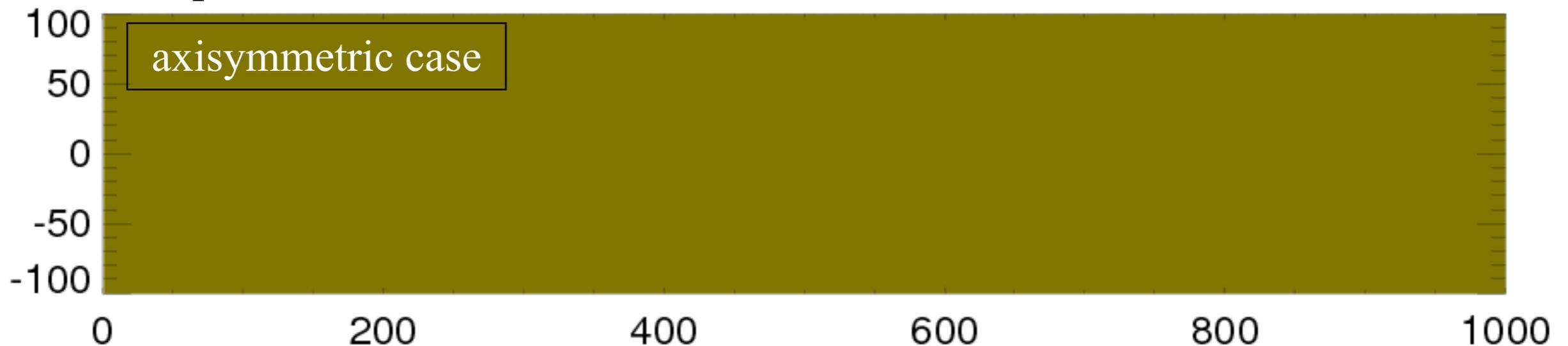
- Growth of the volume-averaged azimuthal velocity is saturated around the 3rd reconfinement region.

non-Axisymmetric vs Axisymmetric

Velocity v_z : $t=0000$



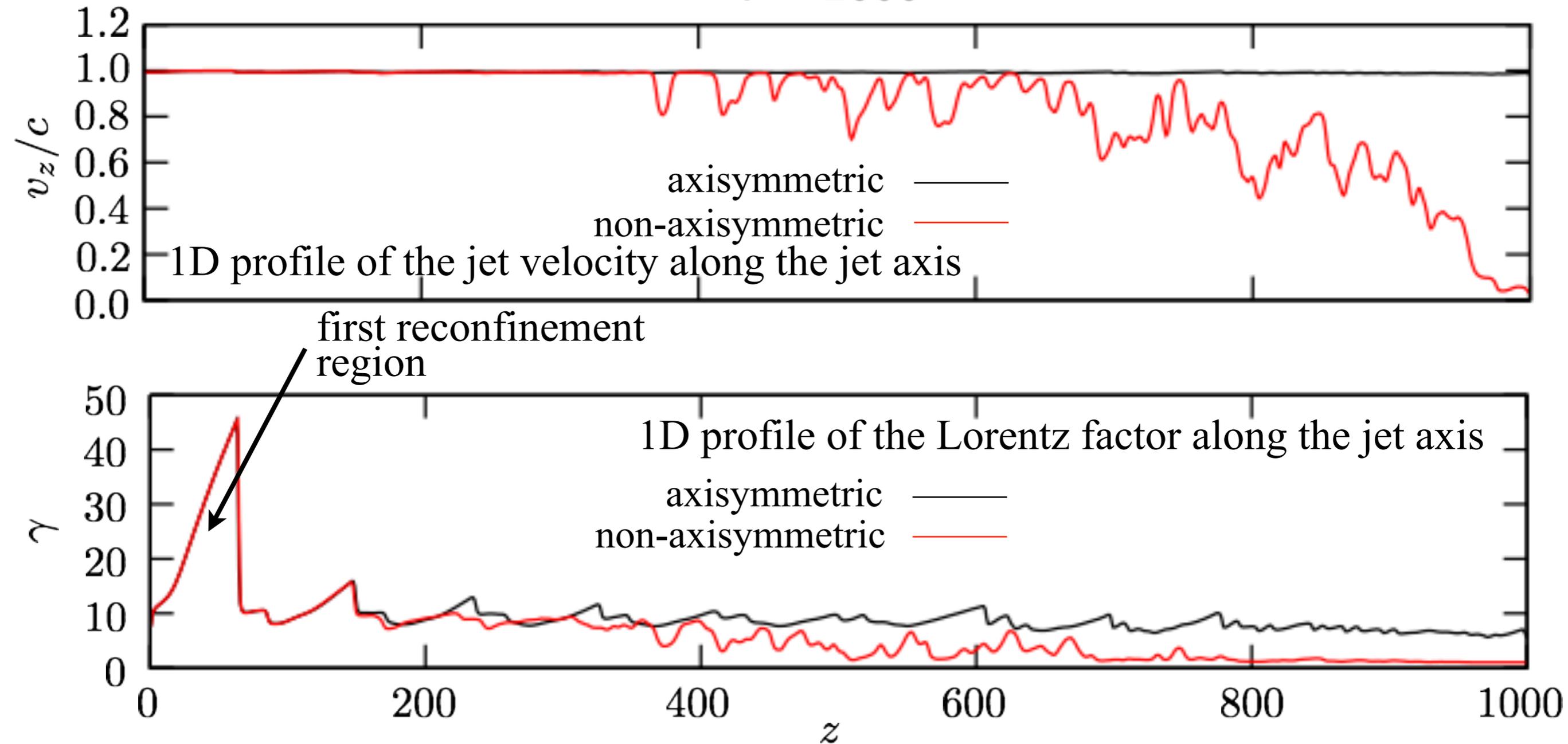
The numerical settings of the axisymmetric case are same as the 3D case except the azimuthal direction.



- deceleration of the jet due to the mixing between the jet and surrounding medium in the non-axisymmetric case.
- Three-dimensionality is essential for the non-linear dynamics and stability of the relativistic jet.

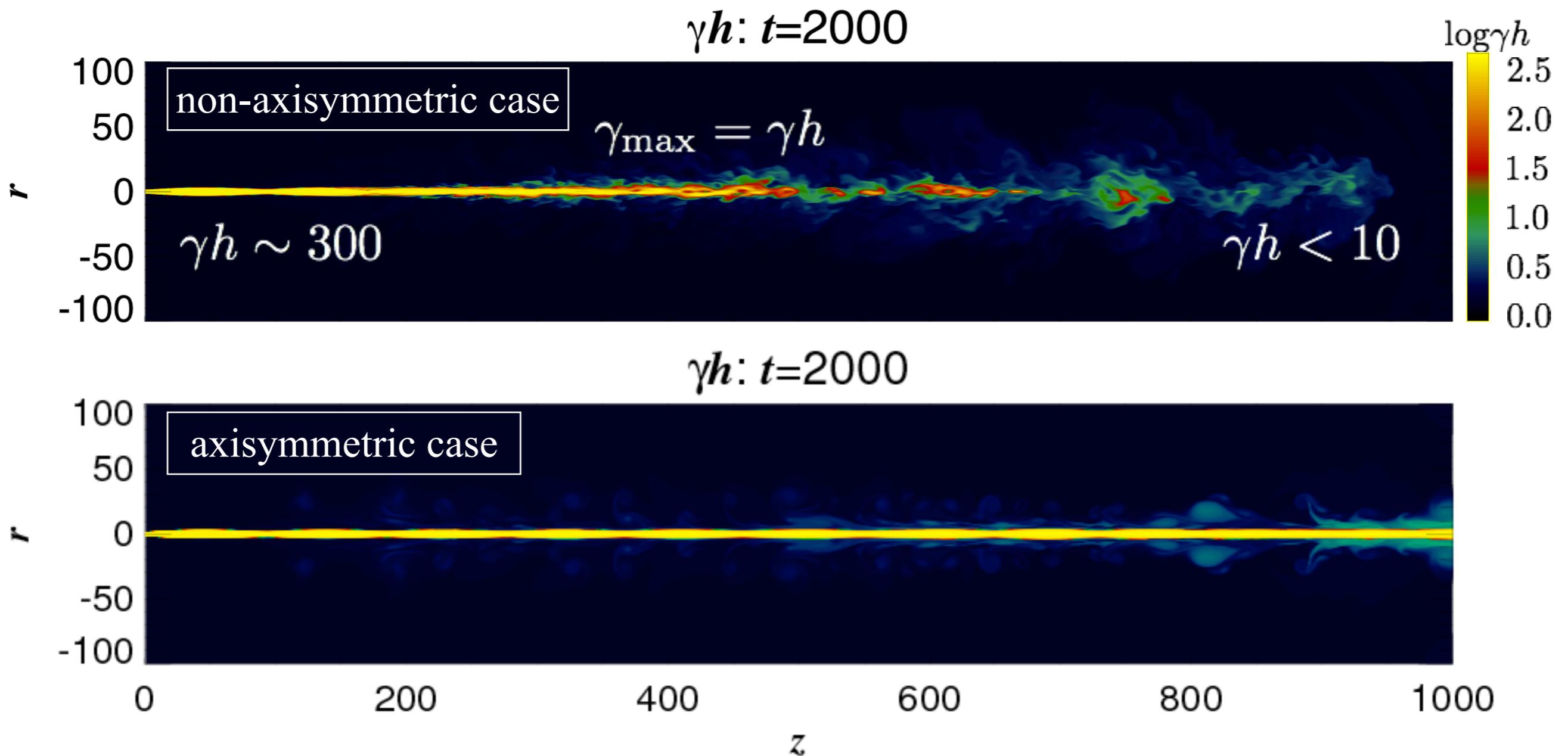
Deceleration of the jet due to mixing

$t = 2000$



- deceleration of the jet due to the mixing between the jet and surrounding medium in the non-axisymmetric case.
- Three-dimensionality is essential for the non-linear dynamics and stability of the relativistic jet.

Deceleration of the jet due to mixing



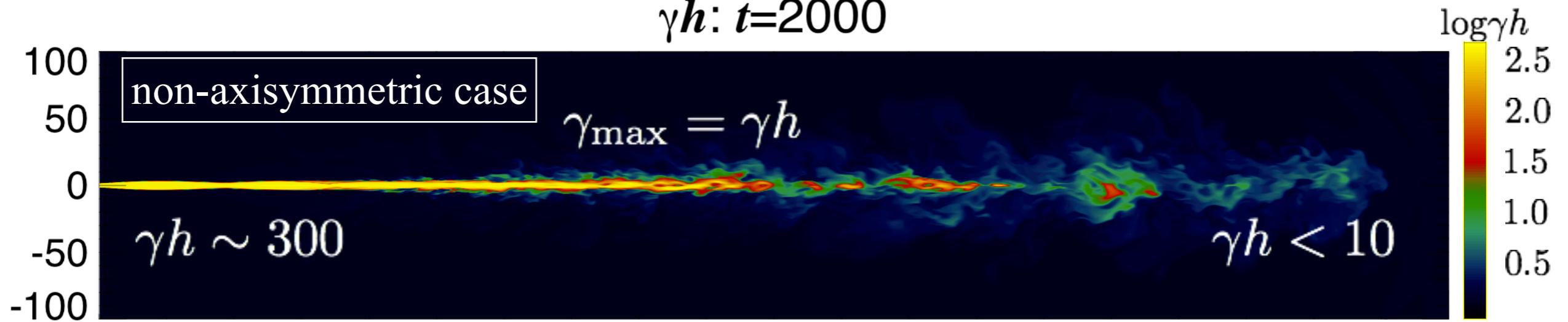
■ relativistic Bernoulli equation: $\gamma h \sim \text{const.}$

γh gives the maximum Lorentz factor of the jet after adiabatic expansion.

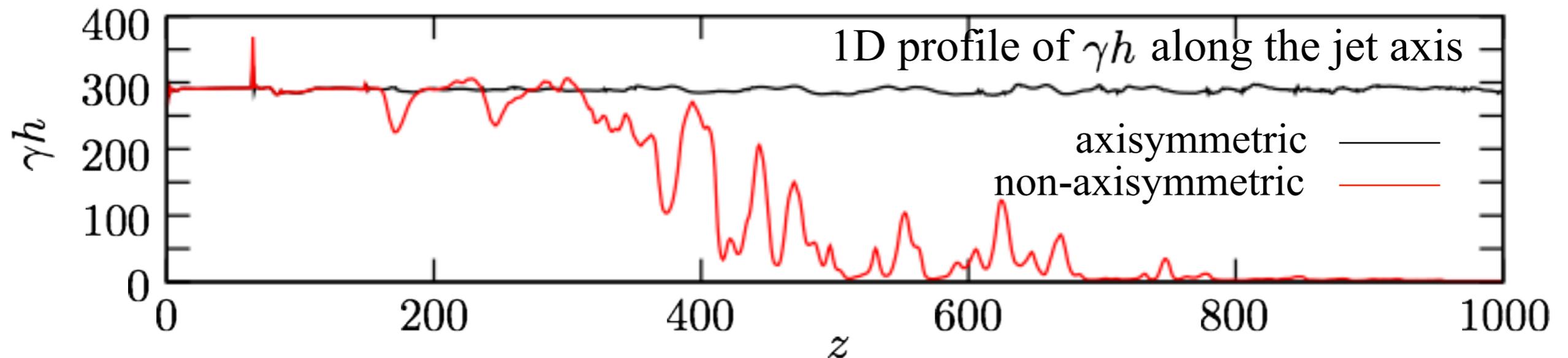
However, γh drops to ~ 10 due to the mixing in this case.

Deceleration of the jet due to mixing

$\gamma h: t=2000$



$t = 2000$



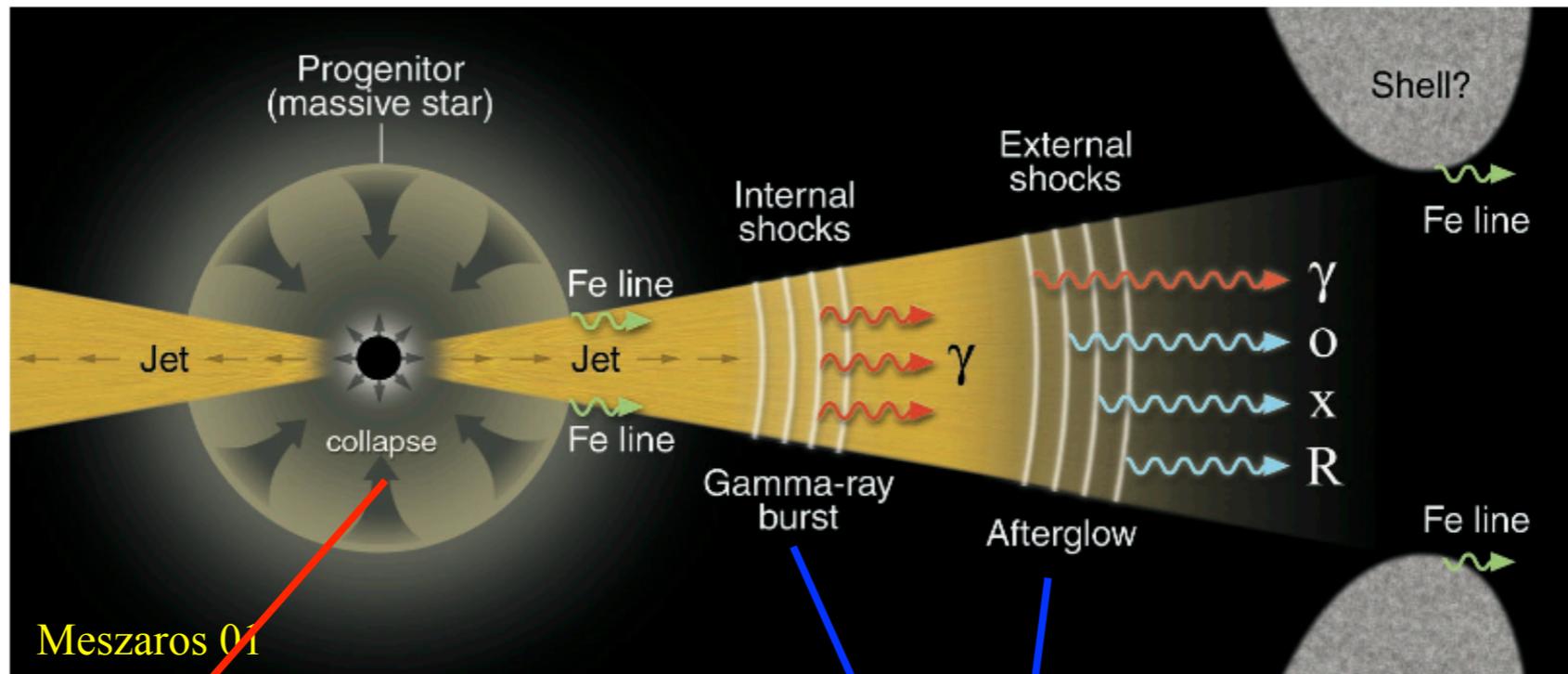
- relativistic Bernoulli equation: $\gamma h \sim \text{const.}$

γh gives the maximum Lorentz factor of the jet after adiabatic expansion.

However, γh drops to ~ 10 due to the mixing in this case.

Magnetically Driven Jet?

schematic picture of the GRB jet



Central engine

■ MHD process

Extraction of the rotational energy of the BH or accretion disk through B-field

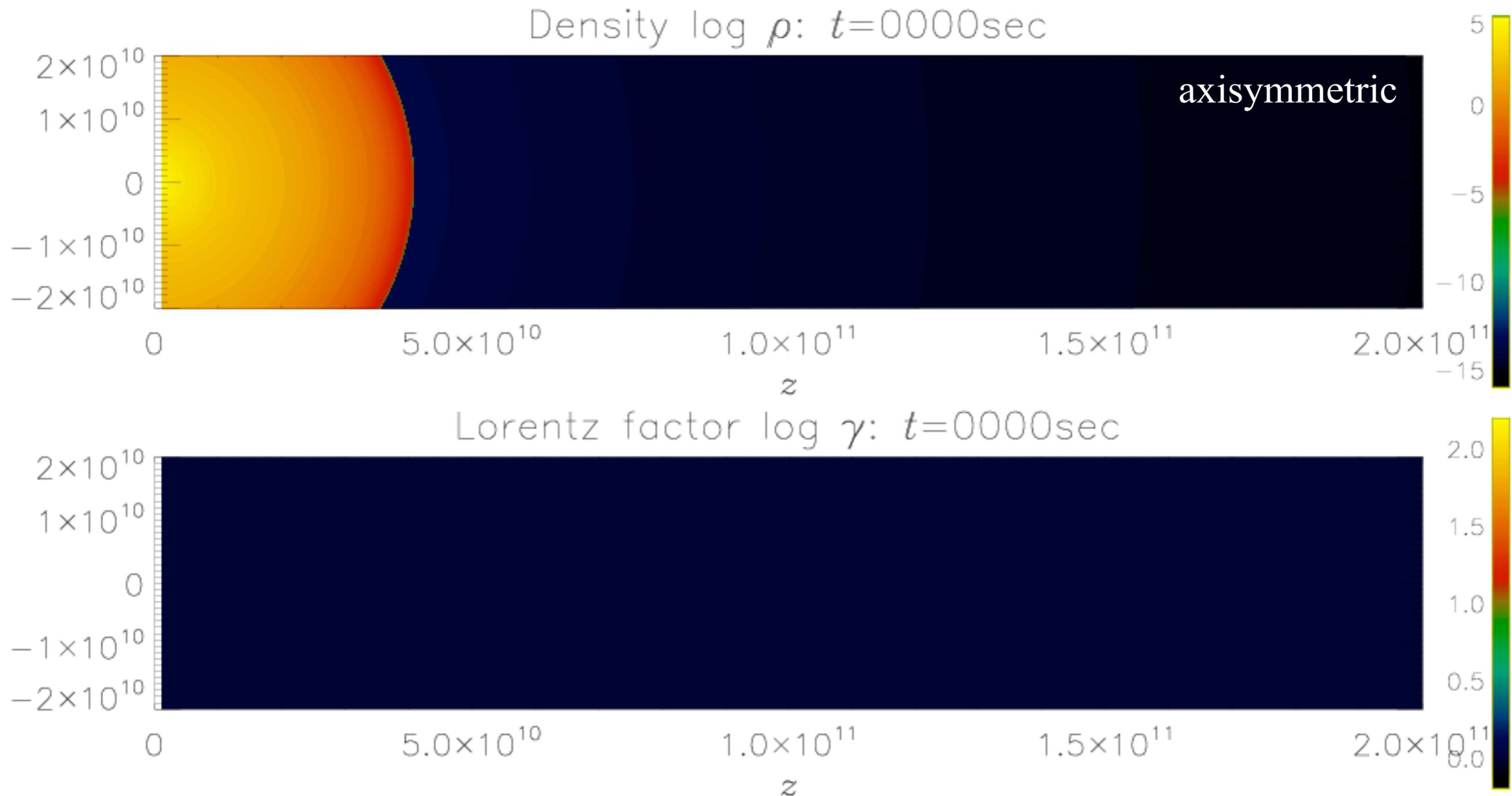
■ thermal process

$\nu\bar{\nu}$ -annihilation $\rightarrow e^+e^-$ -annihilation
 \rightarrow fireball

Observation

- polarization in prompt phase
(Yonetoku et al. 2011, 2012)
- polarization in afterglow phase
(Mundell et al. 2014,
Wiersema et al. 2014)

Propagation of MHD jet for GRB



progenitor: 16TI model (Woosley & Heger 06)

jet:

$$L_{\text{jet}} = 10^{51} \text{ erg/s}$$

$$\gamma_{\text{jet}} = 5$$

$$\beta_{\text{jet}} = P_{\text{gas}}/P_{\text{mag}} = 1$$

$$h_{\text{jet}}/\sigma_{\text{jet}} \sim 2$$

$$B_{\text{jet}} = B_{\phi}$$

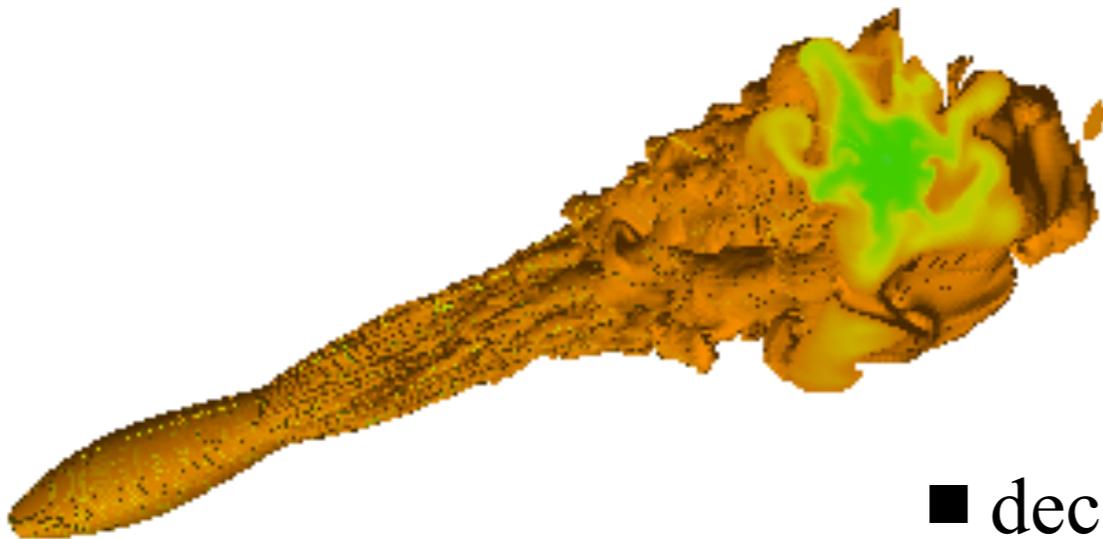
Summary

Basic physics of the propagation of the relativistic jet is investigated through 3D HD/2D MHD numerical simulations.

- A pressure mismatch between the jet and surrounding medium leads to the radial oscillating motion of the jet.

- The jet-external medium interface is unstable due to the oscillation-induced

← { **Rayleigh-Taylor instability**
Richtmyer-Meshkov instability



- deceleration of the jet due to the mixing between the jet and surrounding medium

Next Study:

- more realistic situation for relativistic MHD jets in the context of GRBs