Particle acceleration in superluminal strong waves

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Programs for Junior Scientists

Superluminal strong waves?

Electromagnetic waves with

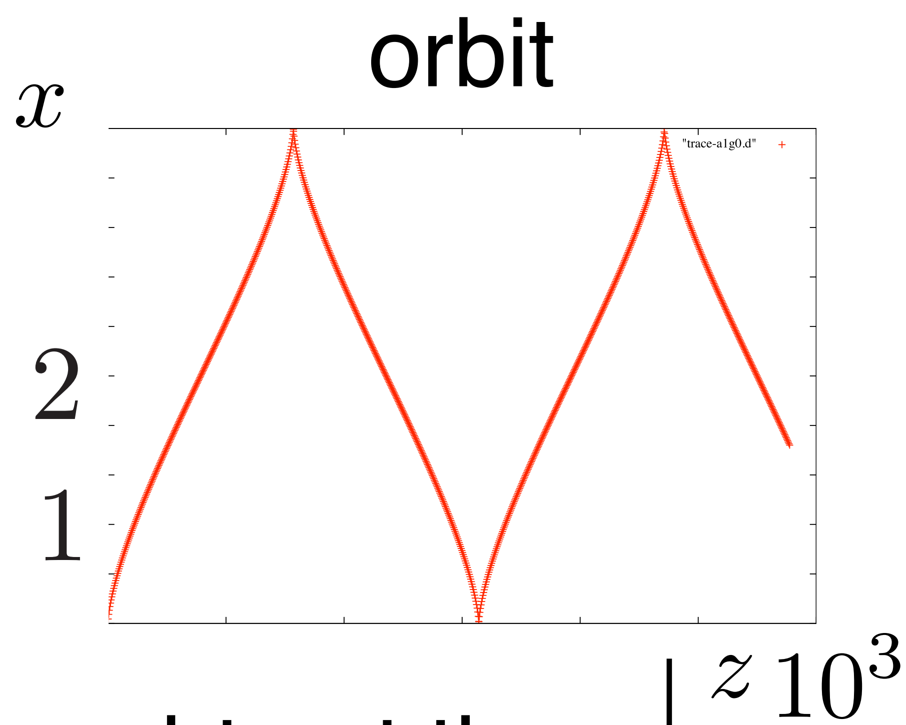
$$a \equiv \frac{eE}{mc\omega} > 1 \quad \& \quad v_{\rm ph} > c$$

$a > 1$ → Particles become relativistic

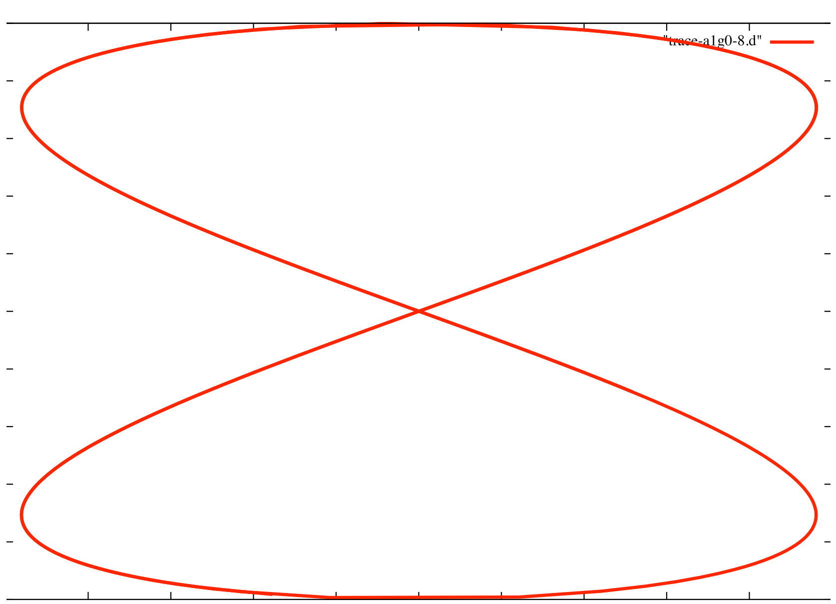
$$\Delta\gamma mc^2 = eE\frac{c}{\omega} \longleftrightarrow \Delta\gamma = \frac{eE}{mc\omega} > 1$$

$\frac{\omega}{k} \equiv v_{\rm ph} > c$ ← Dispersion relation $\omega^2 = \frac{2\omega_{\rm p}^2}{\sqrt{1+a^2}} + k^2c^2$

Trapping effect



subtract the mean motion



Linearly polarized, monochromatic plane wave

$$E = E_x \quad B = B_y \quad k = k_z$$

Not a harmonic oscillation!

$$\frac{d}{dt}(\gamma m_e \vec{v}) = -e(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$$

$$\gamma_{\max} \sim a^2$$

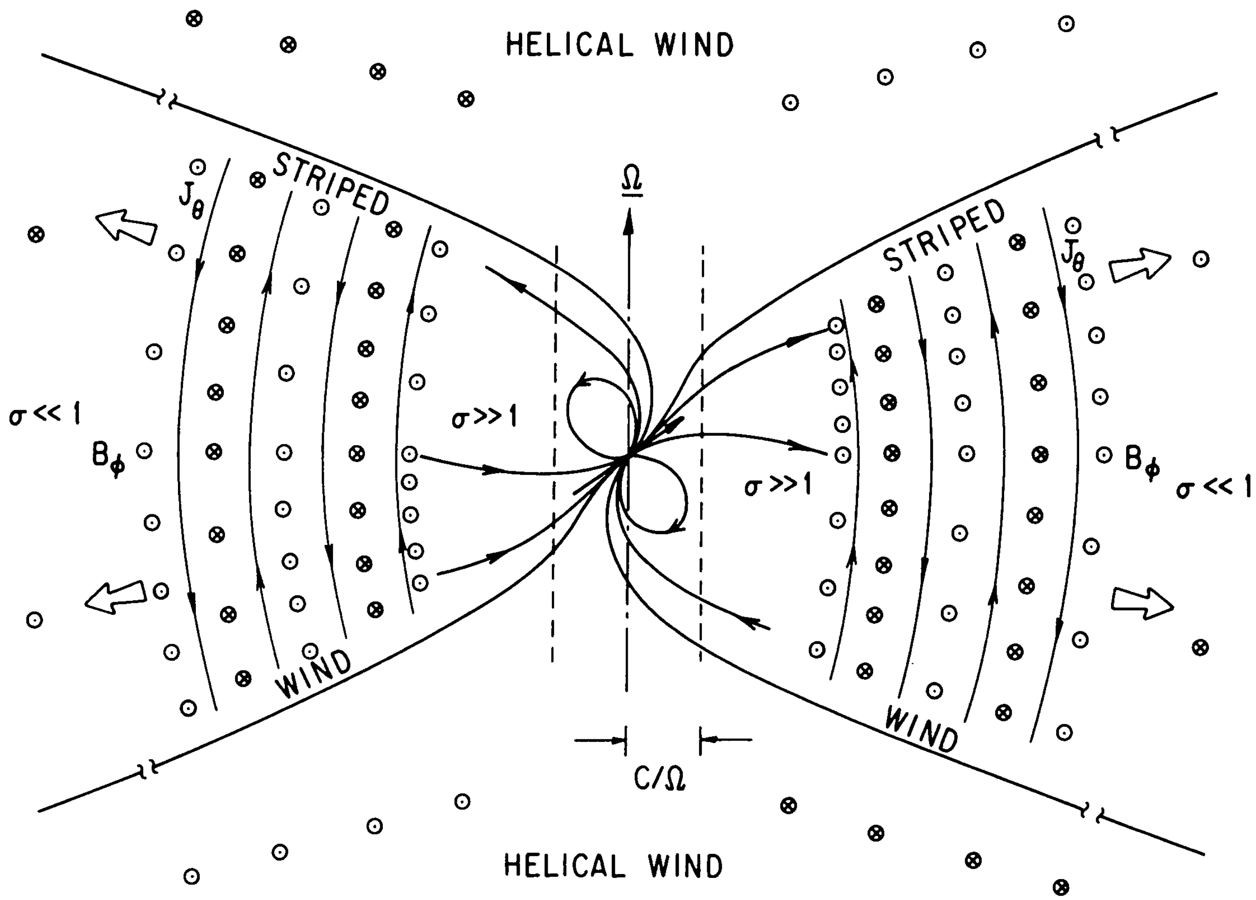
$$T_{\mathrm{particle}} \sim a^2 T_{\mathrm{wave}}$$

Where is this wave?

For example,
Around the termination shock of
the pulsar wind nebulae

There can be the waves
in the GRB jets
(cf. McKinney & Uzdensky 2012)

Striped wind



Coroniti 1990

Crab pulsar

$$T = \frac{2\pi}{\Omega_*} = 33\mathrm{ms}$$

wavelength

$$\longrightarrow \lambda_{\mathrm{sw}} = cT \simeq 10^9 \mathrm{cm}$$

$$a = 3.4 \times 10^{10} \left(\frac{r_{\mathrm{LC}}}{r}\right) \left(\frac{L_{\mathrm{sd}}}{10^{38}\mathrm{erg/s}}\right)^{1/2}$$

(Amano & Kirk 2013)

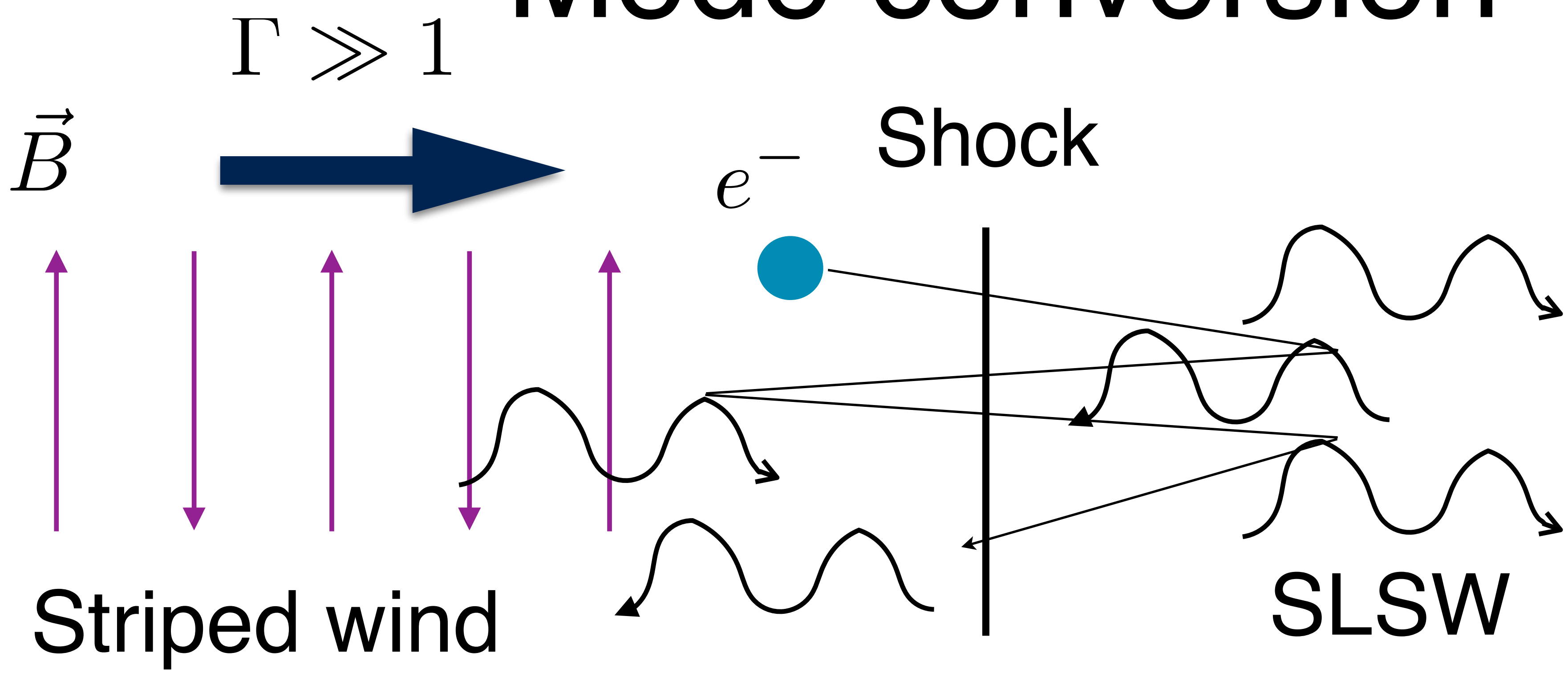
$$r_{\mathrm{TS}} \sim 10^9 \times r_{\mathrm{LC}}$$

$$L_{\mathrm{sd}} = 4.6 \times 10^{38} \mathrm{erg/s}$$

$\Rightarrow$ $a \sim 70 \gg 1$ at the termination shock

Mode conversion

Amano & Kirk 2013



$$\Gamma \lambda_{\rm sw} > \frac{c}{\omega_{\rm p}}$$

MHD

$$\lambda_{\rm sw} < \frac{c}{\omega_{\rm p}}$$

non-MHD

Super Luminal Strong Waves (SLSW) should exist around the termination shock

Aim

Investigation the electron acceleration
in superluminal waves
(and radiation from these electrons)

Method

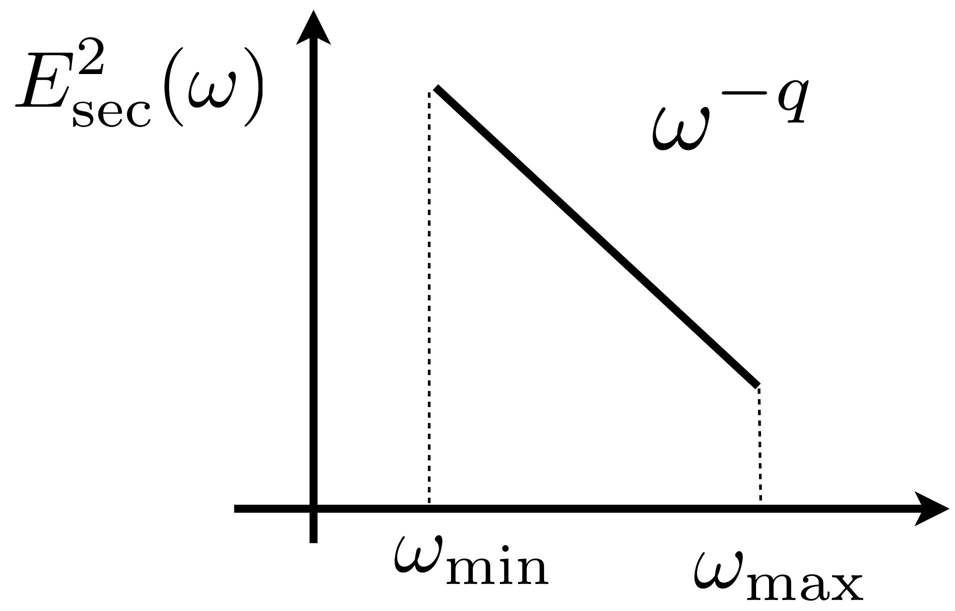
Numerical.

Analytically described waves
and test particles

(Lienard-Wiephert potentail
for the radiation spectra)

Method: SLSWs

$$\vec{E} = \vec{E}_0 + \vec{E}_{\rm sec} \quad \vec{B} = \vec{B}_0 + \vec{B}_{\rm sec}$$



$$\vec{E}_0 = A_0 \cos(\omega_0 t - k_0 z)\hat{e}_z$$

$$\vec{B}_0 = (A_0/\beta_{\rm ph,0}) \cos(\omega_0 t - k_0 z)\hat{e}_z$$

$$\vec{E}_{\rm sec}(\vec{x},t) = \sum_{n=1}^{N} A_n \exp\{i(\vec{k_n}\cdot\vec{x} + \beta_n - \omega_n t)\}\hat{\xi}_{En}$$

$$\vec{B}_{\rm sec}(\vec{x},t) = \sum_{n=1}^{N} \frac{A_n}{\beta_{\rm ph,n}} \exp\{i(\vec{k_n}\cdot\vec{x} + \beta_n - \omega_n t)\}\hat{\xi}_{Bn}$$

$$A_n^2 = \varsigma^2_{\rm sec} G_n \left[\sum_{n=1}^{N} G_n\right]^{-1}$$

$$E_0^2 + \varsigma^2_{\rm sec} = \varsigma^2 = const$$

$$G_n = \frac{4\pi\omega_n^2 \Delta\omega_n}{1 + (\omega_n T_c)^\alpha}$$

$$T_c = 1/\omega_{\rm min}$$

$$a = \frac{e\varsigma}{mc\omega_{\rm min}} = 10 \qquad \beta_{{\rm ph},n} = \omega_n/k_n$$

$$\hat{\xi}_{En} = \cos\psi_n \hat{e}'_x + i\sin\psi_n \hat{e}'_y$$

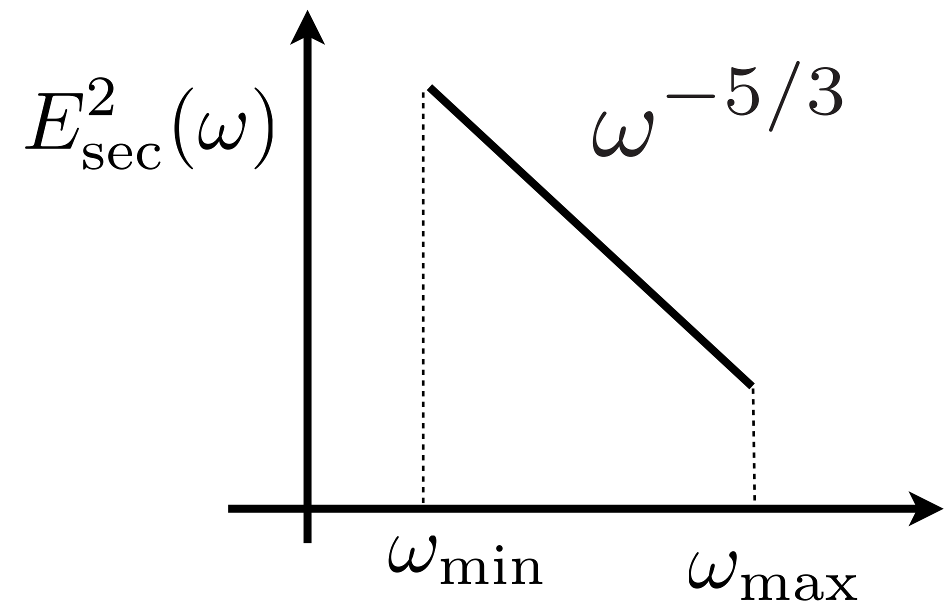
$$N = 100 \qquad \omega_{\rm p} = \sqrt{5}\times 10^{-2}\omega_{\rm min}$$

$$\hat{\xi}_{Bn} = -i\sin\psi_n \hat{e}'_x + \cos\psi_n \hat{e}'_y$$

$$\omega_n^2 = \frac{2\omega_{\rm p}^2}{\sqrt{1+a^2}} + k_n^2 c^2 \quad \omega_{\rm max} = 10^2\omega_{\rm min}$$

$$\hat{e}'_z = \frac{\vec{k_n}}{|k_n|} \qquad q = 5/3$$

Summary of the situation



Upstream rest frame.

Waves

primary wave (1 wave)

+

secondary waves (isotropic)

without entropy mode

Constraint

$$E_0^2 + \varsigma_{\rm sec}^2 = \varsigma^2 = const$$

ς : mean electric field strength

Parameter

$$a = \frac{e\varsigma}{mc\omega_{\rm min}} = 10$$

unit time $\frac{mc}{e\varsigma} = 1$

Acceleration and radiation spectra

1. solve the equation of motion

$\vec{v}_{\rm init}$: isotropic

$\gamma_{\rm init} = 10$

$$\frac{d}{dt}(\gamma m_{\rm e}\vec{v}) = -e(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$$

2. Calculate the radiation spectrum from the Lienard-Wiephert potential

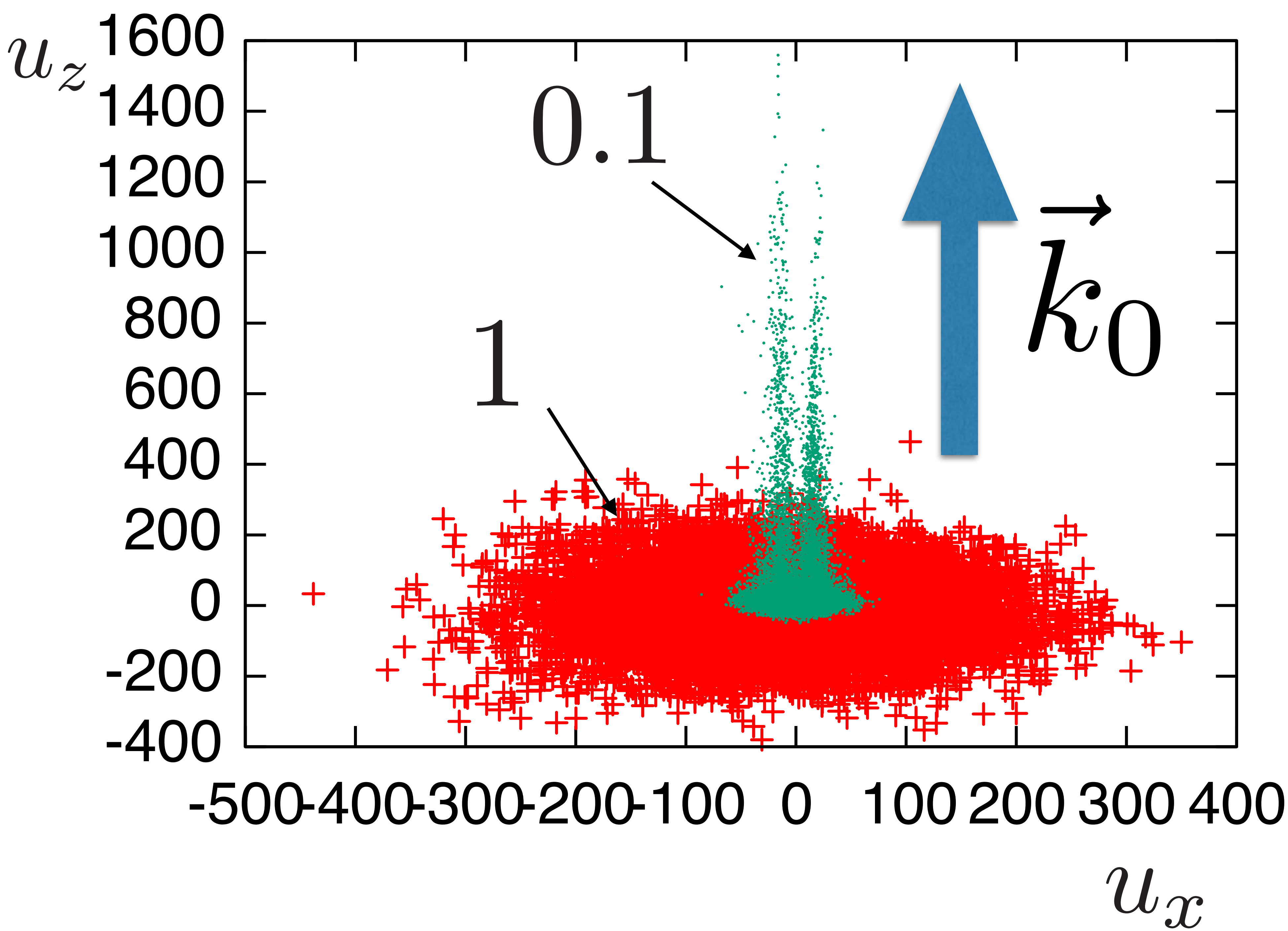
$$\frac{dW}{d\omega d\Omega} = \frac{e^2}{4\pi c^2}\left|\int_{-\infty}^{\infty} dt' \frac{\vec{n} \times \left[(\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}}\right]}{(1 - \vec{\beta} \cdot \vec{n})^2} \exp\{i\omega(t' - \frac{\vec{n} \cdot \vec{r}(t')}{c})\}\right|^2$$

$\vec{n}$ unit vector toward the observer

t' retarded time

Results: 4-velocities $t = 3 \times 10^4 \omega_0^{-1}$

$\frac{e\varsigma_{\rm sec}}{mc} = \underline{0.1}$ (primary domi.) & $\frac{e\varsigma_{\rm sec}}{mc} = \underline{1}$

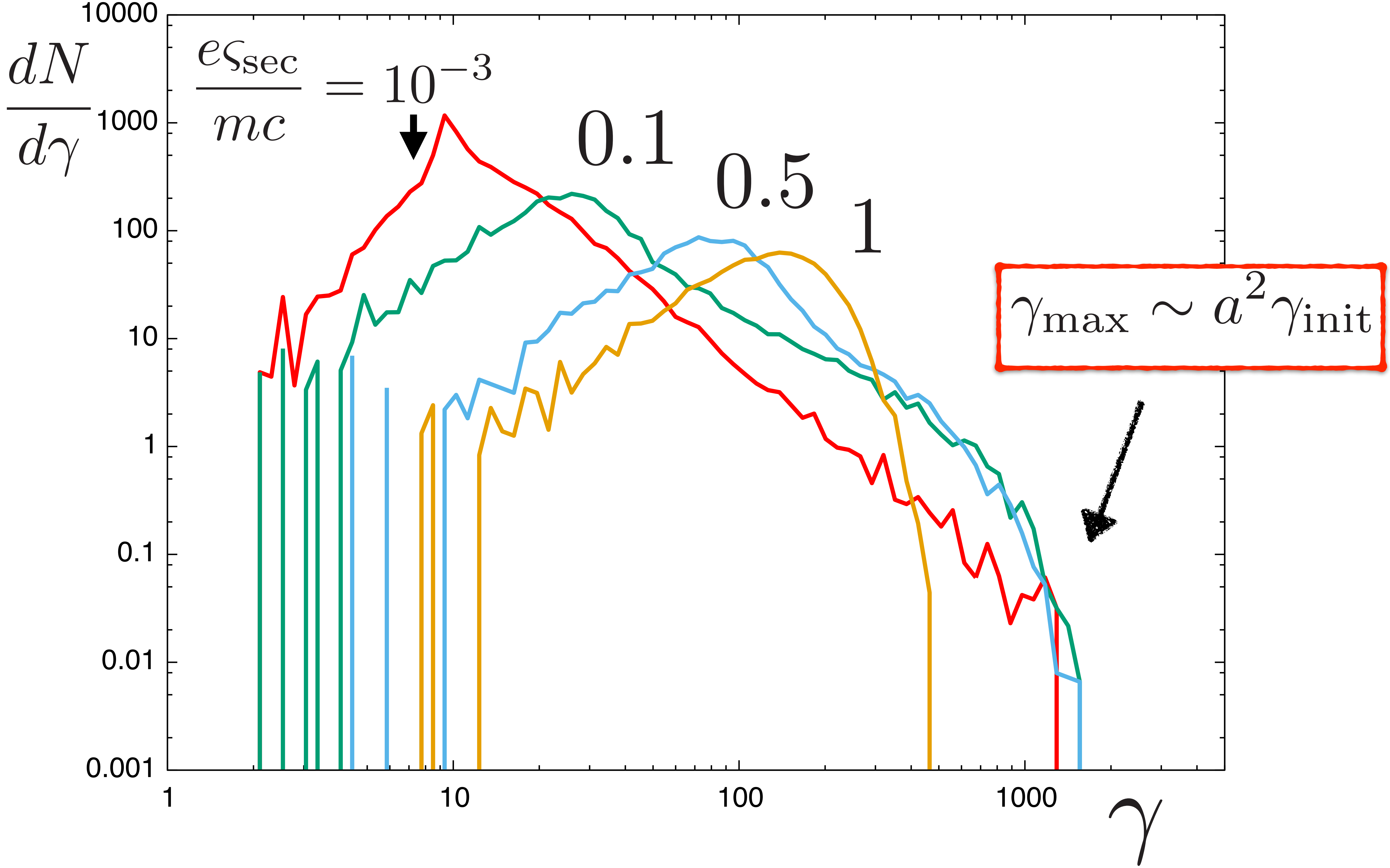


$\vec{v} \parallel \vec{k}_0$

trapped & selectively accelerated

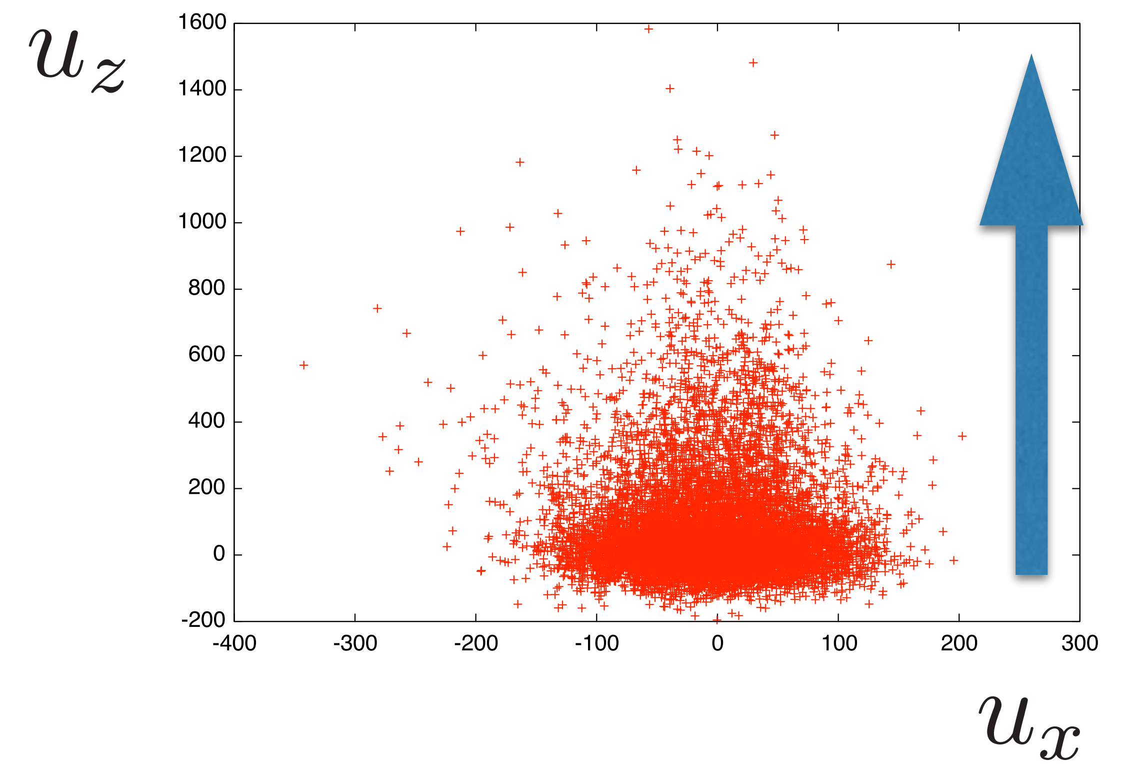
Energy spectra

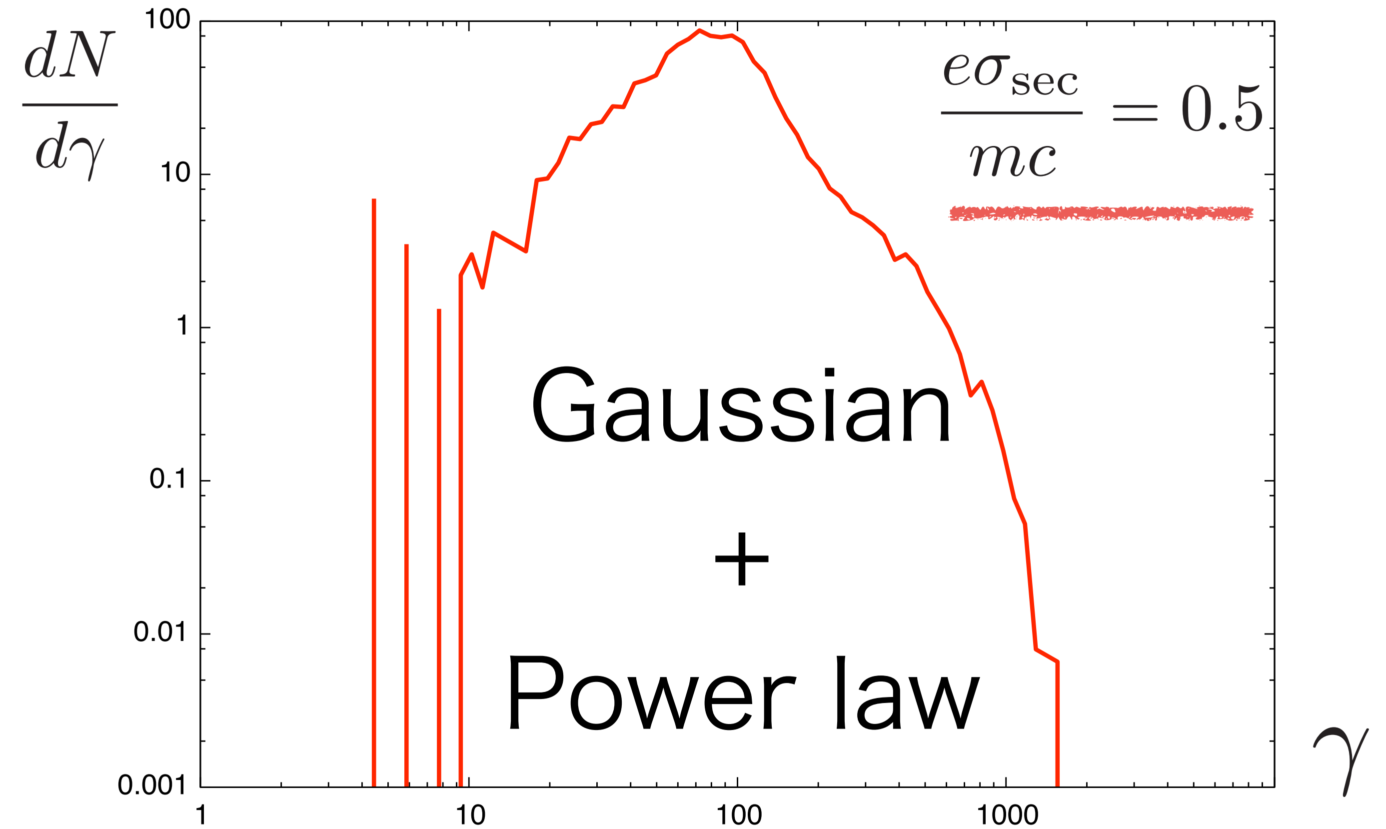
$t = 3 \times 10^4 \omega_0^{-1}$

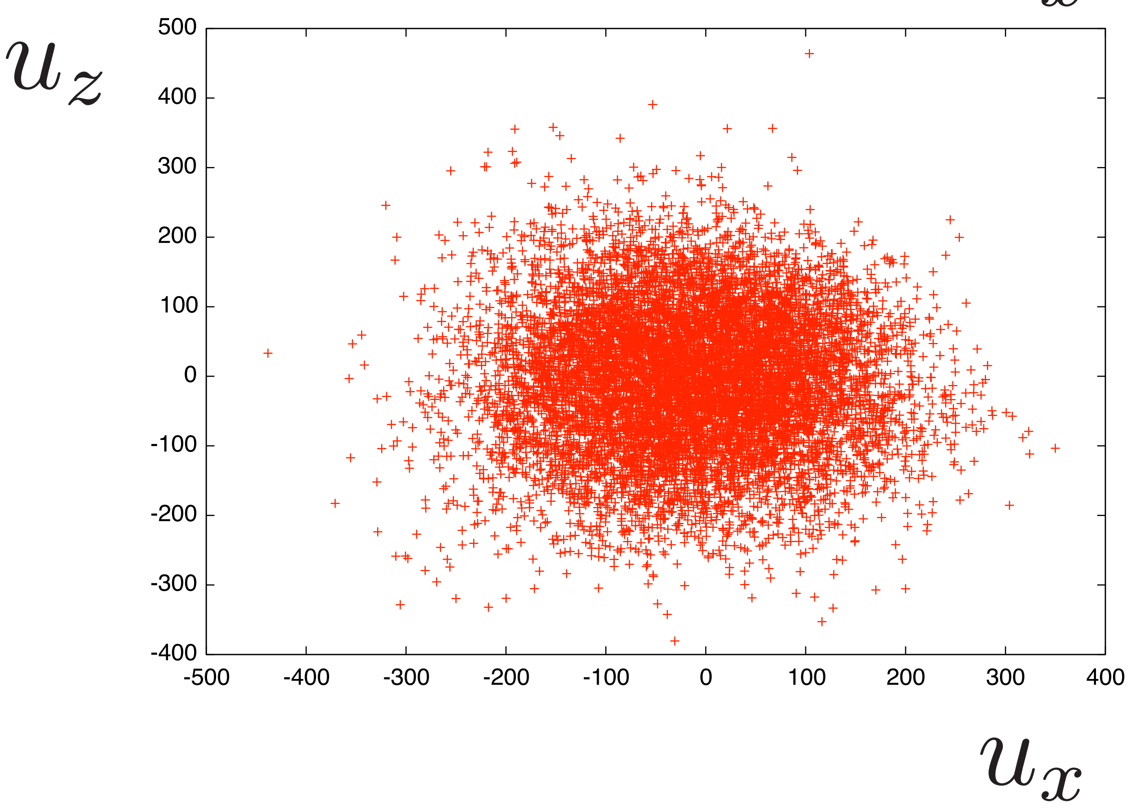


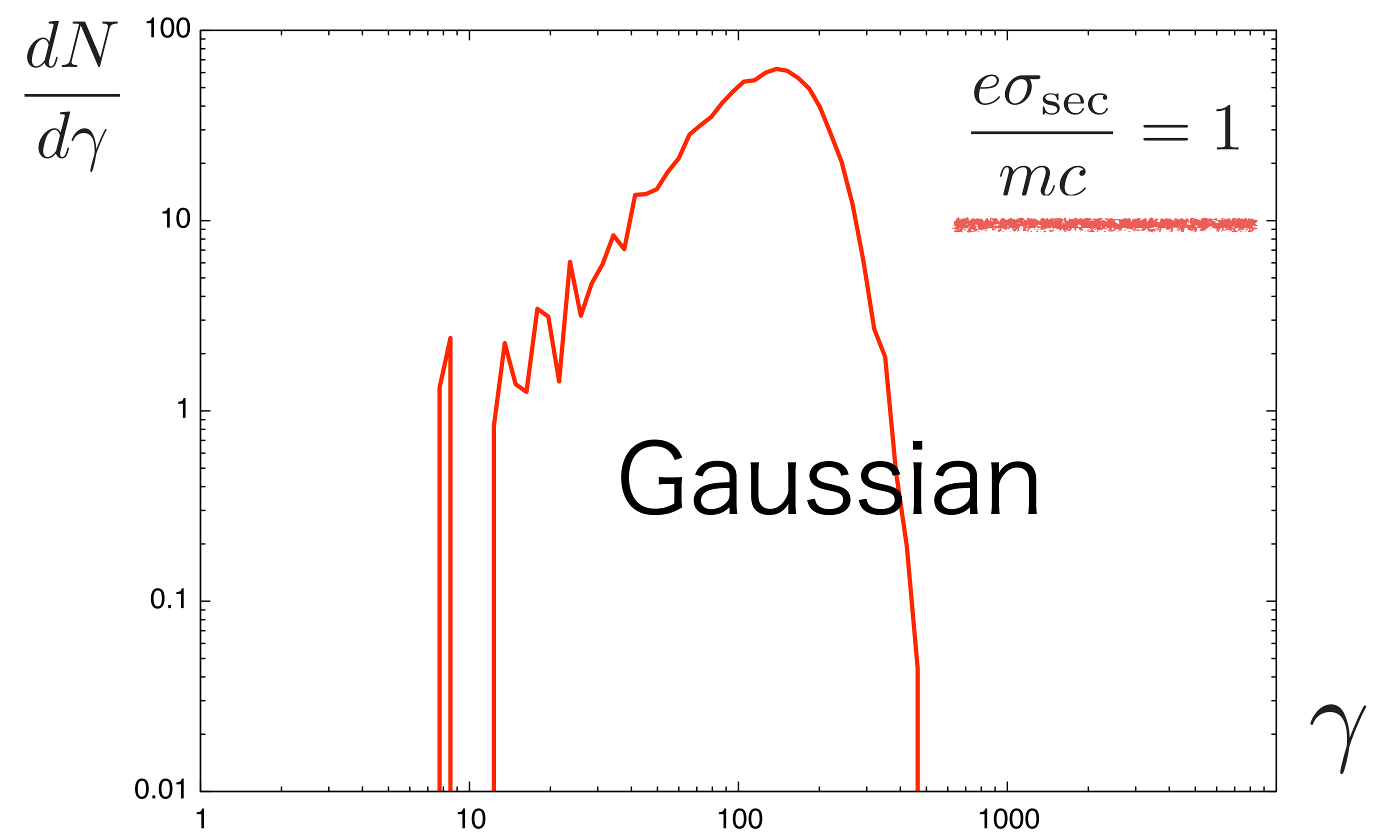
Results: particle acceleration $t = 3 \times 10^4 \omega_0^{-1}$

4-velocities







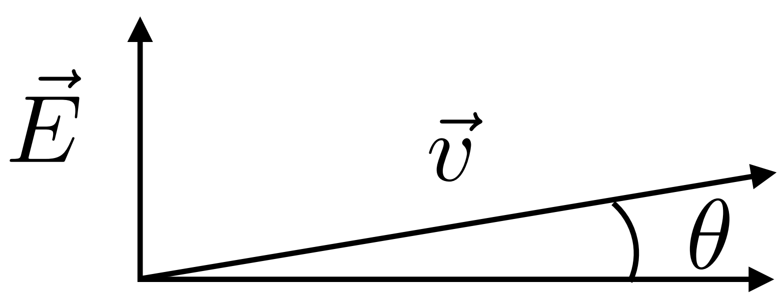


Discussion: Efficiency of acceleration

DSA Bohm limit

$$\Delta\gamma mc^2 \simeq \gamma_0 mc^2 \quad \text{in gyro time} \quad T_g = \frac{\gamma_0 mc}{eB}$$

SLSW acceleration (strongly strapped)



$\theta \ll 1$ is realized

This acceleration mechanism
is important only for the pre-acceleration
for the DSA in the upstream

Energy

$$\tau = \frac{\gamma_0 mc}{eE\theta} = T_{\rm g}/\theta \gg T_{\rm g}$$

much slower than DSA

Discussion: 2nd order acceleration?

transport equation

$$\frac{\partial f}{\partial t} = -\frac{1}{p^2}\frac{\partial}{\partial p}\left\{p^2\left[A(p)f - D(p)\frac{\partial f}{\partial p}\right]\right\} - \frac{f}{t_{\rm esc}} + \frac{S}{4\pi p^2},$$

For resonant scattering, $D(p) \propto p^q$ q : power index of the turbulence

Non resonant scattering ➡ D does not depend on p

➡ $$\frac{\partial f}{\partial t} = \frac{D}{p^2}\frac{\partial}{\partial p}\left(p^2\frac{\partial f}{\partial p}\right)$$ for $t \neq 0$

f is a Gaussian function.

➡ $$\frac{dN}{d\gamma} \propto p^2 f$$

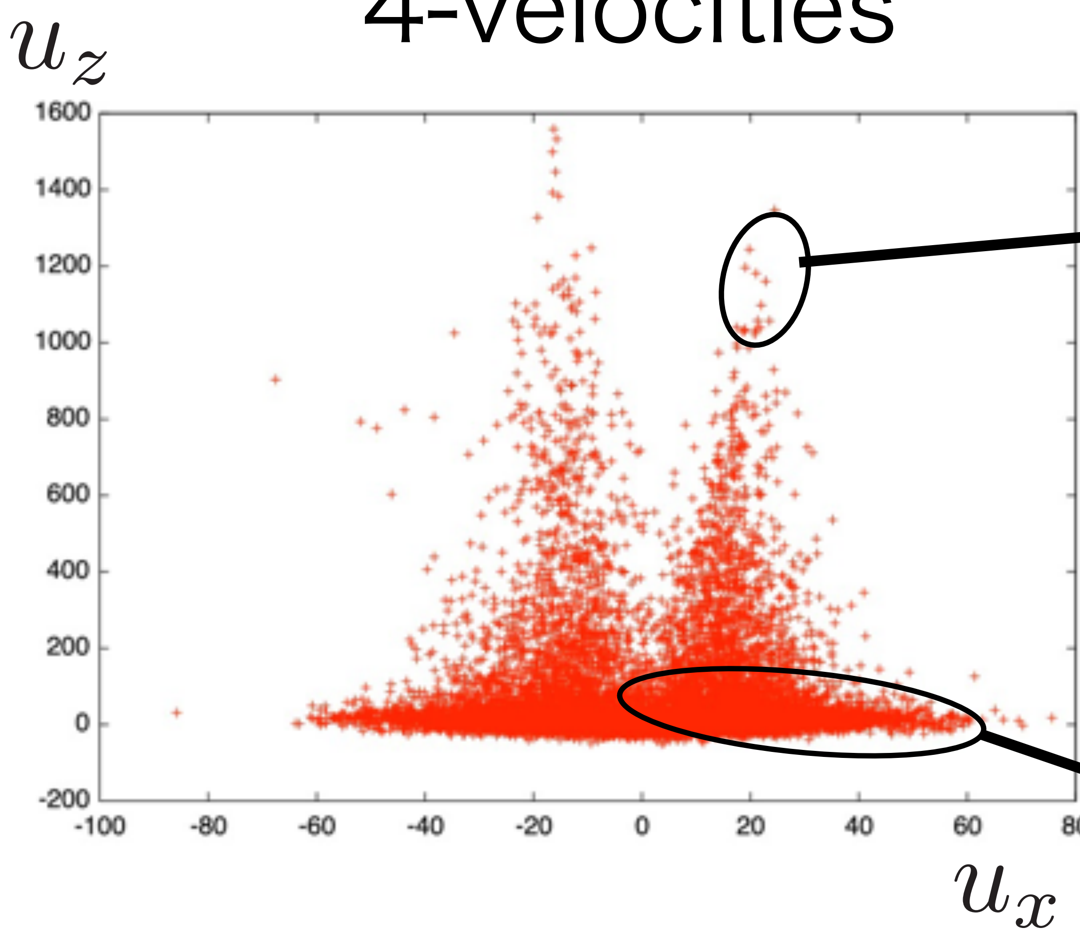
power law with index 2
+ exponential cutoff

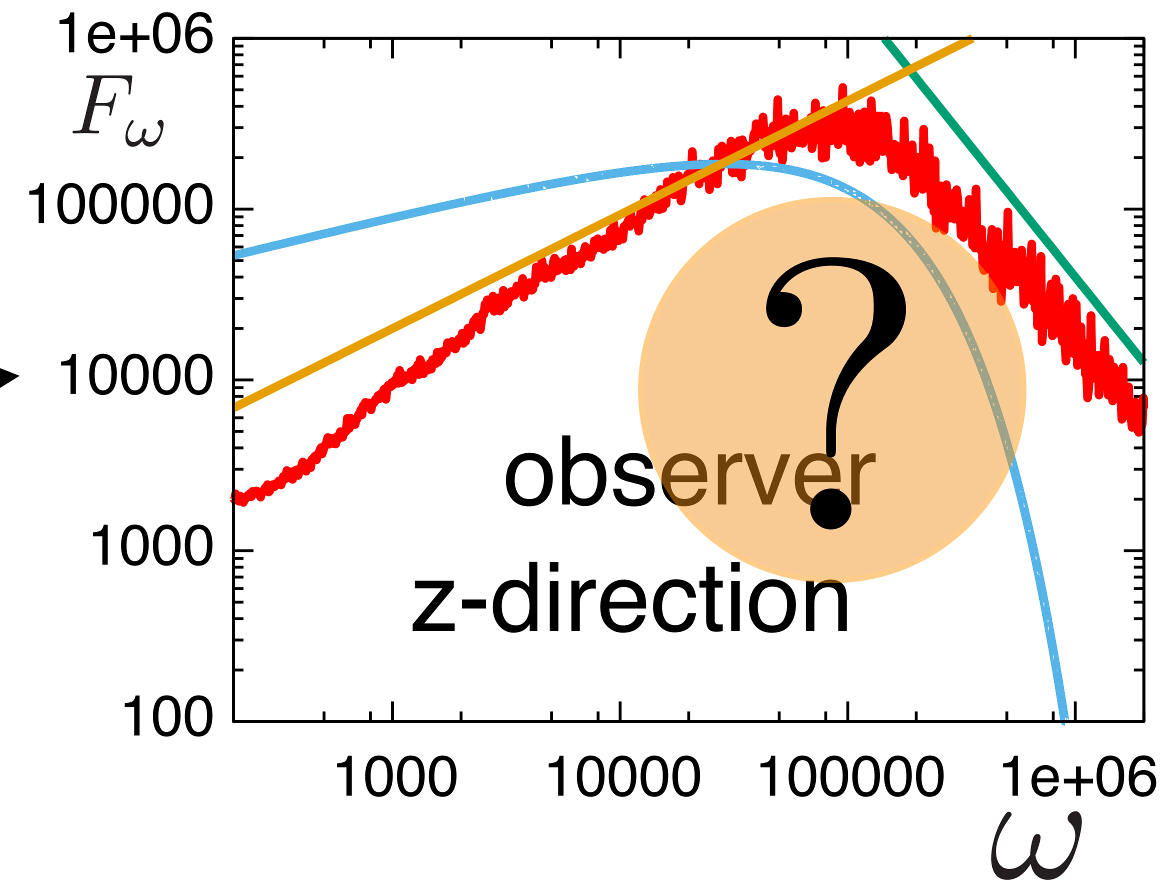
Summary for particle acceleration

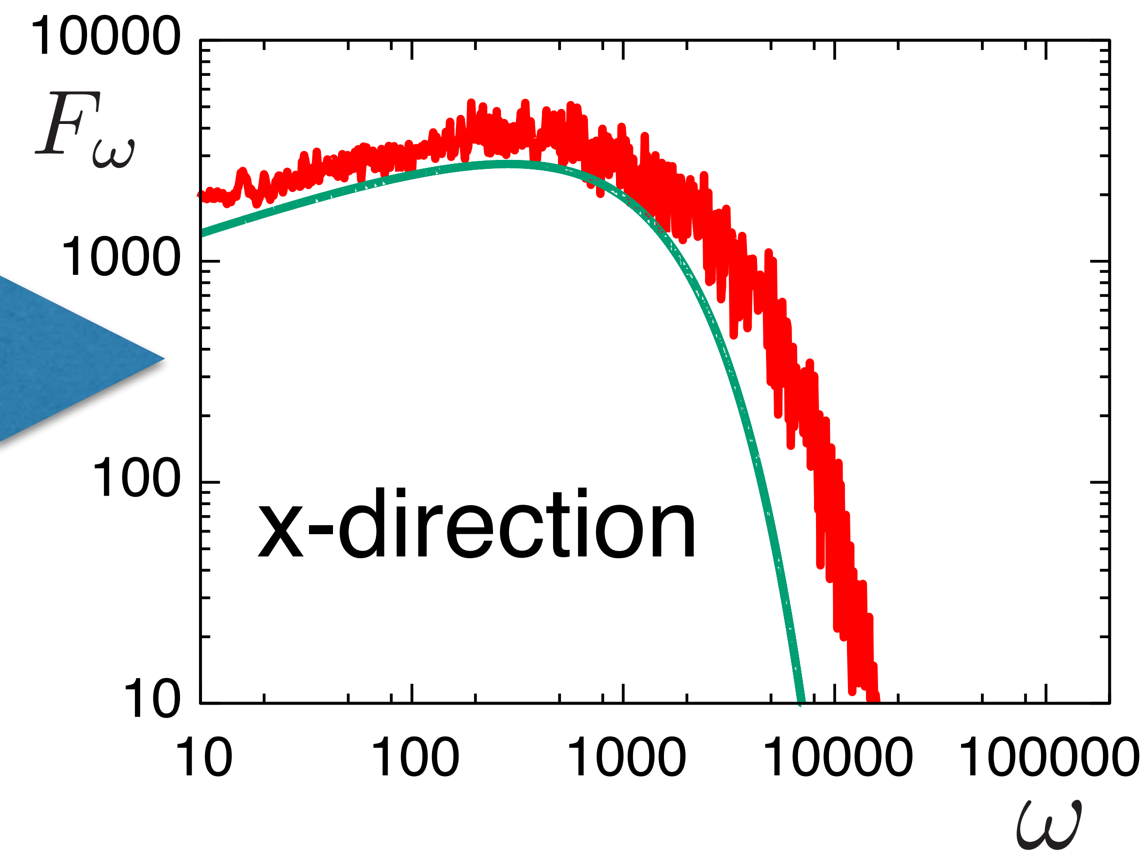
- We calculate the electron acceleration in superluminal strong waves and radiation from them.
- When the primary wave is dominant (or even comparable to the secondary waves), selective acceleration occurs.
  It form the power law energy distribution.
- This acceleration mechanism can play a crucial role for the injection to the shock acceleration (DSA) in the upstream of the termination shock.

Radiation

Results: Radiation spectra $\frac{e\varsigma_{\rm sec}}{mc} = 0.1$



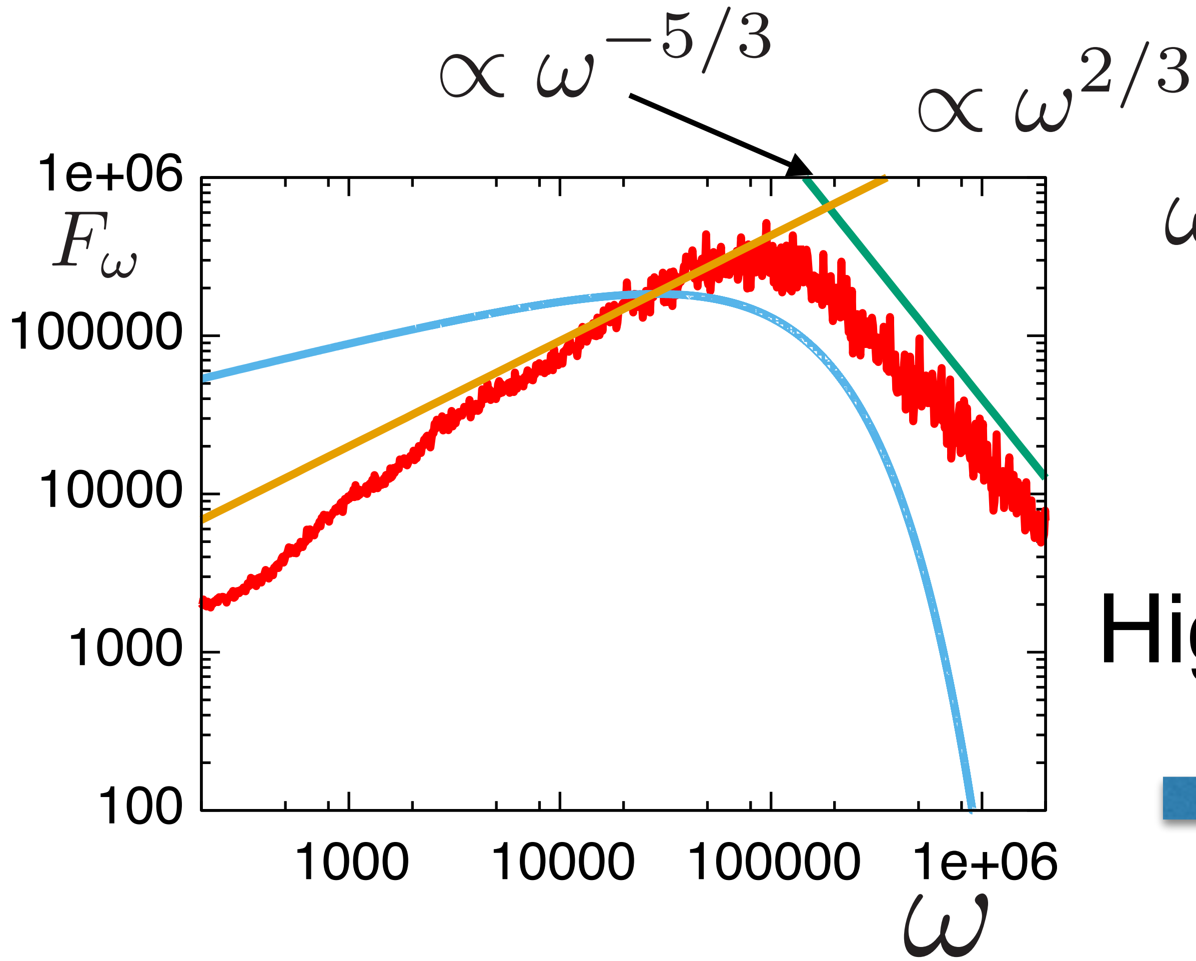




Synchro-Compton radiation

$$\omega_{\rm peak} \simeq \gamma^2 \frac{eB_{0y}}{mc} \sim 10^3$$

Results: Radiation spectra $\frac{e\varsigma_{\rm sec}}{mc} = 0.1$



$$\omega_{\rm peak} \sim \gamma_{\rm max}^2 \frac{e\varsigma_{\rm sec}}{mc} = \gamma_{\rm max}^2 \omega_{\rm min}$$

$$\sim (10^3)^2 \times 0.1$$

Higher frequency than peak

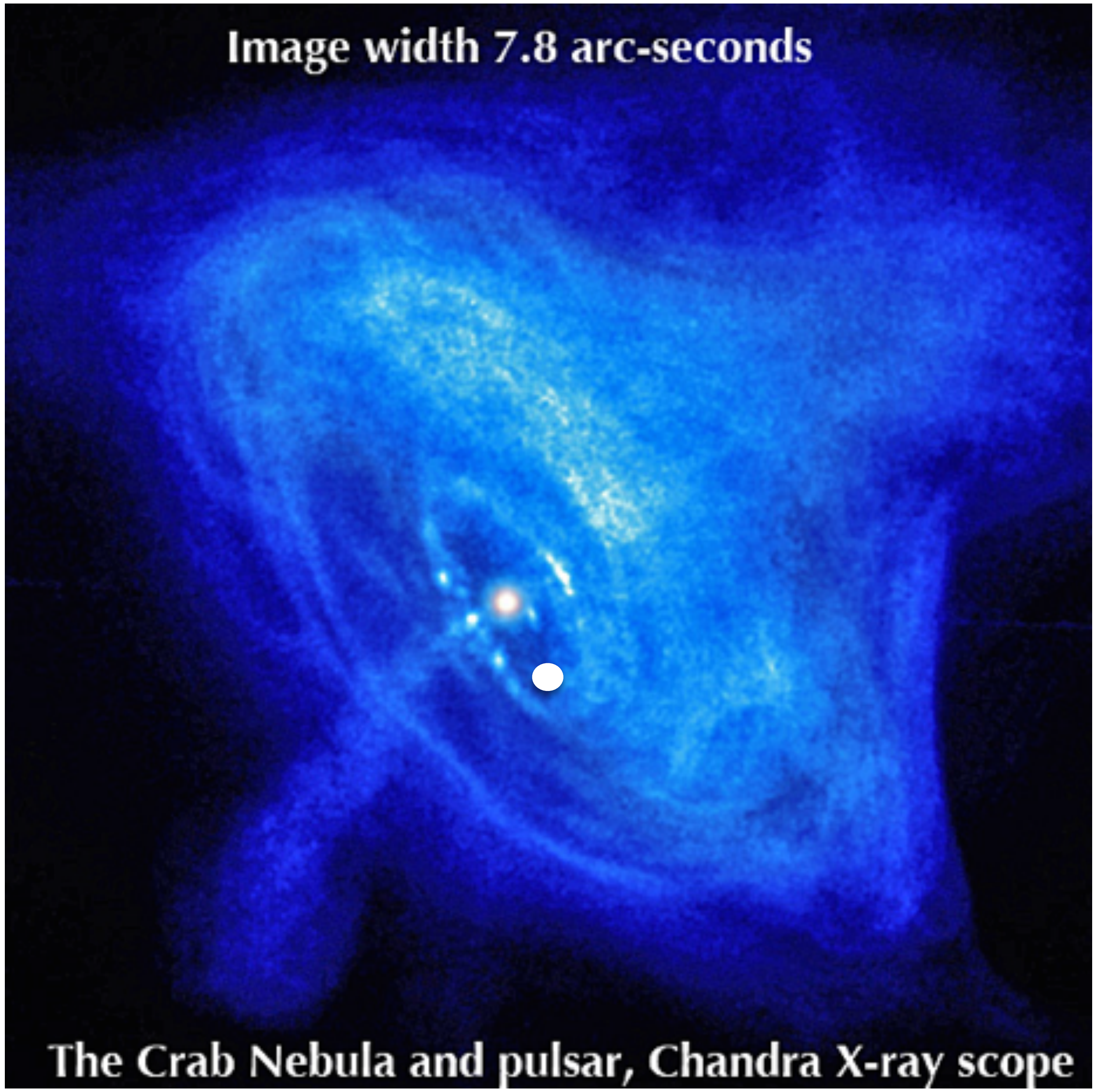
→ jitter radiation contribution

$$\omega_{\rm rad} = \gamma_{\rm max}^2 \omega_{\rm turb}$$

Energy gain → SLSW acceleration

Deflection → Secondary wave origin

Observation



Radio emission

$$\omega \sim \Gamma \gamma'^2_{\rm max} \frac{eB/\Gamma}{mc} \simeq 2 \times 10^{10} \left(\frac{\gamma'_{\rm max}}{10^3}\right)^2 \left(\frac{B}{10^{-3}{\rm G}}\right) {\rm s}^{-1}$$

constrained by

$$L \sim 6 \times 10^{16} \left(\frac{\Gamma}{10^2}\right)^2 \left(\frac{\gamma'_0}{10}\right)^2 {\rm cm}.$$

$$L_{\rm TS} = 3 \times 10^{17} {\rm cm}$$

Area

$$A \sim \frac{L^2}{\Gamma^2}$$

from beaming effect

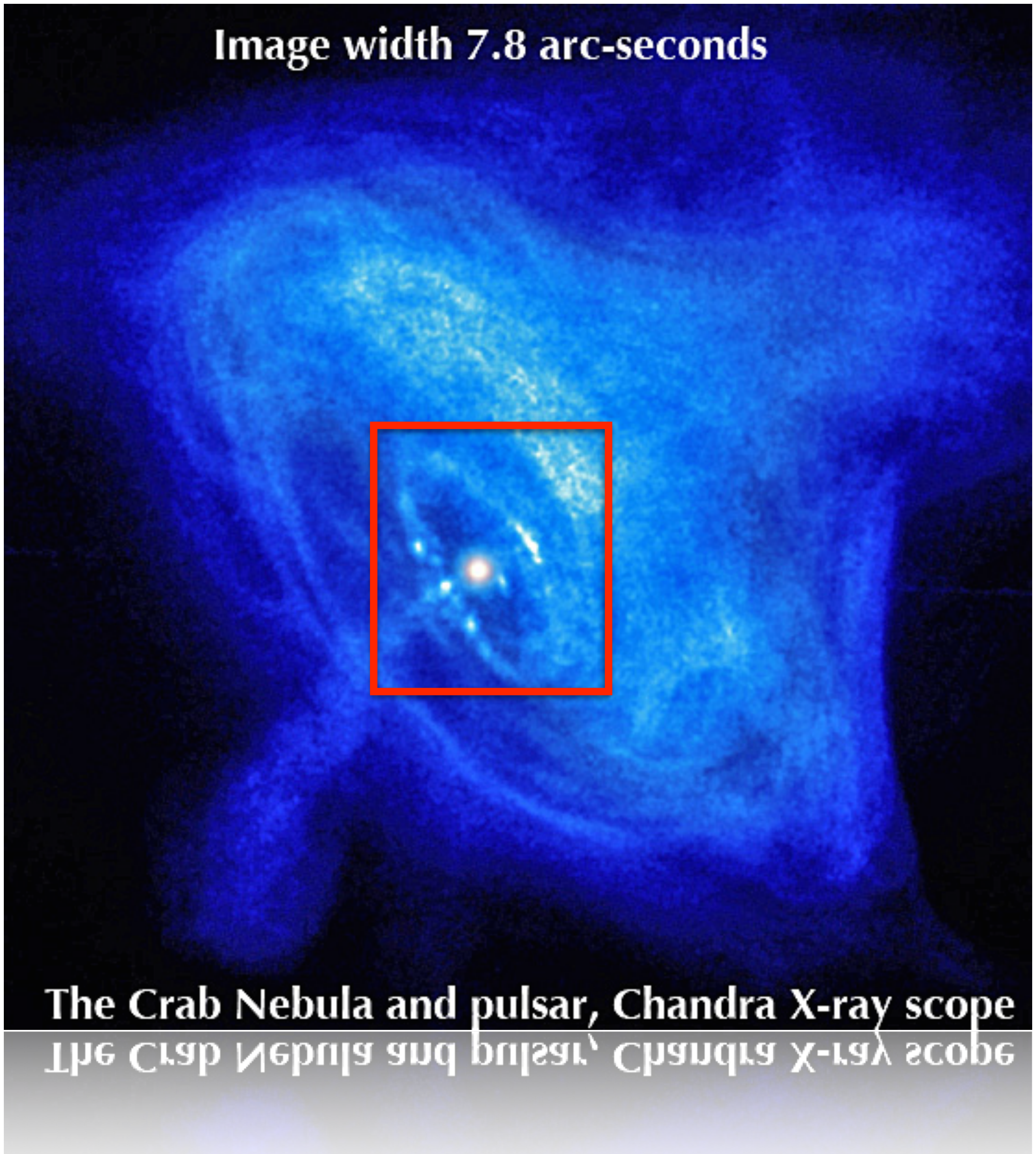
Summary

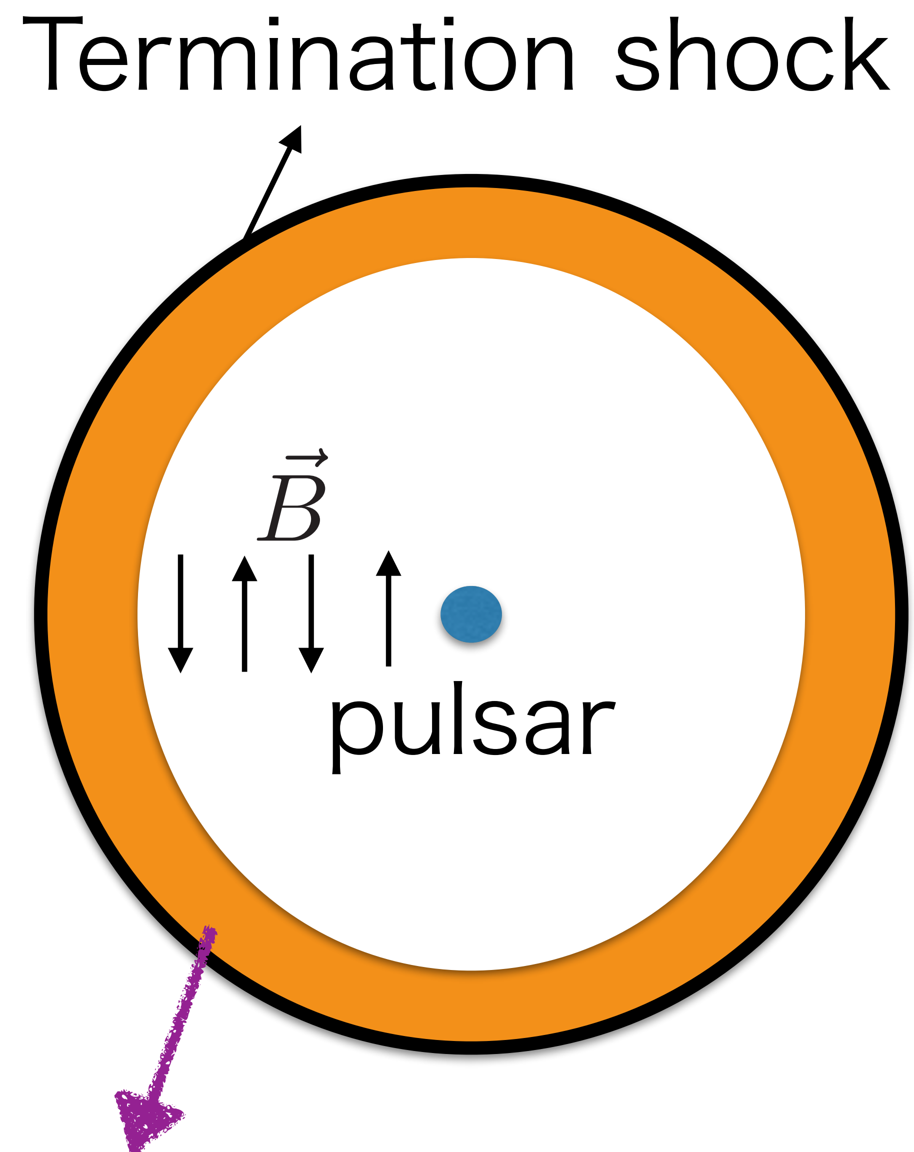
- We calculate the electron acceleration in superluminal strong waves and radiation from them
- When the primary wave is dominant, selective acceleration occurs. It can be important for the injection into DSA
- Radiation features are understood by considering the synchro-Compton & jitter radiation in the secondary waves.

End

Back up

Location





SLSWs exist?

Generated from Striped wind

Estimation of a of entropy wave

$$L = \frac{c\vec{E}\times\vec{B}}{4\pi}4\pi r^2$$

isotropic approximation

pure toroidal wind

$$E = \frac{vB}{c} \sim B$$

$$B \propto r^{-1}$$

$$r_{\rm LC} = \frac{c}{\omega}$$

$$\Rightarrow \quad L = \frac{m^2c^3\omega^2a^2r^2}{e^2}$$

$$\Rightarrow \quad a = \frac{r_{\rm LC}}{r}\left(\frac{e^2L}{m^2c^5}\right)^{1/2}$$

$$\sim 3.4\times10^{10}\left(\frac{r_{\rm LC}}{r}\right)\left(\frac{L}{10^{38}{\rm erg/s}}\right)^{1/2}$$

Entropy wave in upstream

In upstream frame,

wavelength of entropy mode $\Gamma_u \lambda_{\rm sw}$

inertial length $c/\omega_{\rm p}$

$$\eta_{\rm up} \equiv \frac{\Gamma_u \lambda_{\rm sw}}{2\pi c/\omega_{\rm p,u}}$$

Substituting Crab parameters, → $\eta_{\rm up} \simeq 3 \times 10^2 \sqrt{\Gamma_u^2 n_{\rm u}}$

$L_{\rm sd}$ = isotropic kinetic flux + Poynting flux

*their ratio $\equiv \sigma$

→ $\Gamma_{\rm u}^2 (1+\sigma) n_{\rm u} = 2 \times 10^{-2}$ @TS

$\therefore$ $\eta_{\rm up} \simeq 40 \sigma^{-1/2}$

$\sigma \sim 10^4$ @LC

maybe, $\sigma \ll 10^4$ @TS → $\eta_{\rm up} > 1$

Entropy wave in downstream

$$\eta_{\rm down} = \lambda_{\rm sw}\sqrt{\frac{4\pi e^2}{mc}}\sqrt{\frac{\Gamma_{\rm d}^2 n_{\rm d}}{\gamma_{\rm th}}}$$

* $\gamma_{\rm th}$: thermal Lorentz factor

$$\frac{\eta_{\rm down}}{\eta_{\rm up}} = \sqrt{\frac{\Gamma_{\rm d}^2 n_{\rm d}}{\gamma_{\rm th}\Gamma_{\rm u}^2 n_{\rm u}}}$$

Rankine-Hugoniot relation

$$\Gamma_{\rm d} n_{\rm d} \sim \Gamma_{\rm u} n_{\rm u}$$

$$= \frac{\sqrt{\Gamma_{\rm d}}}{\sqrt{\gamma_{\rm th}\Gamma_{\rm u}}}$$

$$\Gamma_{\rm d} \sim 1$$

kinetic energy dominant

$$\gamma_{\rm th} \sim \Gamma_{\rm u}$$

$$\sim \frac{1}{\Gamma_{\rm u}}$$

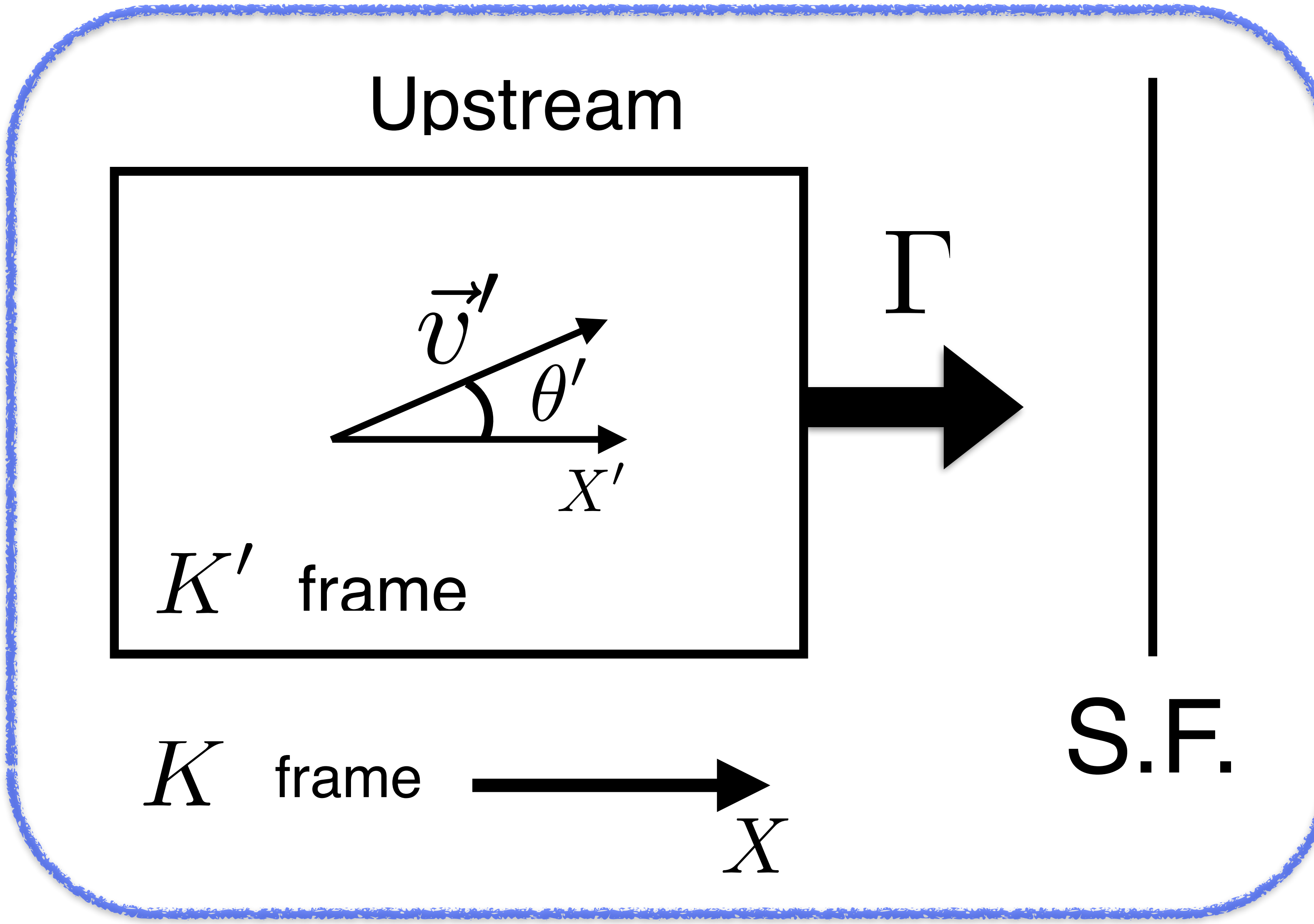
$\eta_{\rm up} < O(10)$ & $\Gamma_{\rm u} > O(10^2)$

$$\therefore \eta_{\rm down} \ll 1$$

Energy in shock rest frame

Non-thermal electrons

around $\vec{k'}_0$



Thermal electrons

$\longrightarrow$ isotropic

$\gamma_{\mathrm{th,max}} \simeq 2\Gamma\gamma'_{\mathrm{th}}$

condition for

$\gamma_{\mathrm{nth}} > \gamma_{\mathrm{th,max}}$

$$\cos\theta' > \frac{2\gamma'_{\mathrm{th}}}{\gamma'_{\mathrm{nth}}} - 1$$

satisfied by wide range θ'

since $\gamma'_{\mathrm{nth}} \gg \gamma'_{\mathrm{th}}$

Acceleration efficiency: weakly trapped population

random walk — one step $\delta E = amc^2 \quad t = 1/\omega_{\rm min}$

For electrons accelerating by DSA, $\gamma_0 \gg a$

$$< E^2 >= E_0^2 + N(amc^2)^2 \qquad \& \qquad N = \omega_0 T_{\rm g} = \gamma_0 / a$$

$$\longrightarrow \quad < \gamma^2 > - \gamma_0^2 = \gamma_0 a$$

Mean energy gain in gyro time

$$\Delta E \equiv (< \gamma^2 >^{1/2} - \gamma_0) mc^2 = \frac{a}{2} mc^2 \ll \gamma_0 mc^2$$

➡ SLSW acceleration is important only before cross the shock (upstream)

This acceleration can be important for injection into DSA

Condition for Maxwell distribution

- Distribution for some direction does not depend on
  the distribution for other direction
- Distribution does not change if we rotate the axes
- Distribution is a dependent only on the absolute of the momentum
- Total energy does not change
- Equipartition of energy

Total energy does not conserved.

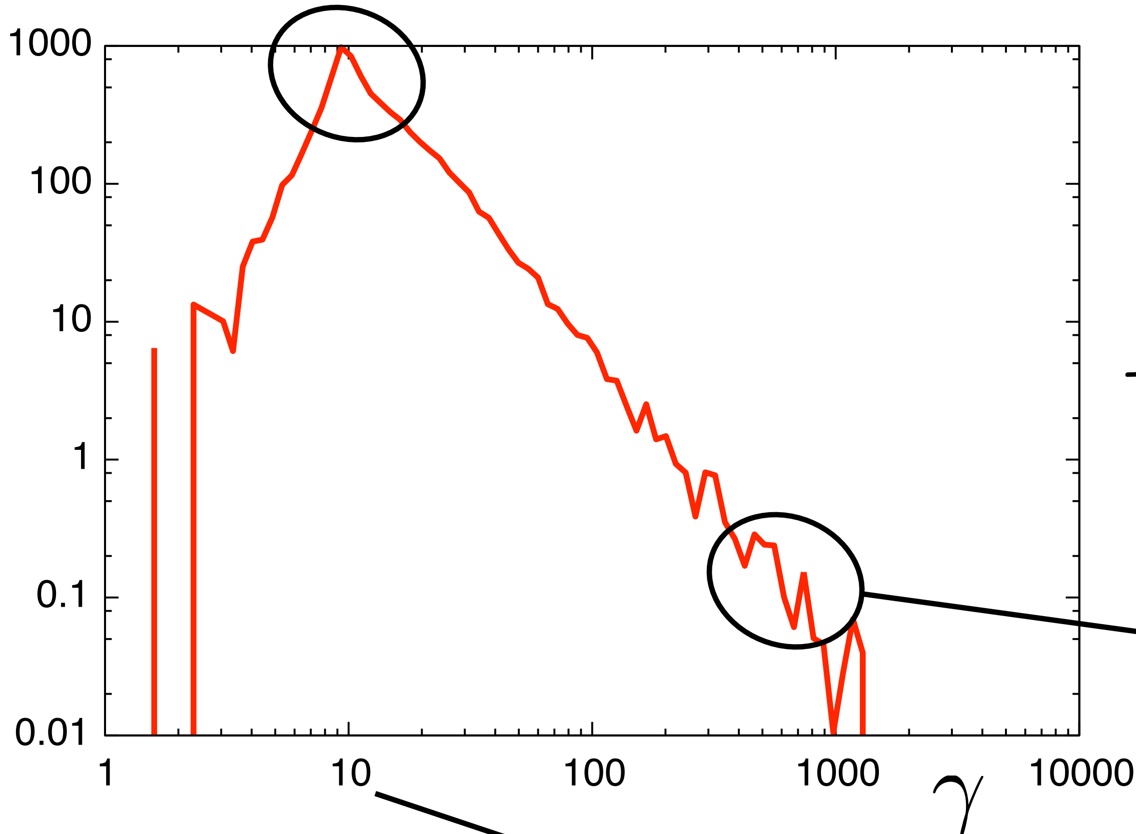
However, other conditions are nearly achieved.

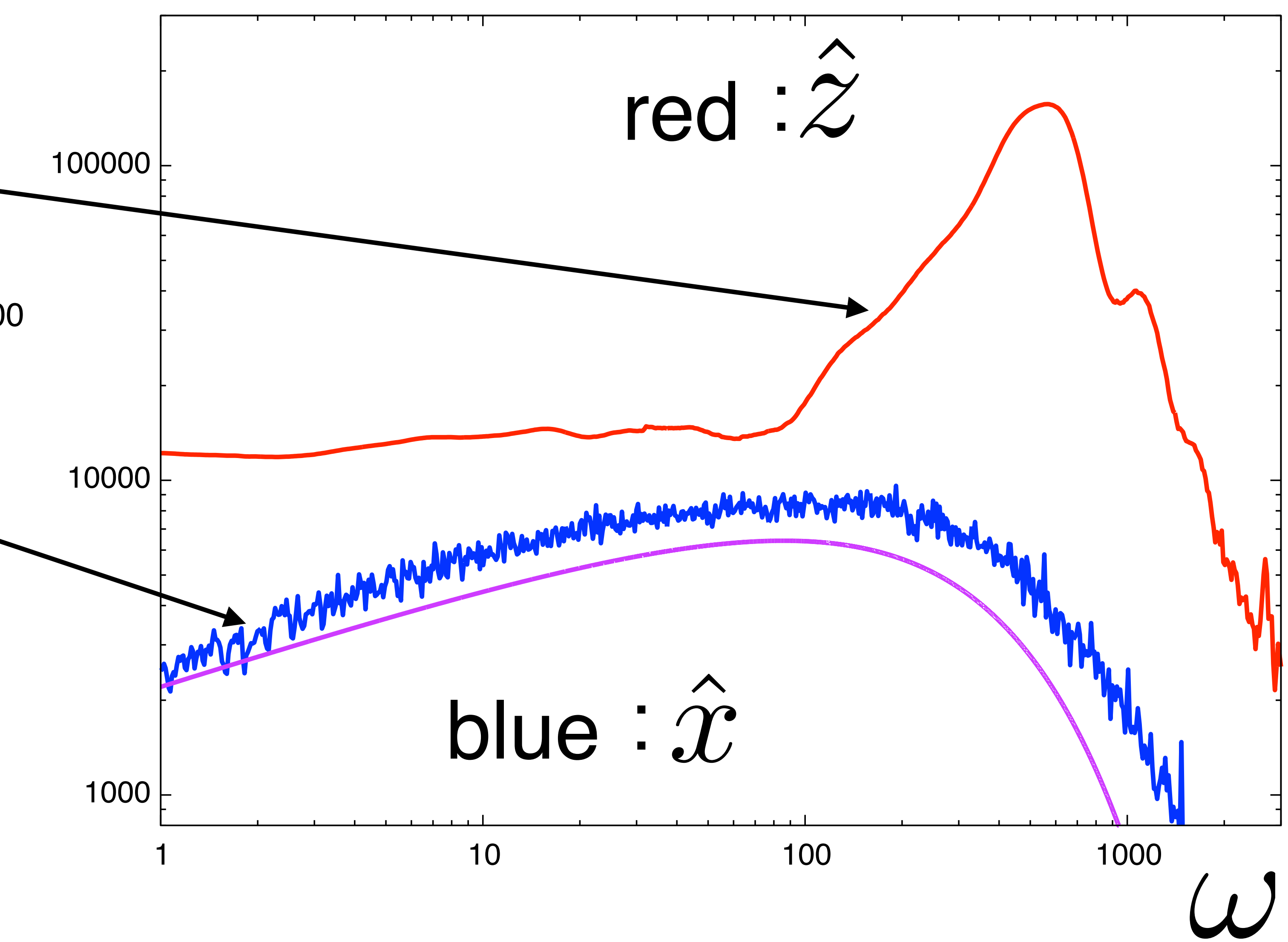
Thus, quasi Maxwellian can be realized

Radiation spectra for $e\varsigma_{\rm sec}/mc = 10^{-3}$

primary wave deflect the electron very slowly

$$\omega_{\rm peak} \sim 500 \ll \gamma_{\rm max}^2 \frac{e\varsigma}{mc} = 10^6$$





$$\omega_{\rm peak} \simeq \gamma^2 \frac{2\pi}{T_{\rm particle}}$$

$$= \gamma^2 \frac{\omega_0}{a^2 \gamma_{\rm init}^2} = 10^2$$

1-wave test calculation

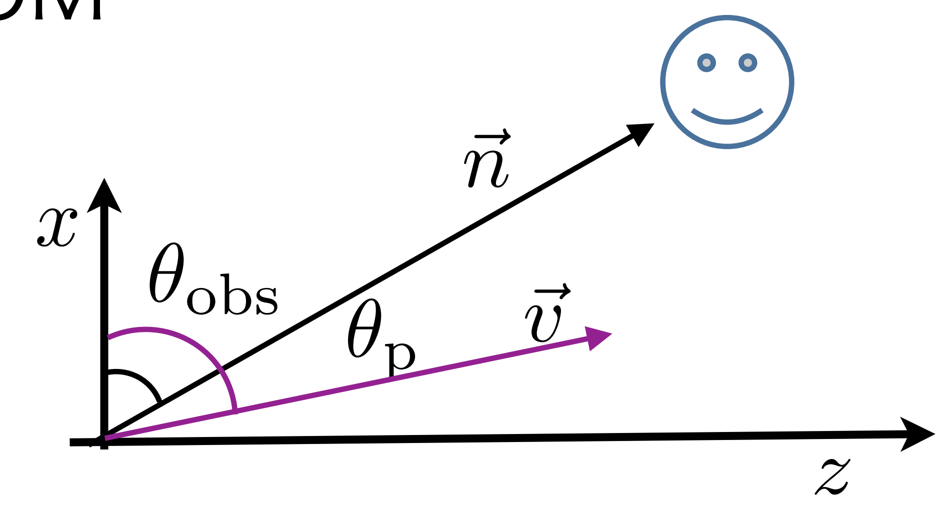
1、A SLS wave with the approximation $v_{\mathrm{ph}} = c$

$$\begin{cases} E_x = E_0 \cos(\omega_{\mathrm{sw}} t - k_{\mathrm{sw}} z) \\ B_y = B_0 \cos(\omega_{\mathrm{sw}} t - k_{\mathrm{sw}} z) \end{cases} \quad \text{or} \quad \begin{cases} E_x = -E_0 \cos(\omega_{\mathrm{sw}} t + k_{\mathrm{sw}} z) \\ B_y = B_0 \cos(\omega_{\mathrm{sw}} t + k_{\mathrm{sw}} z) \end{cases}$$

2、Inject an electron and solve the EOM

$$\vec{v}_{\mathrm{init}} = (0, 0, v_z)$$

$$\frac{d}{dt}(\gamma m_{\mathrm{e}} \vec{v}) = -e(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$$

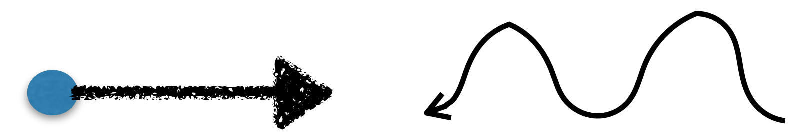


3、Calculate the radiation spectrum using Lienard-Wiechert potential

$$\frac{dW}{d\omega d\Omega} = \frac{e^2}{4\pi c^2} \left| \int_{-\infty}^{\infty} dt' \frac{\vec{n} \times [(\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{(1 - \vec{\beta} \cdot \vec{n})^2} \exp\{i\omega(t' - \frac{\vec{n} \cdot \vec{r}(t')}{c})\} \right|^2$$

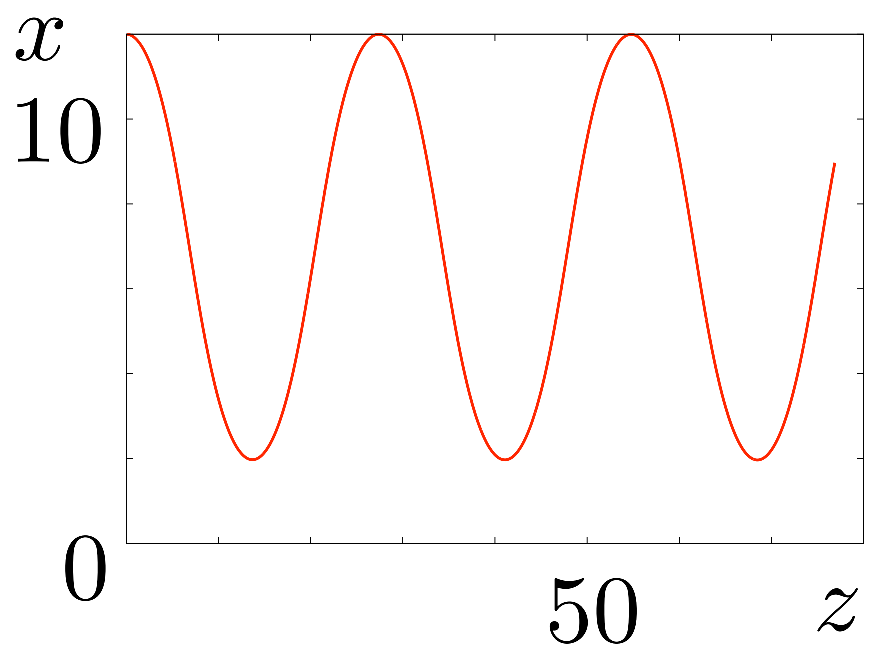
$\vec{n}$ unit vector toward the observer $\quad$ t' retarded time

Head-on collision

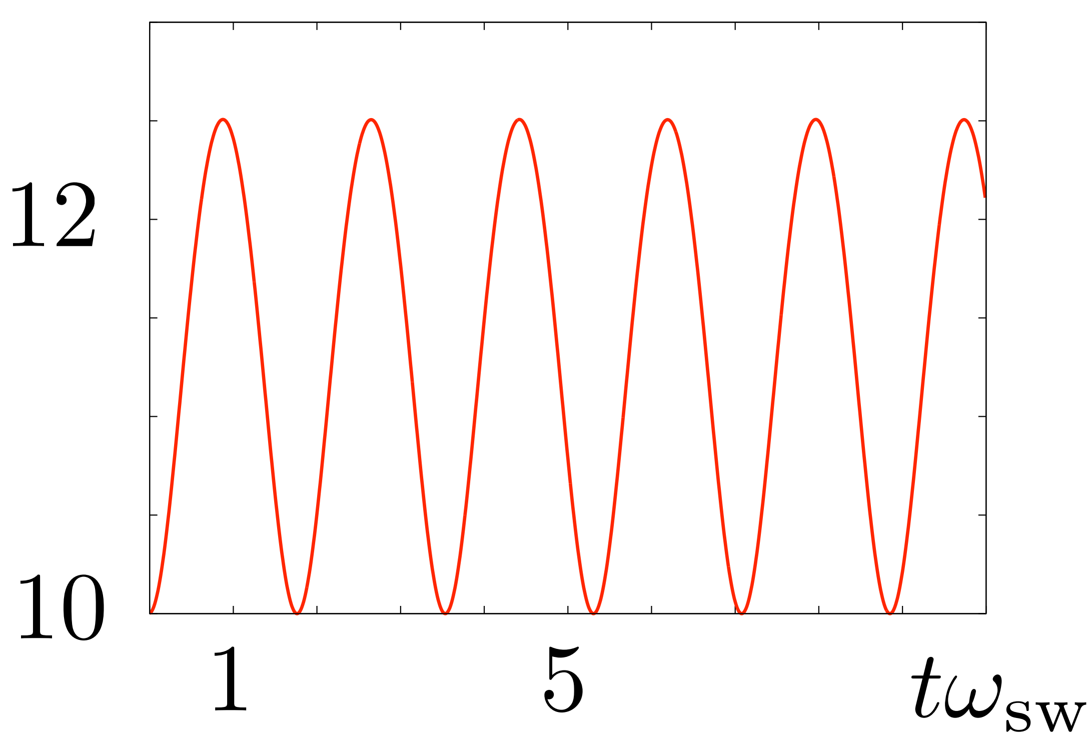


It is consistent with synchro-Compton theory

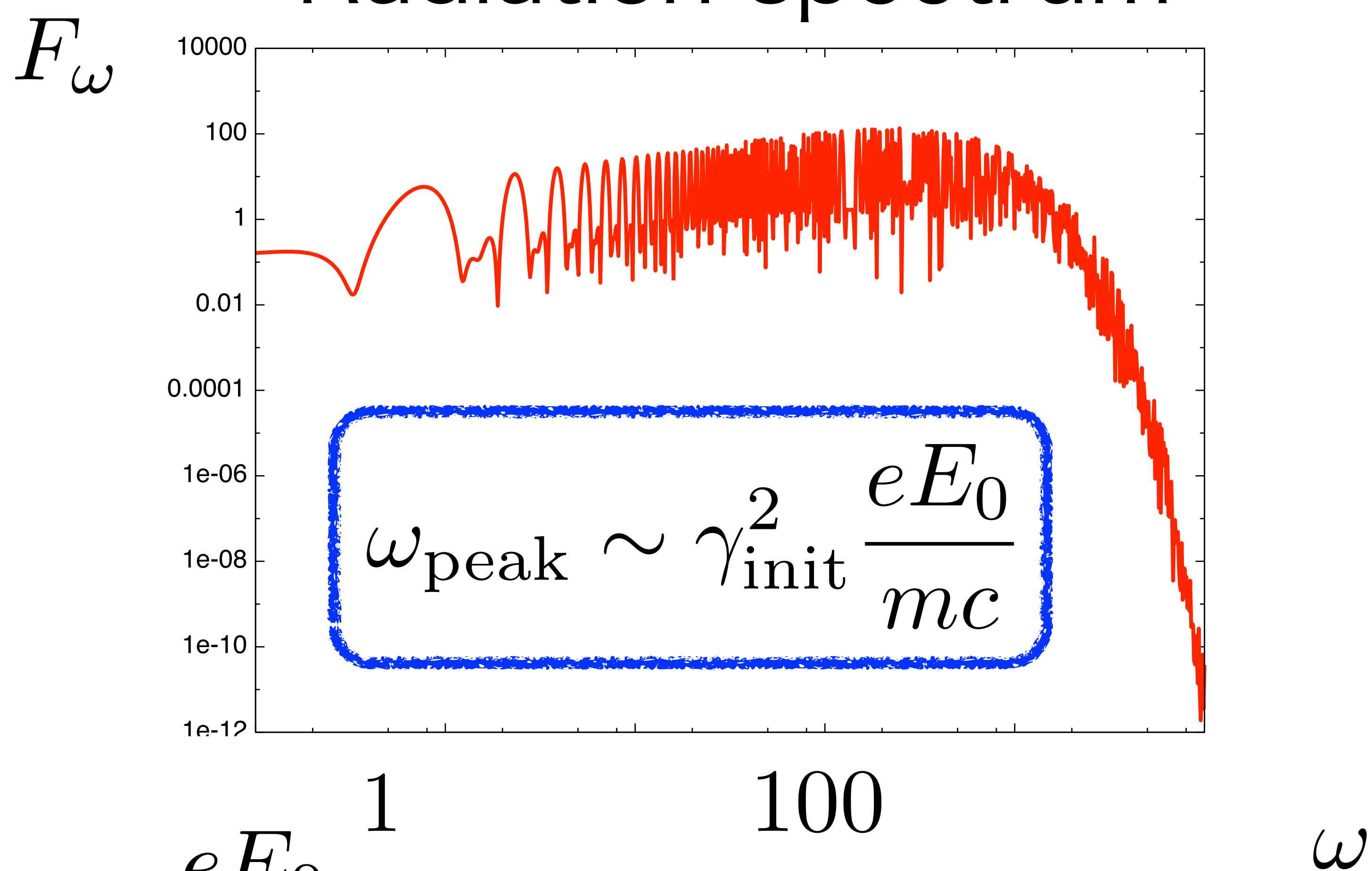
orbit



Lorentz factor



Radiation spectrum

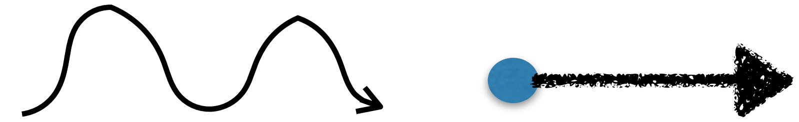


$$\omega_{\rm peak} \sim \gamma_{\rm init}^2 \frac{eE_0}{mc}$$

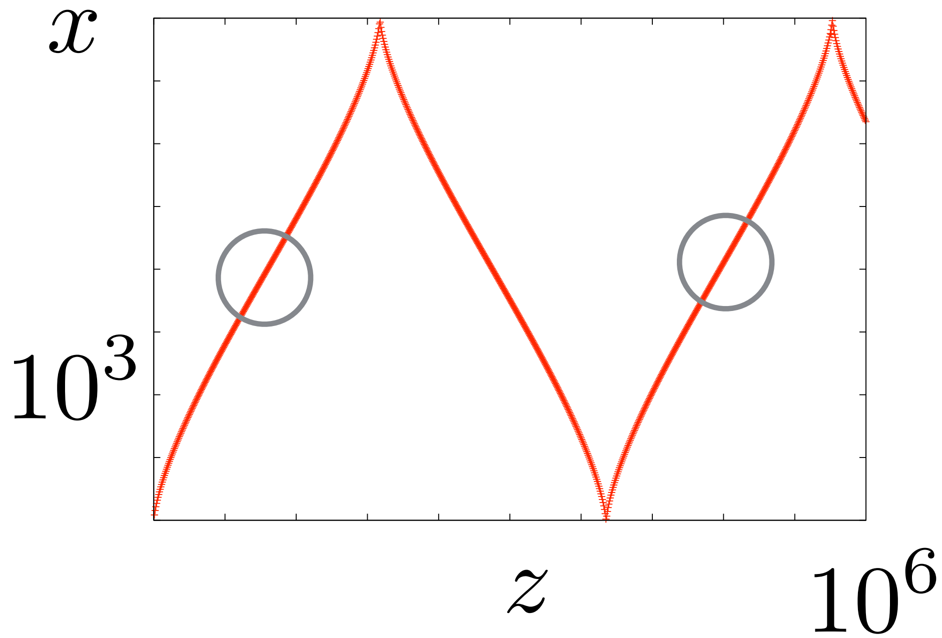
$$\frac{eE_0}{mc} = 1 \qquad \gamma_{\rm init} = 10$$

$$\omega_{\rm sw} = 0.1 \qquad \vec{n} = (0, 0, 1)$$

Rear-end collision



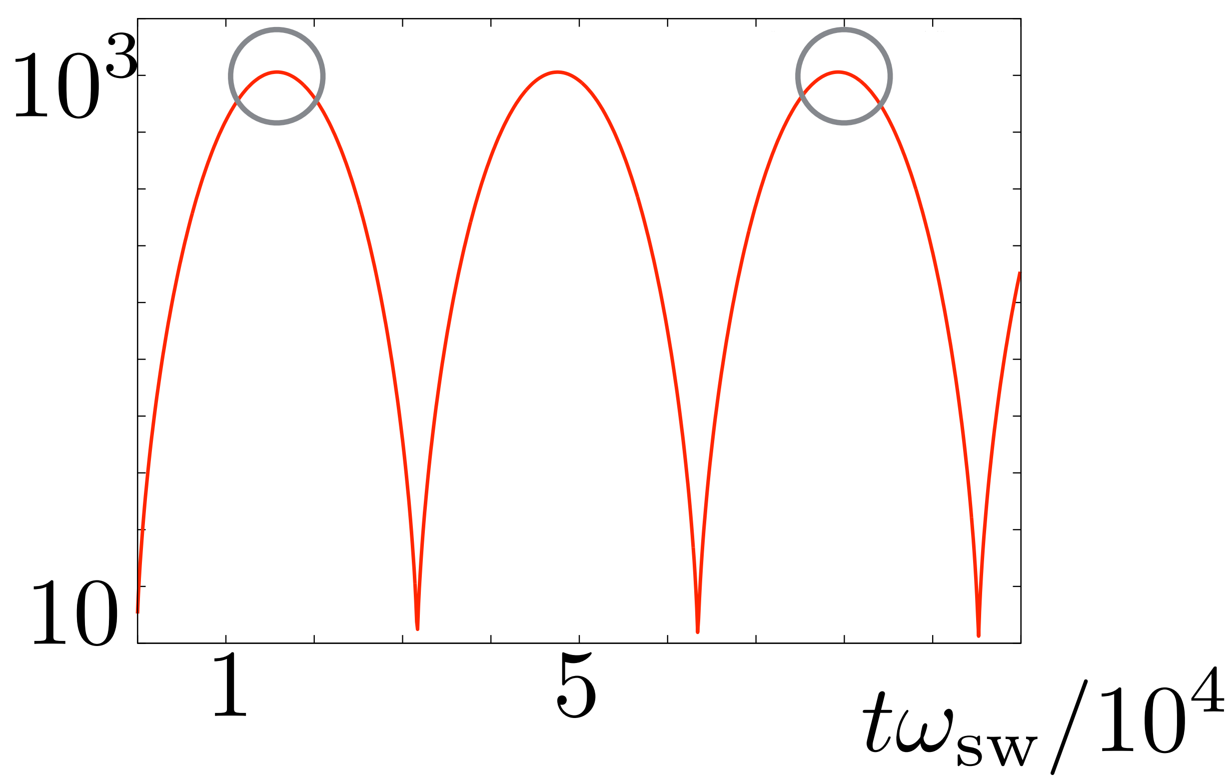
orbit



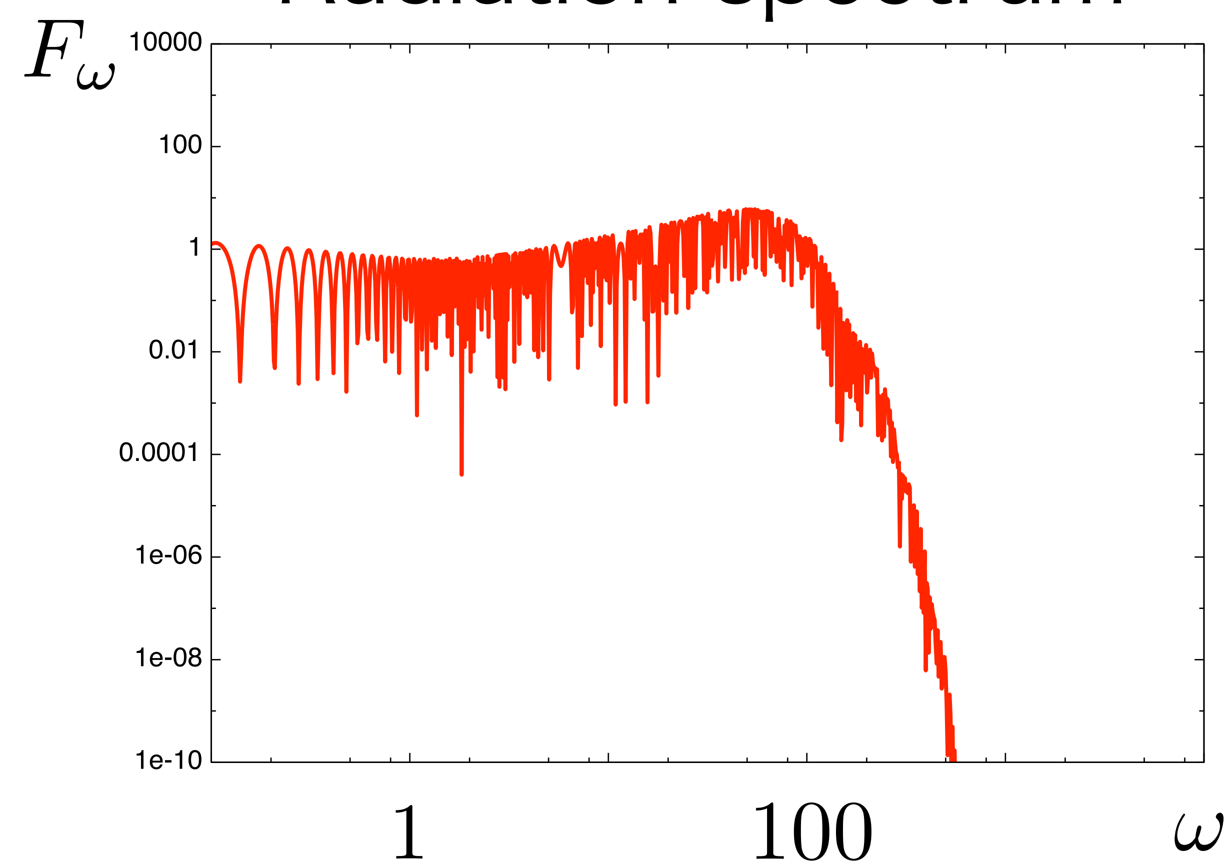
$$\omega_{\rm peak} \lesssim 100 \quad \Longleftrightarrow \quad \gamma_{\rm max}^2 \frac{eE_0}{mc} = 10^6$$

$$\omega_{\rm sw} = 0.1$$

Lorentz factor



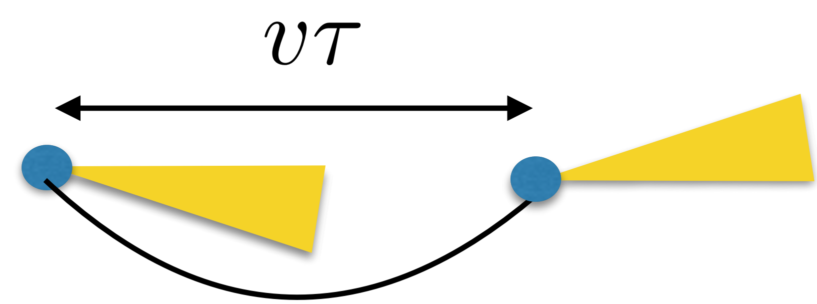
Radiation spectrum

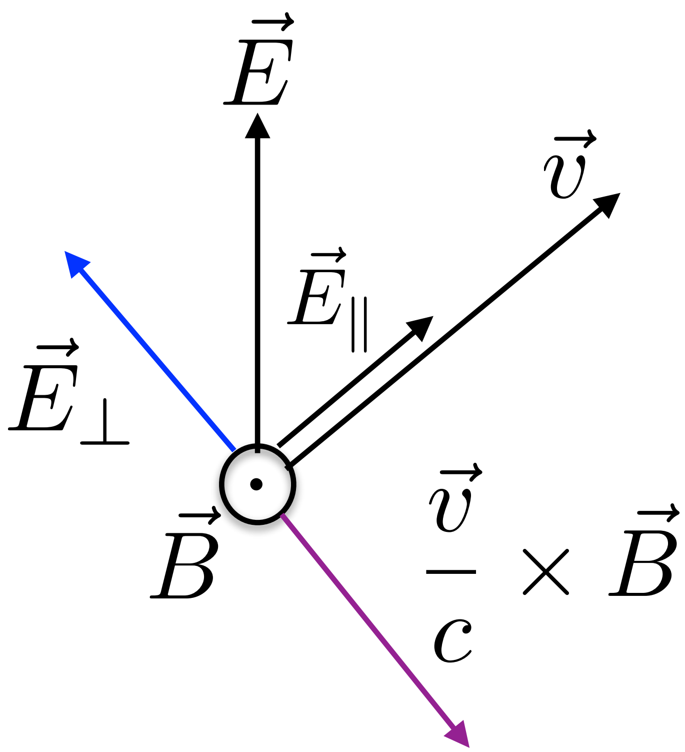


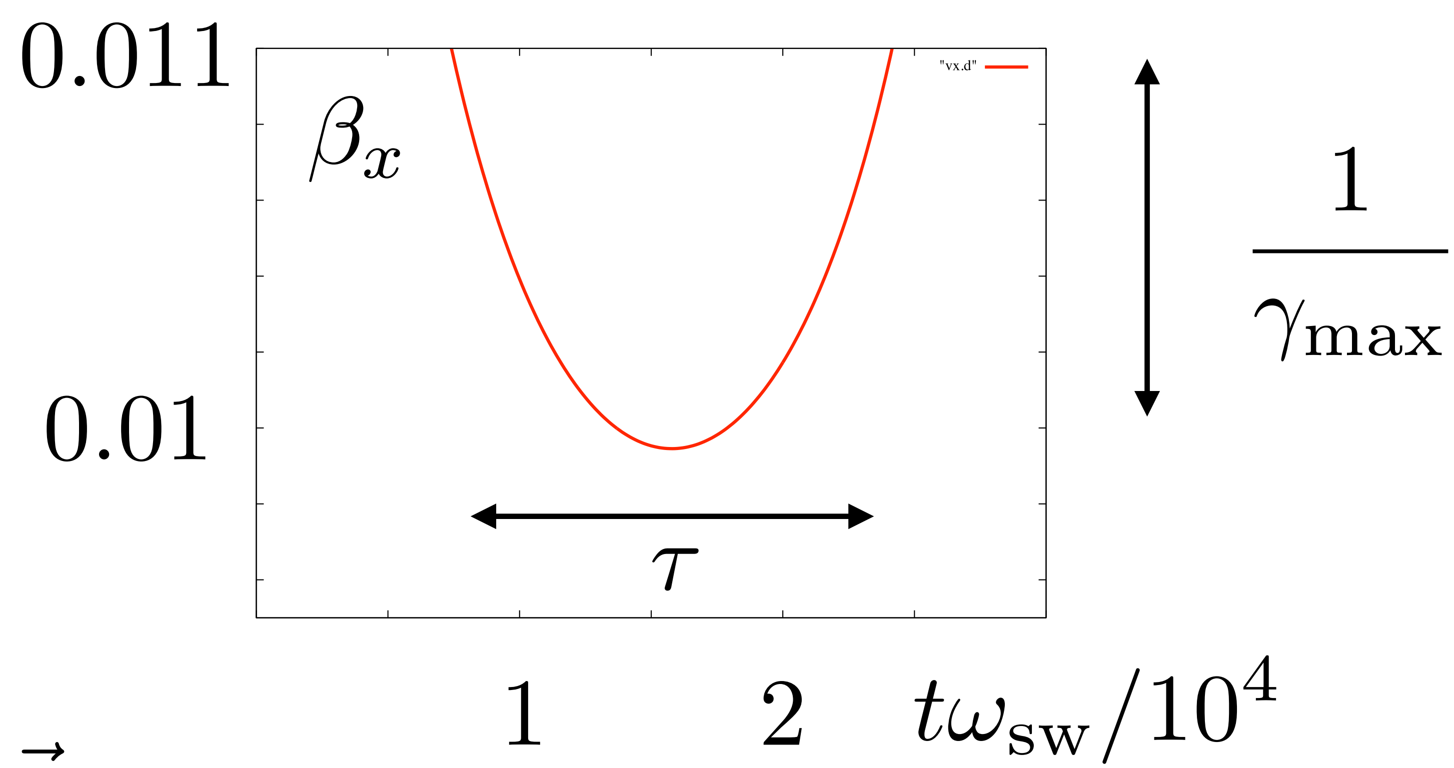
$$\vec{n} = (0.1, 0, 0.995)$$

Interpretation of the peak frequency 1

Electron is accelerated
with being trapped







$\vec{k}$ is nearly parallel with $\vec{v}$

$$\gamma_{\mathrm{max}} \sim \gamma_{\mathrm{init}} a^2 = 10^3$$

$$T_{\mathrm{particle}} \sim T_{\mathrm{sw}} a^2 \gamma_{\mathrm{init}}^2 = 10^4 T_{\mathrm{sw}}$$

$$\tau = T_{\mathrm{particle}}/K$$

$$K = O(1)$$

Interpretation of the peak frequency 2

$$\omega_{\rm peak} \sim \gamma_{\rm max}^2 \frac{2\pi}{\tau}$$

$$= (\gamma_{\rm init} a^2)^2 \frac{K\omega_{\rm sw}}{a^2\gamma_{\rm init}^2} = \underline{K a^2 \omega_{\rm sw}} = K \times 10 \lesssim 100$$

$$* \; K = O(1)$$

$$\frac{\text{Peak frequency for rear-end}}{\text{Peak frequency for head-on}} = \frac{K a^2 \omega_{\rm sw}}{\gamma_{\rm init}^2 \frac{eE_0}{mc}} = \frac{K a}{\gamma_{\rm init}^2}$$

Necessary condition for peak frequency is determined by a

$$\underline{a > \gamma_{\rm init}^2}$$

Crab flare as a jitter radiation of a SLSW

$\gamma_{\max} \simeq 10^{10}$ → Maximum Lorentz factor by DSA

$\lambda_{\mathrm{SLSW}} \sim 10^9\mathrm{cm}$ in quiescent state $B = 10^{-4}\mathrm{G}$

$$\nu_{\mathrm{jit}} \simeq \gamma_{\max}^2 c/\lambda_{\mathrm{SLSW}} = 3 \times 10^{21}\mathrm{Hz}$$

$$\nu_{\mathrm{flare}} = 10^{22-23}\mathrm{Hz}$$

$\lambda_{\mathrm{SLSW}} < 10^8\mathrm{cm}$
for $a < 1$

Peculiar cascading may invoke a flare

$$E^2(\omega) \propto \omega^{-\alpha} \quad \alpha < 1$$