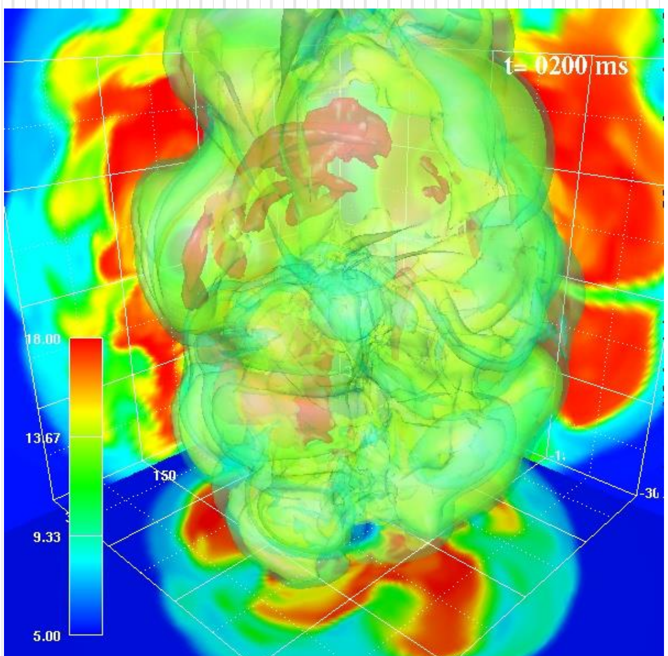
2013/9/26 RIKEN

Neutrino Transport for Supernovae Simulations from Petaflops era to Exaflops era



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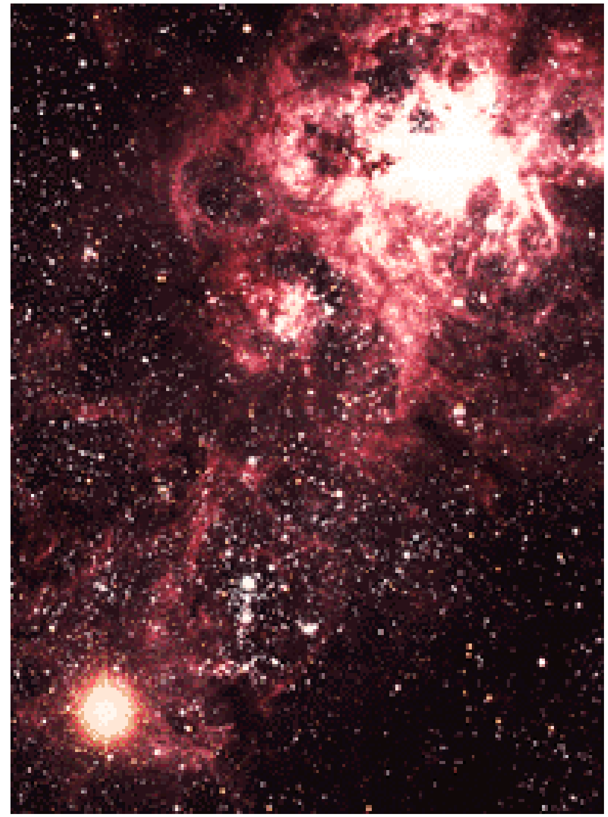
Most luminous object in Universe



Visible light

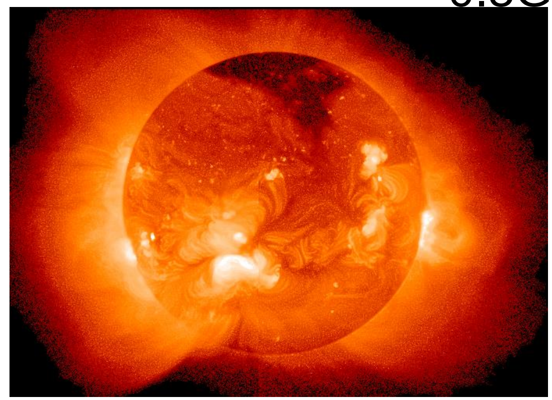
Supernova 1987A in the Large Magellanic Cloud (1987 Oct)
D_LMC=48.8kpc=12 k ly

Supernovae(10pc)M=-18
1 billion of Sun（4.8）
Milkyway（M=-20.5）1/6
Decay 100day

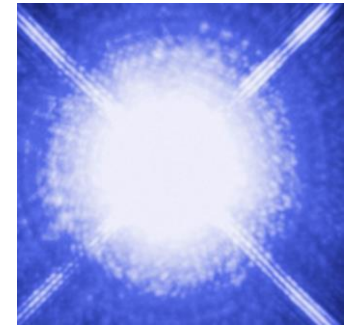
Death of Massive Star

Gas Clould

~0.5Gm



Main Sequence(Sun)

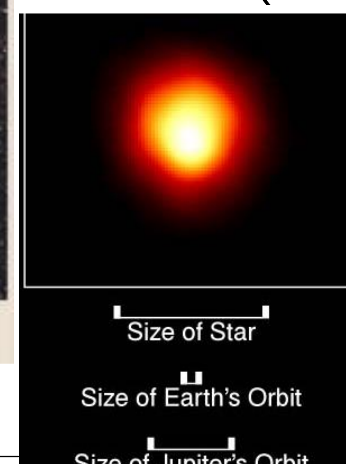


White dwarf(Sirius)

Light star
~100 M year

Heavy star
~10 k year

Red Giant(Betergius)



~100Gm

Supernova(1987A)

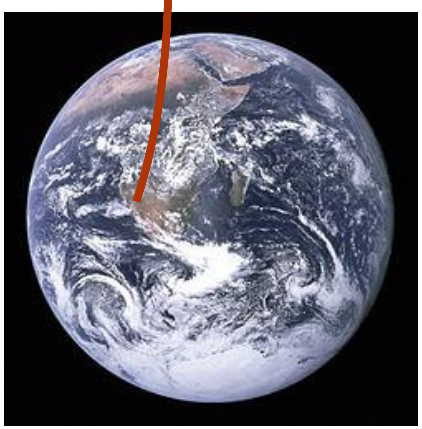
Supernova Remnant
(Cas A)

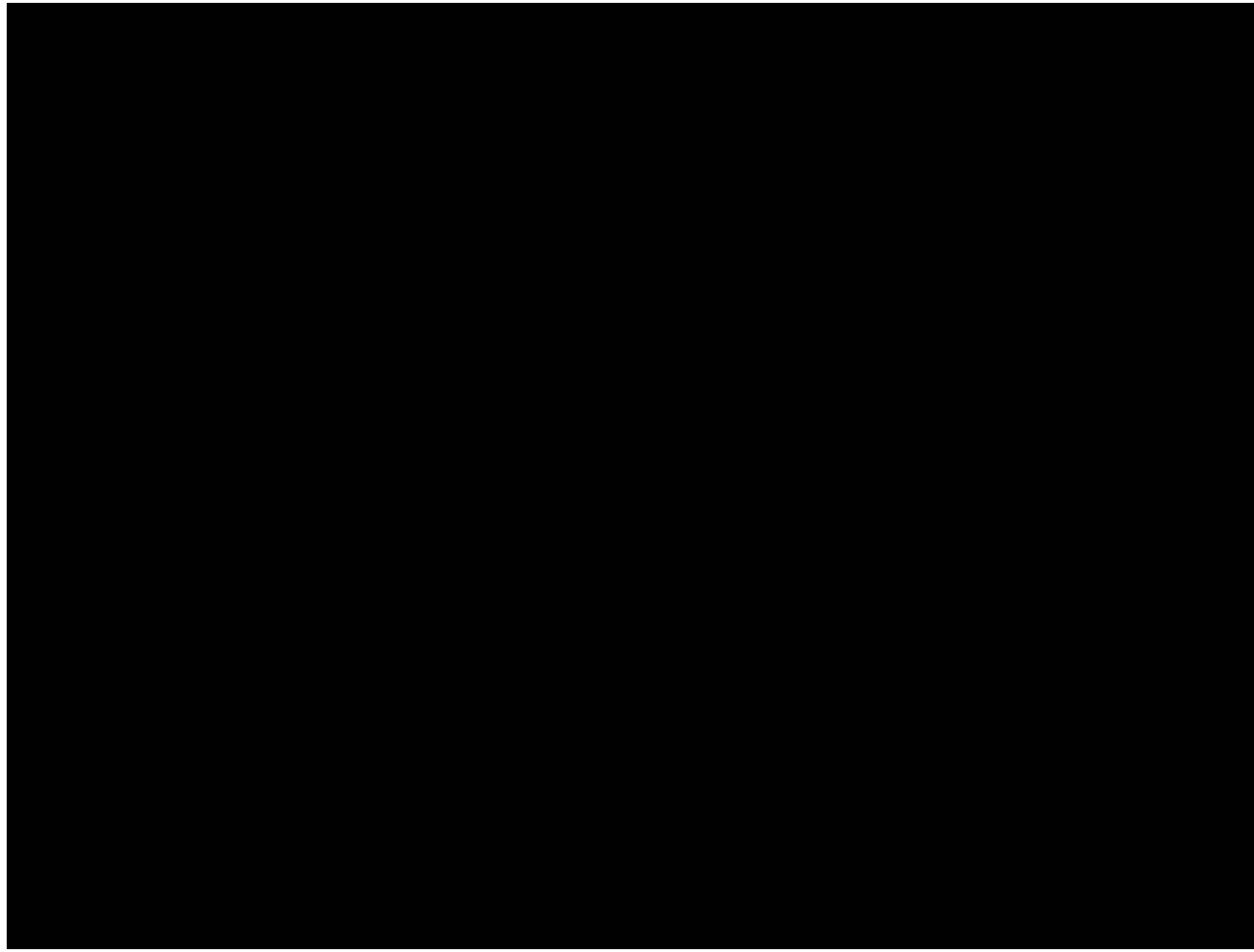
~10km

Neutron Star

Radate Cosmic Ray

Fermi acceleration

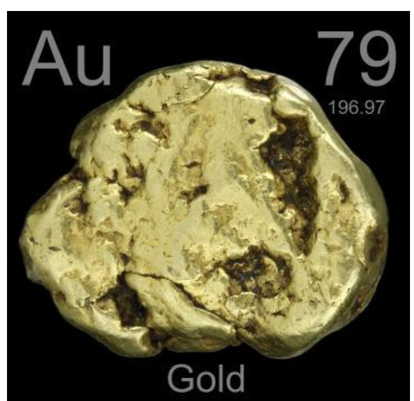




- Particles are accelerated in the strong magnetic fields and propagate to the earth

Origin of Heavy element

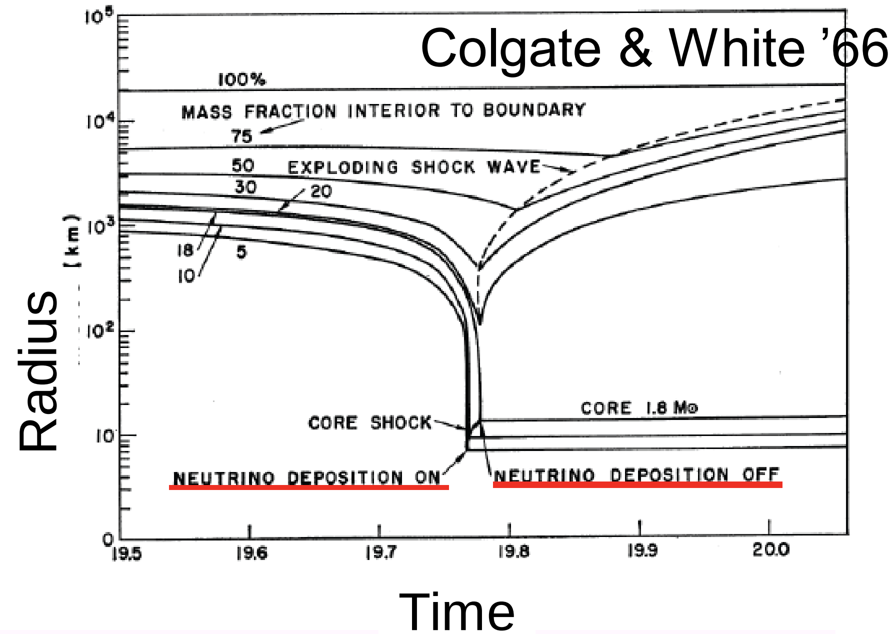
- Heavy elements like iron is distributed by supernovae
- It is the origin of other stars.

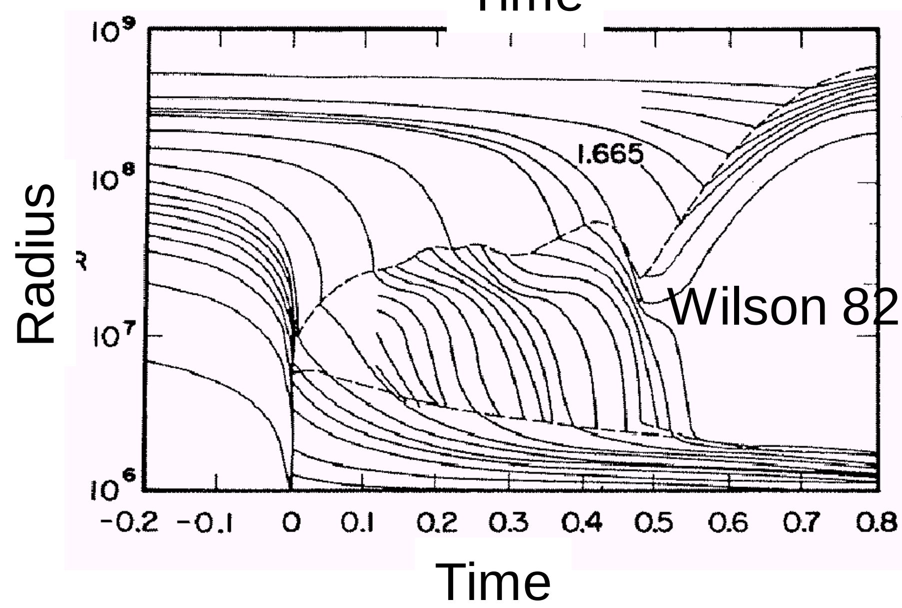


History of supernovae simulations

- Colgate & White ’66
  began simulation
- Wilson’ 82
  shows explosion by neutrino heating
- Even today, the mechanism is not fully understood yet!

The explosion mechanism is everlasting question during 50 years.





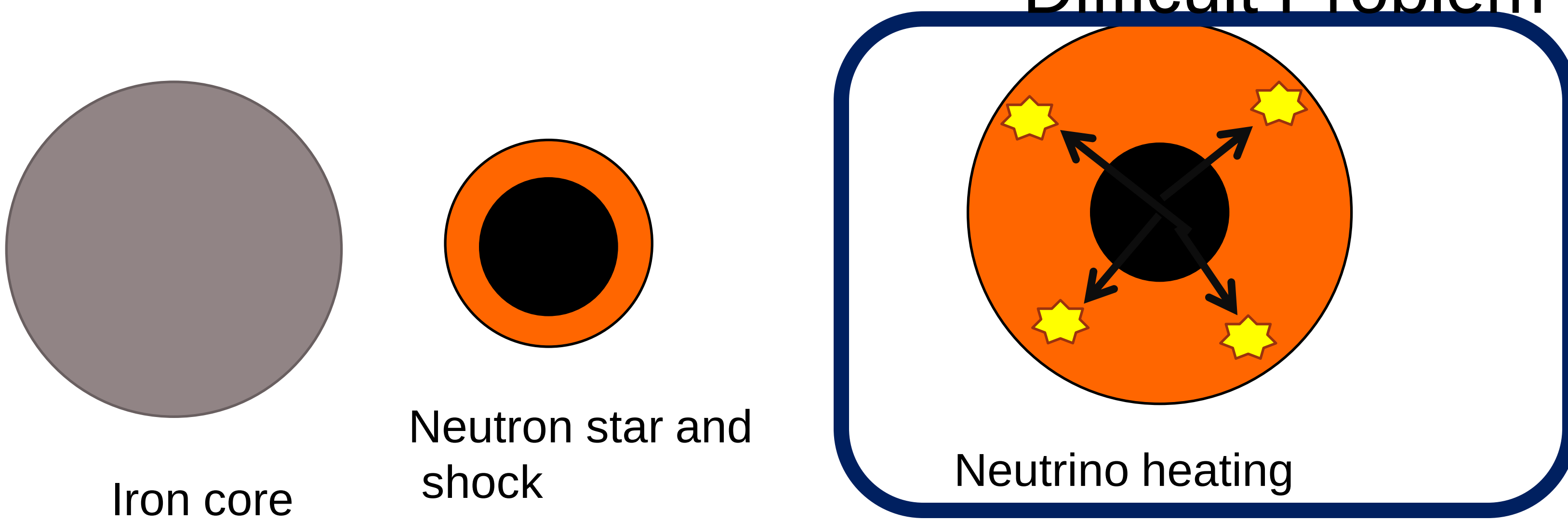
Brief intro. of Neutrino heating Mechanism

(1) Iron core begin to shrink by the strong gravitational force

(2) Density increase and neutron star is produced.
Iron core stops to shrink and desacceleration makes shock wave

(3) the shock is heated by high energy neutrino radiated from the neutron star and finally blow up the outer layer.

Difficult Problem !

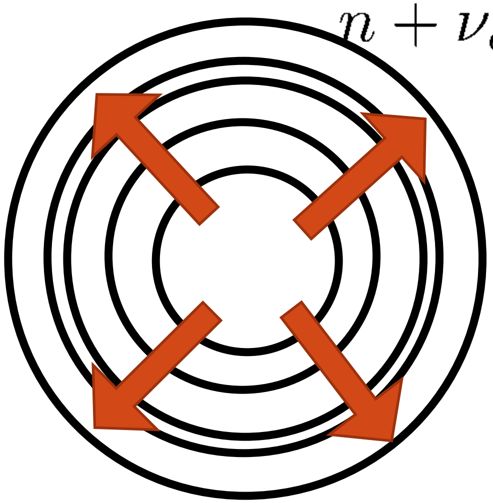


Past of supernovae study

○2000 -2005

- Assume spherical symmetry and solve sophisticated neutrino transport

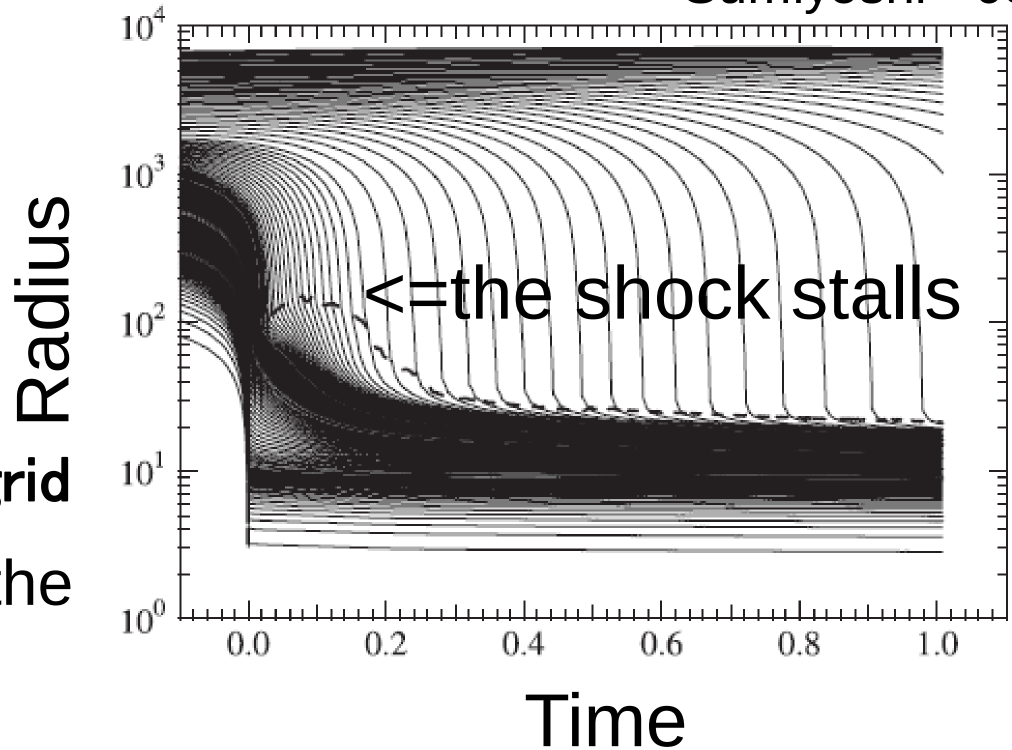
$$n + \nu_e \to p + e^-$$



Spherically symmetric grid

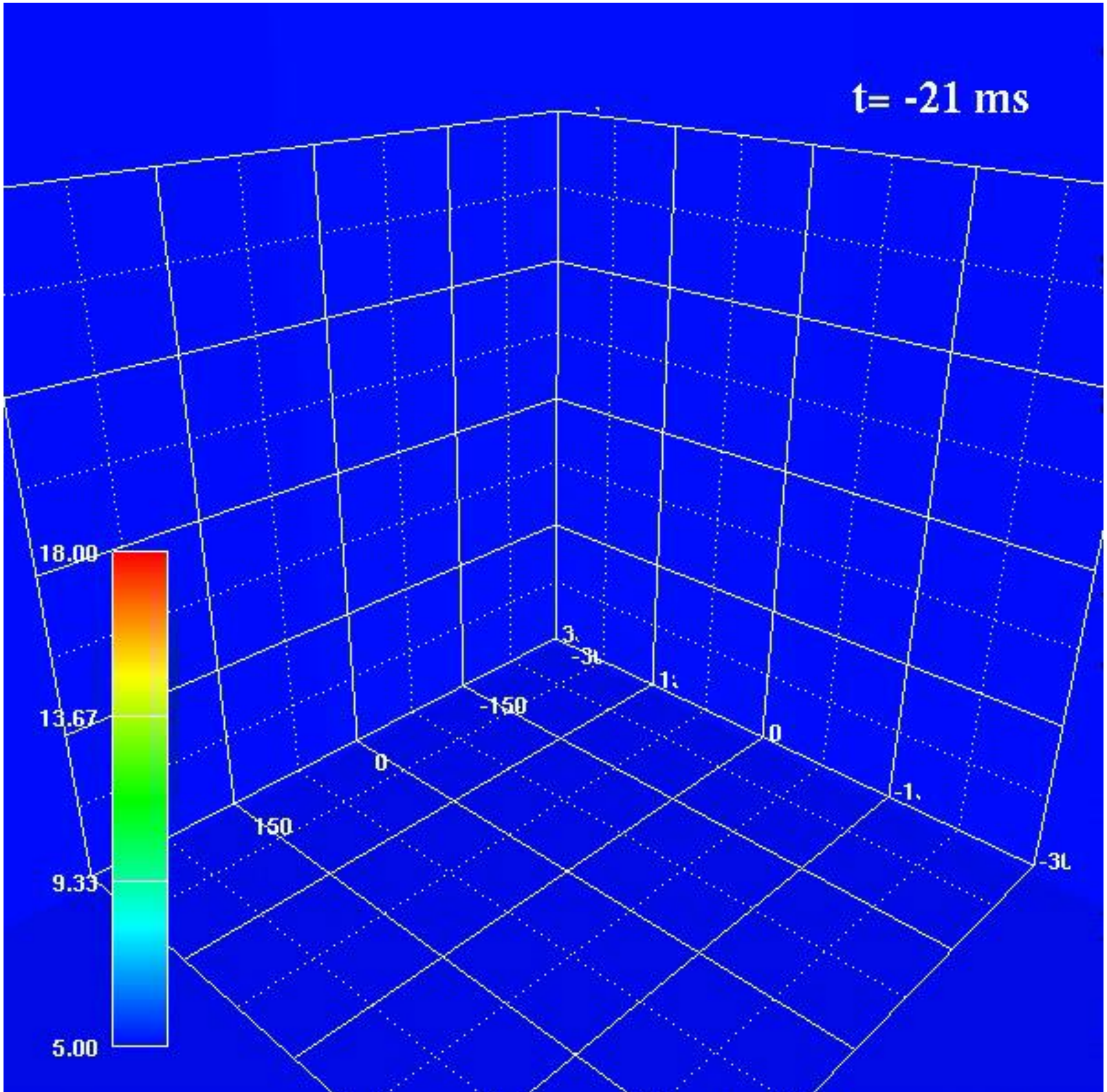
Neutrino is going from the center and head outer layor

Sumiyoshi+ 05



＝＞Why it fails? What is missing?

A example of no explosion



1D-spherical symmetry

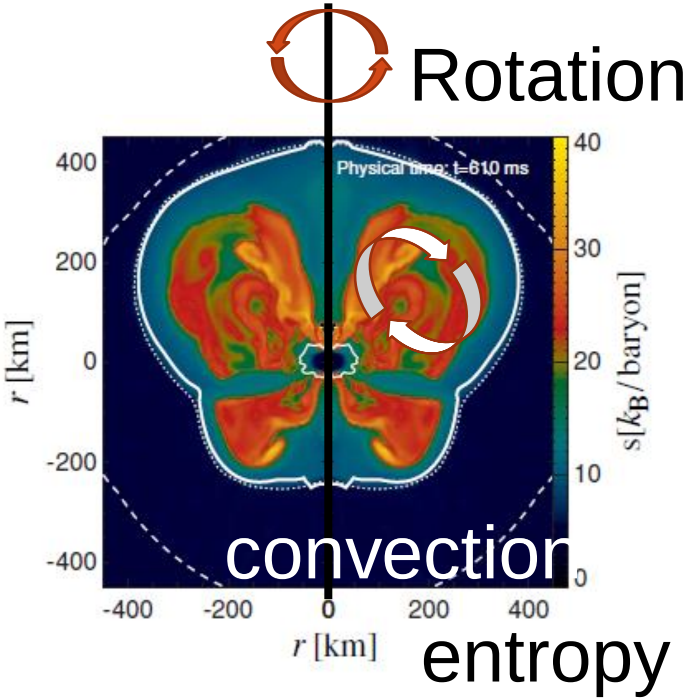
Entropy is visualized

Supernova fails

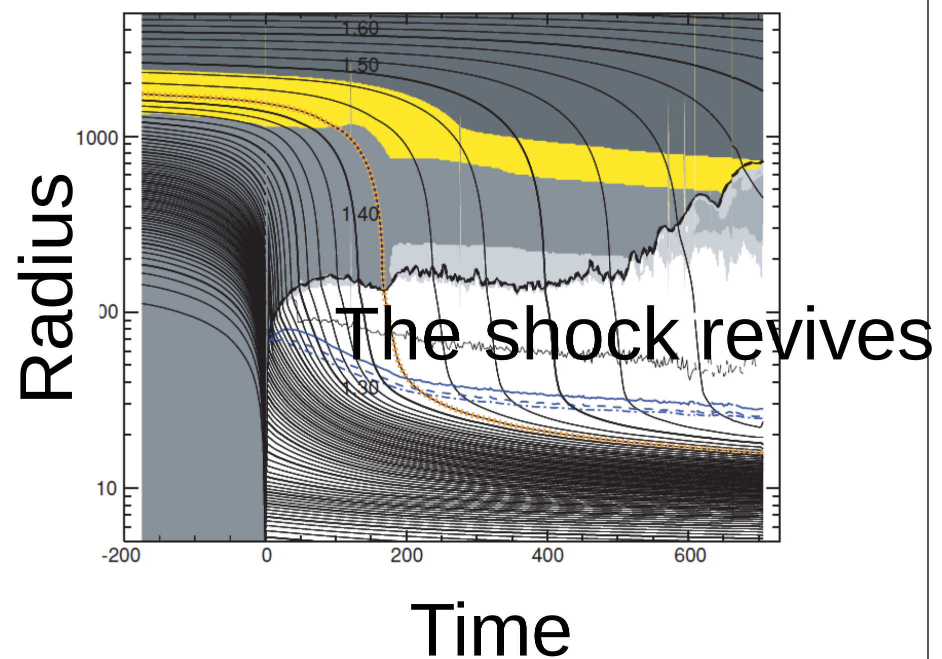
Past of supernovae study（2）

○2005–2010

- Assume axi-symmetry and investigate effect of the convection



Marek & Janka 2009



- Convection enhances neutrino heating rate and found successful explosion. But.. 2D is not natural.

Question and Motivation

Nature is in 3D

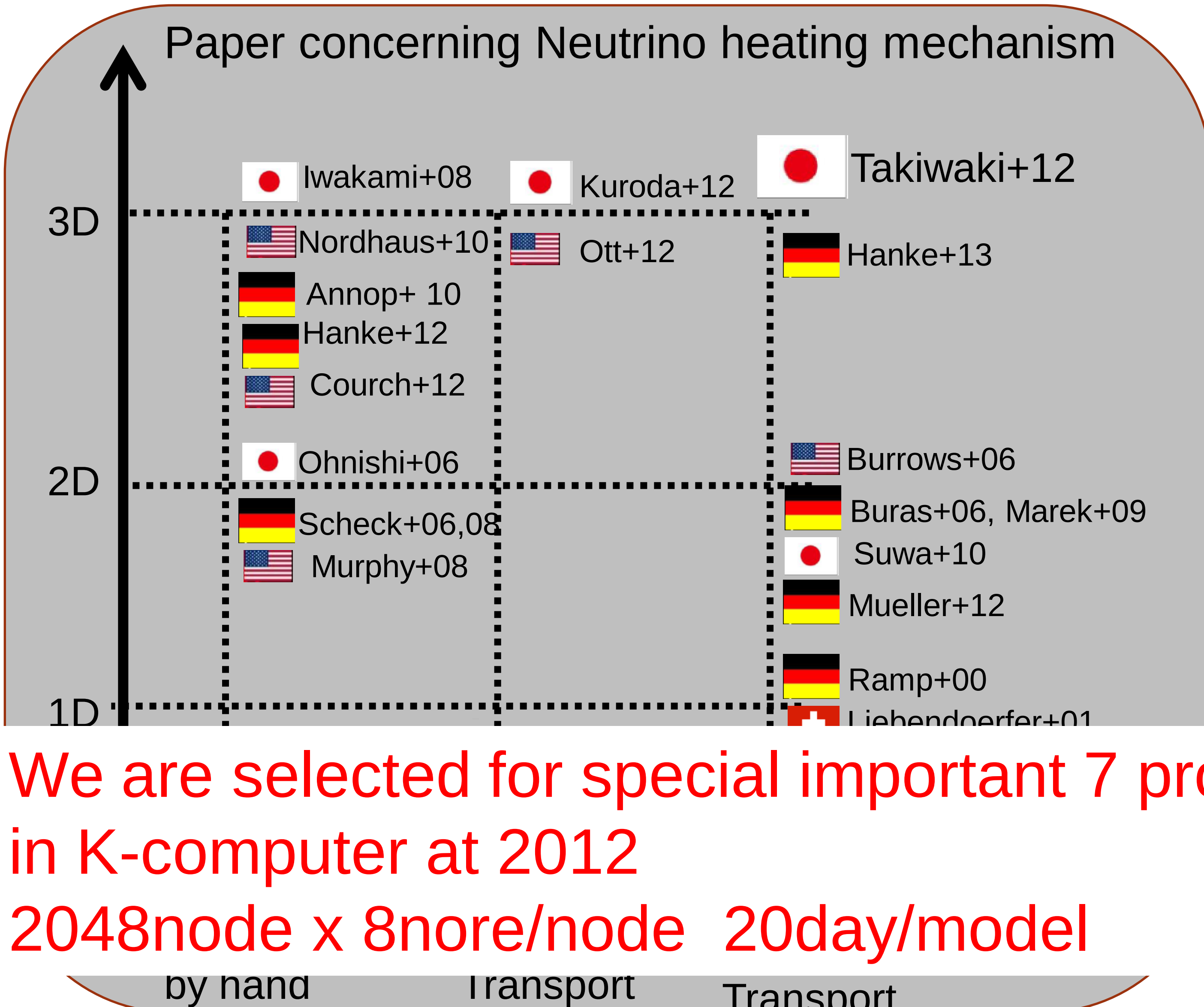
Does Neutrino heating mechanism works in 3D simulation?

Q1 Explosion or No explosion?

Q2 How energetic that is?

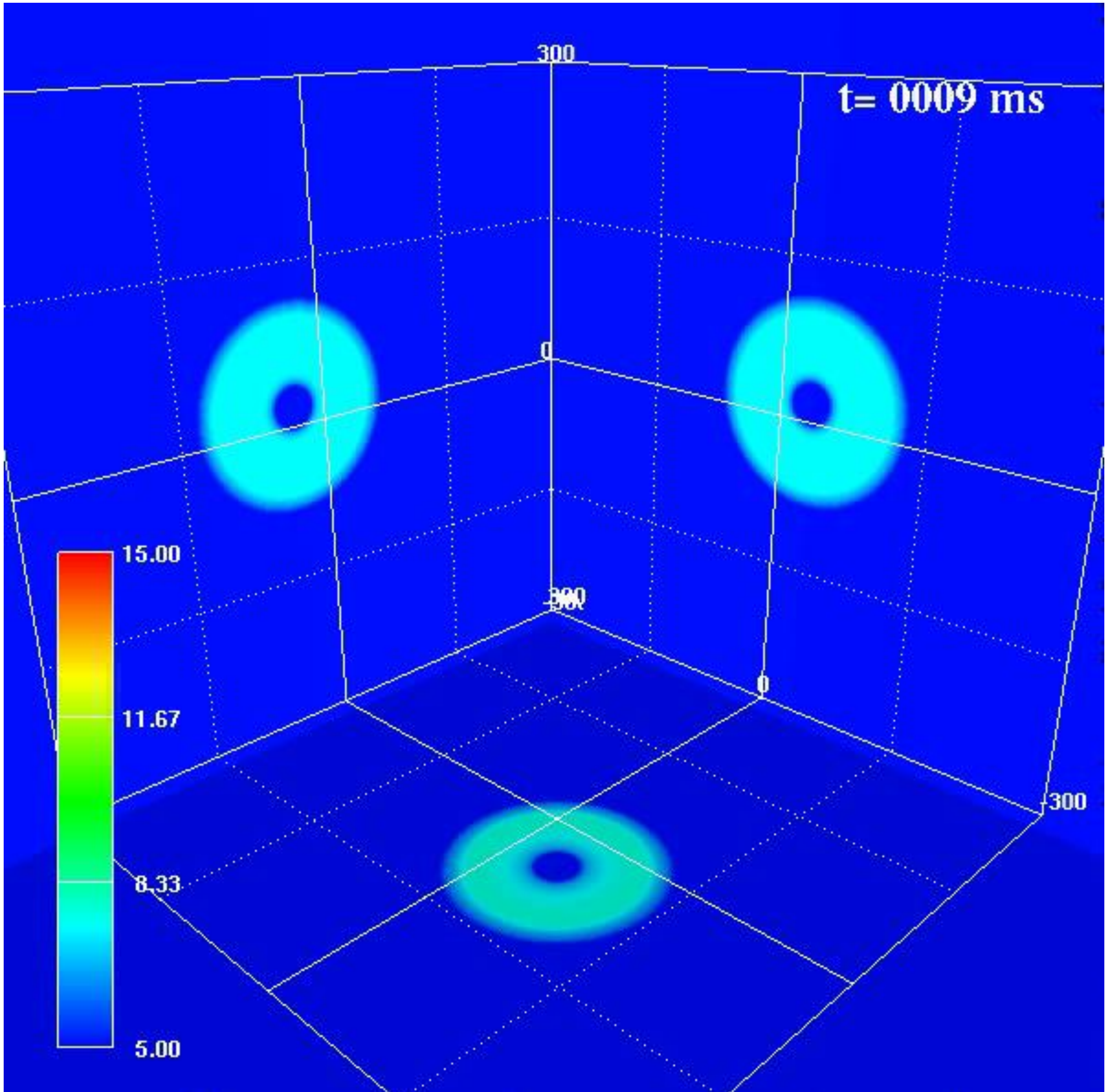
Q3 How is the shape of the shock?

Competition in the world



We are selected for special important 7 problems
in K-computer at 2012
2048node x 8nore/node 20day/model

Q1：Explosion or No explosion?



Explode ！

progenitor:s11.2

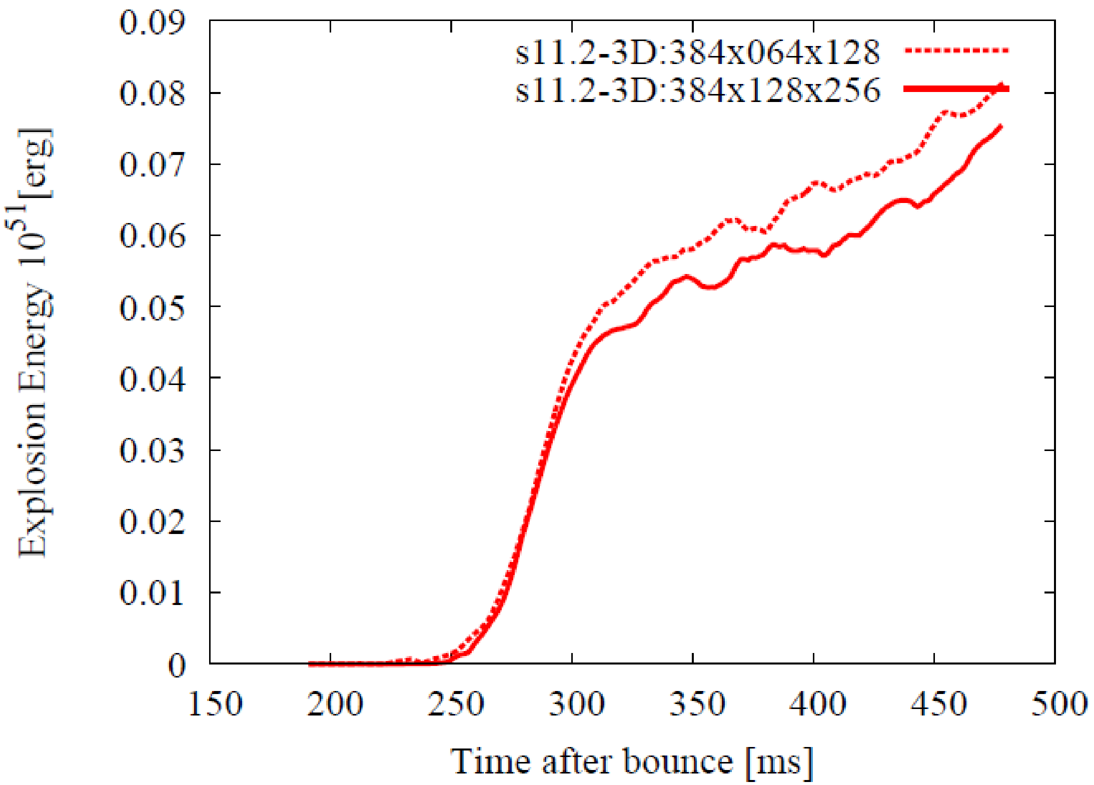
EoS：LS-K220

resolution：
384(r)x128(θ)x256(φ)
The finest grid

Neutrino Trasport：
Ray-by-Ray:IDSA
+Leakage

Hydro:
HLLE, 2nd order

Q２ ：How energetic that is?

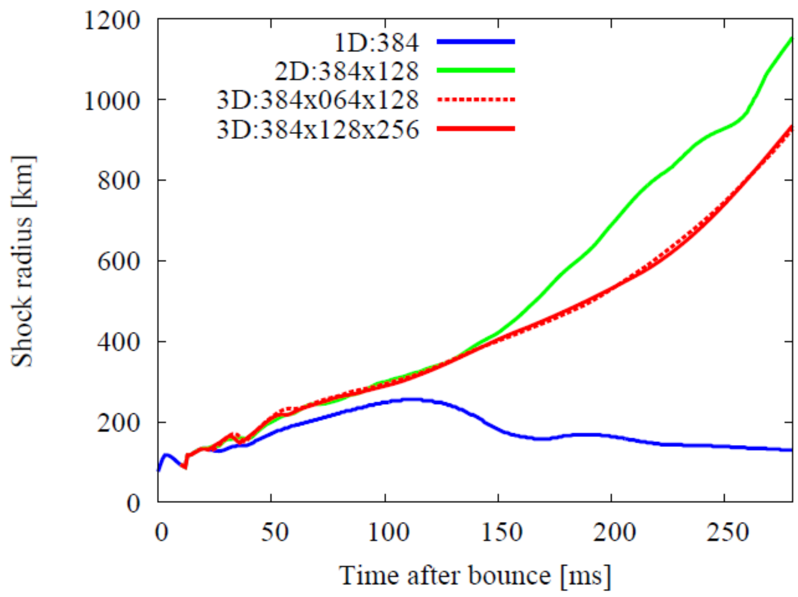


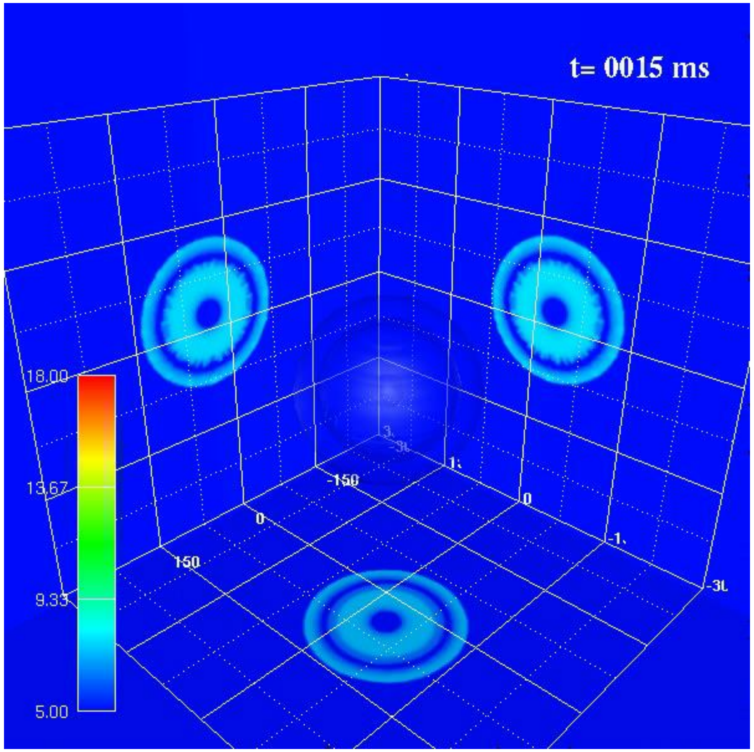
E_exp~a few 10^50erg

(still increasing but approximately.)

Typical supernovae~10^51erg, a little smaller than the observation

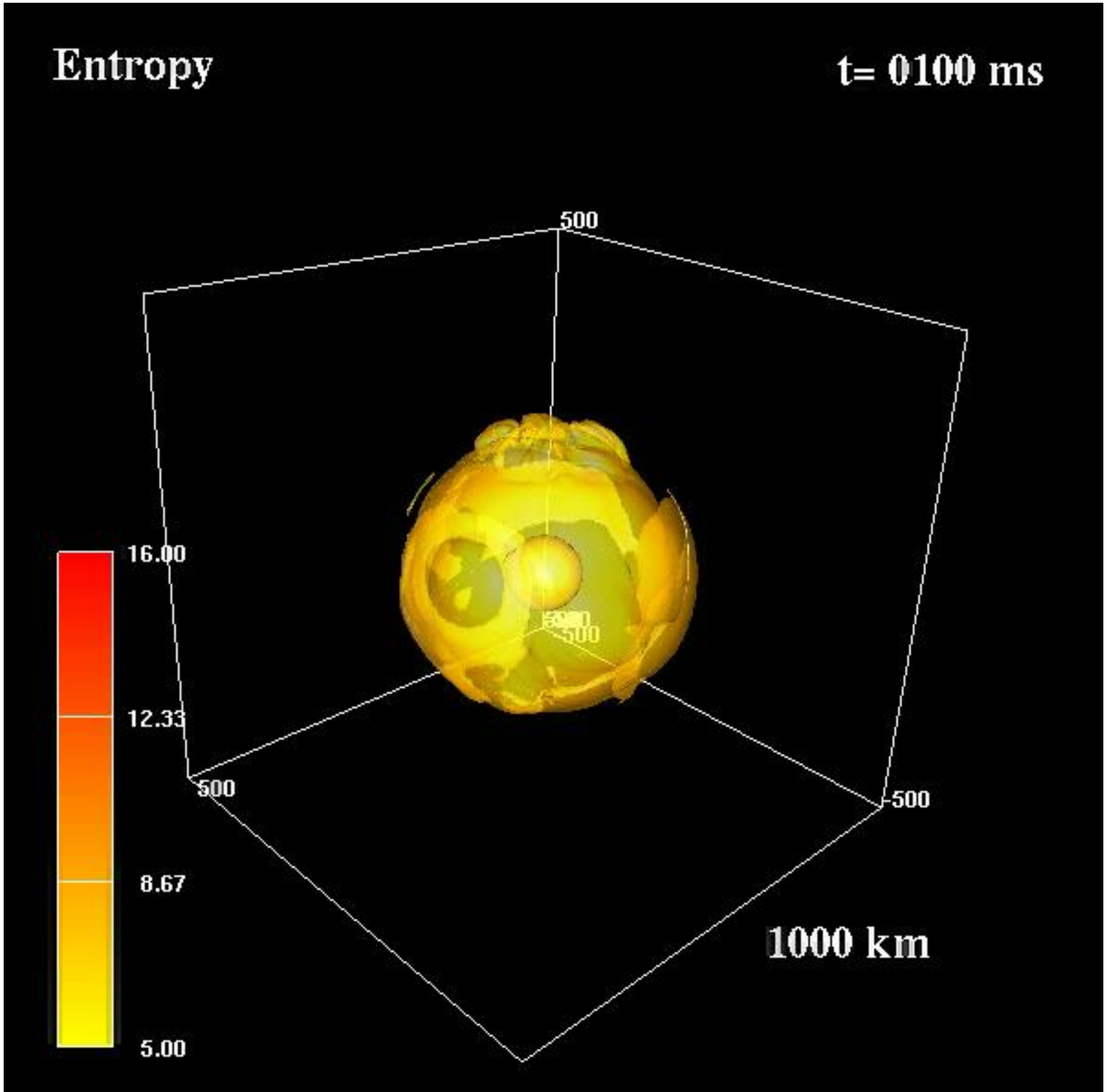
2D vs 3D





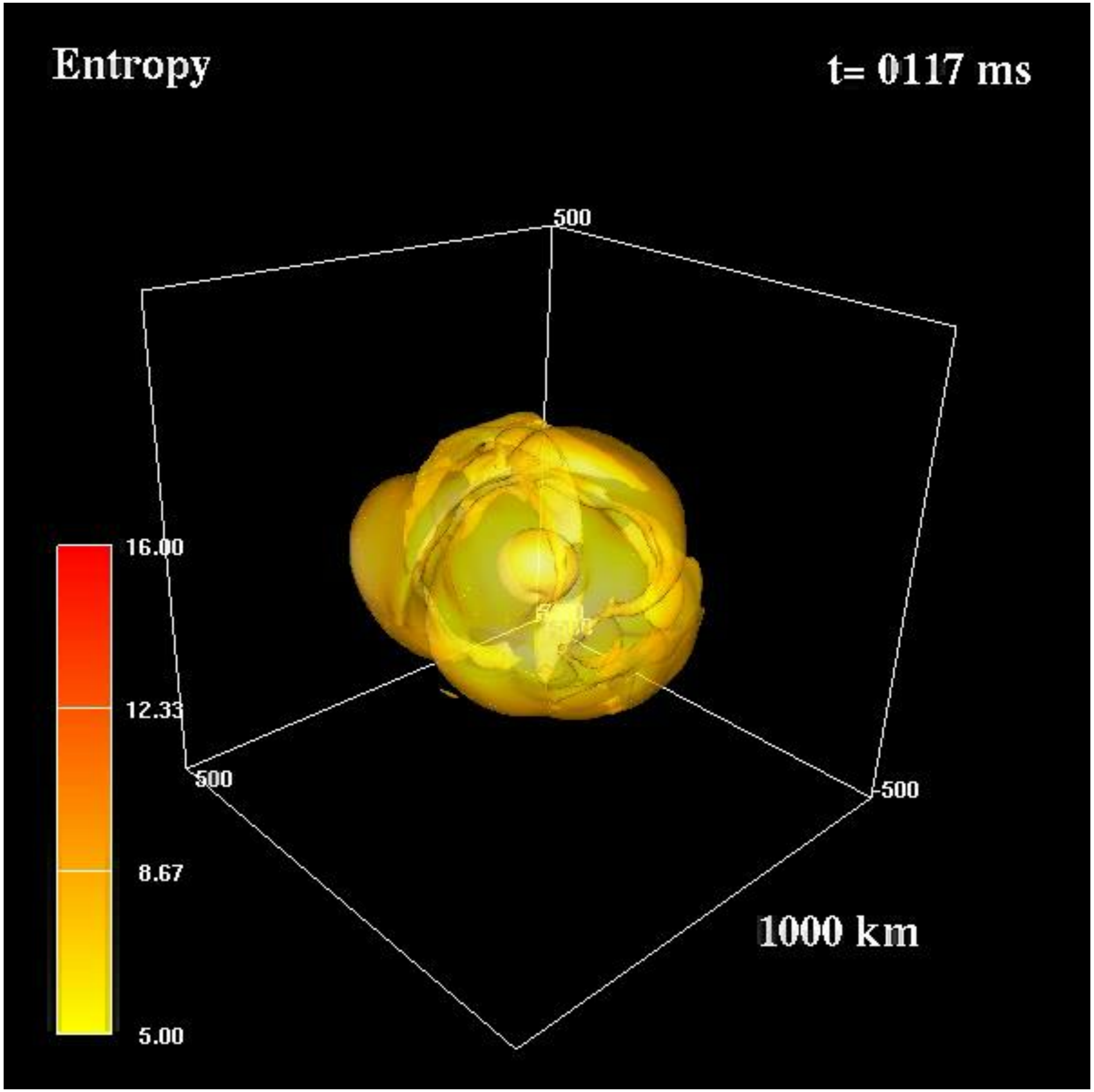
2D simulation overestimate the strength of the shock.
Large convection tend to remain by artificial assumption.

Q3:Shape？



Approximately spherical. Small bubbles are found.

Q3:Shape（with rotation）？

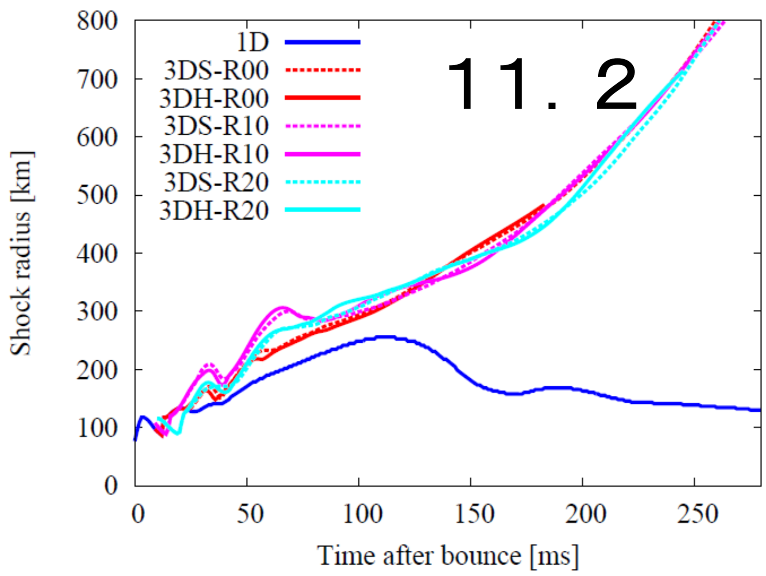


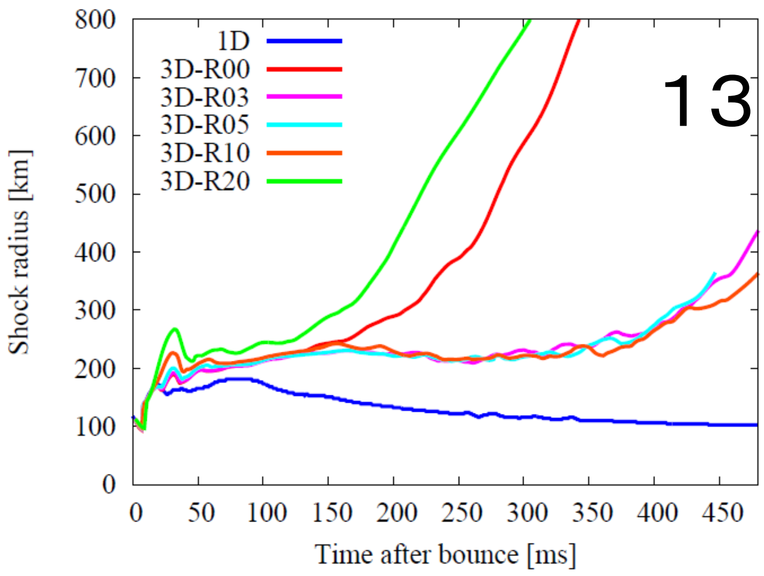
Strong shock is found at equator

First found in 3D model.

Does rotation affect the shock revival?

↓回転





For 11.2M_s, light progenitor, it does not.
For 13 M_s it does. Rapid rotation makes the shock oblate and the shock expansion begins from the eqator.

We perform 3D simulation with self-consistent manner and answer three typical questions.

Q1 Explosion? → A1 : Found explosion at low-mass progenitor

Q2 How energetic? → A2: smaller than the typical value by factor

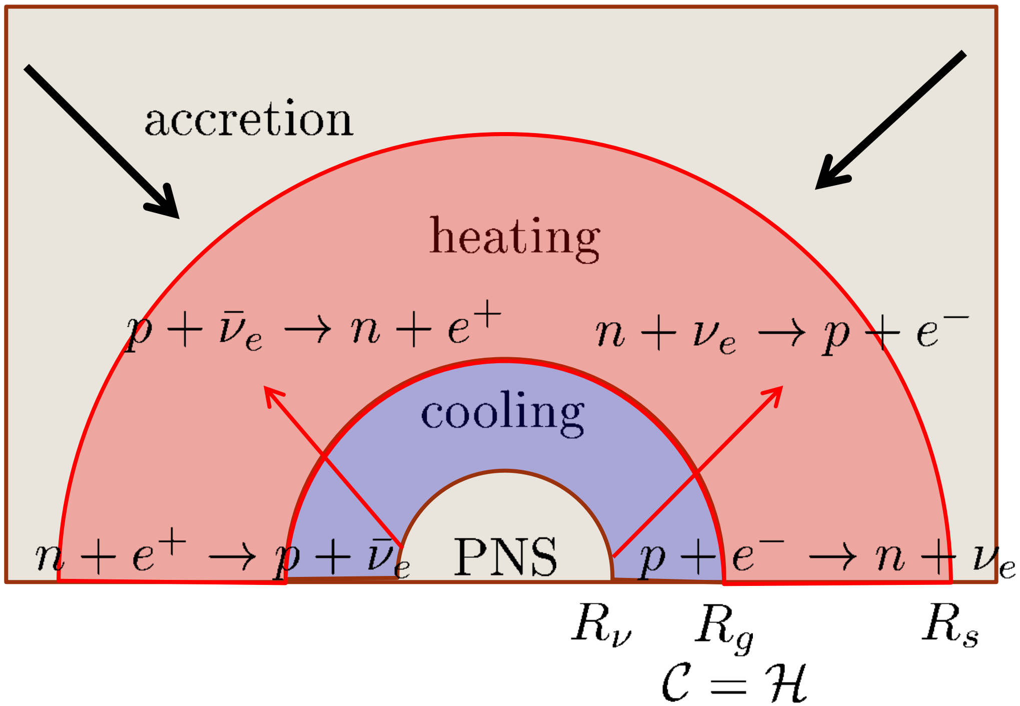
Q3 How is the shape? → A3: no rotation=>spherical
+ small bubbles
rapid rotation => oblate

Supernova modeling begins to succeed by supercomputers.

Method of Neutrino Transport in Petaflops era and Exaflops era

Basics of Neutrino Heating Mechanism

Janka 01



-Cooling term

$$p+e^- \rightarrow n+\nu_e$$

$$Q_\nu^- = (3\alpha^2+1)\frac{\pi\,\sigma_0\,c\,(kT)^6}{(hc)^3(m_e c^2)^2}\frac{\rho}{m_u} \times [Y_p\mathcal{F}_5(\eta_e)+Y_n\mathcal{F}_5(-\eta_e)] \approx 145\,\frac{\rho}{m_u}\left(\frac{kT}{2\,\mathrm{MeV}}\right)^6 \left[\frac{\mathrm{MeV}}{\mathrm{s}}\right]$$

$$\rho_{\mathrm{proton}} \times \rho_{\mathrm{electron}}(\propto T^3) \times \sigma(\propto T^2) \times \bar{E}(\propto T)$$

-Heating term

$$n+\nu_e \rightarrow p+e^-$$

$$Q_\nu^+ = \frac{3\alpha^2+1}{4}\frac{\sigma_0\langle\epsilon_{\nu_e}^2\rangle}{(m_e c^2)^2}\frac{\rho}{m_u}\frac{L_{\nu_e}}{4\pi r^2\langle\mu_\nu\rangle}(Y_n+2Y_p) \approx 160\,\frac{\rho}{m_u}\frac{L_{\nu_e,52}}{r_7^2\langle\mu_\nu\rangle}\left(\frac{kT_{\nu_e}}{4\,\mathrm{MeV}}\right)^2\left[\frac{\mathrm{MeV}}{\mathrm{s}}\right].$$

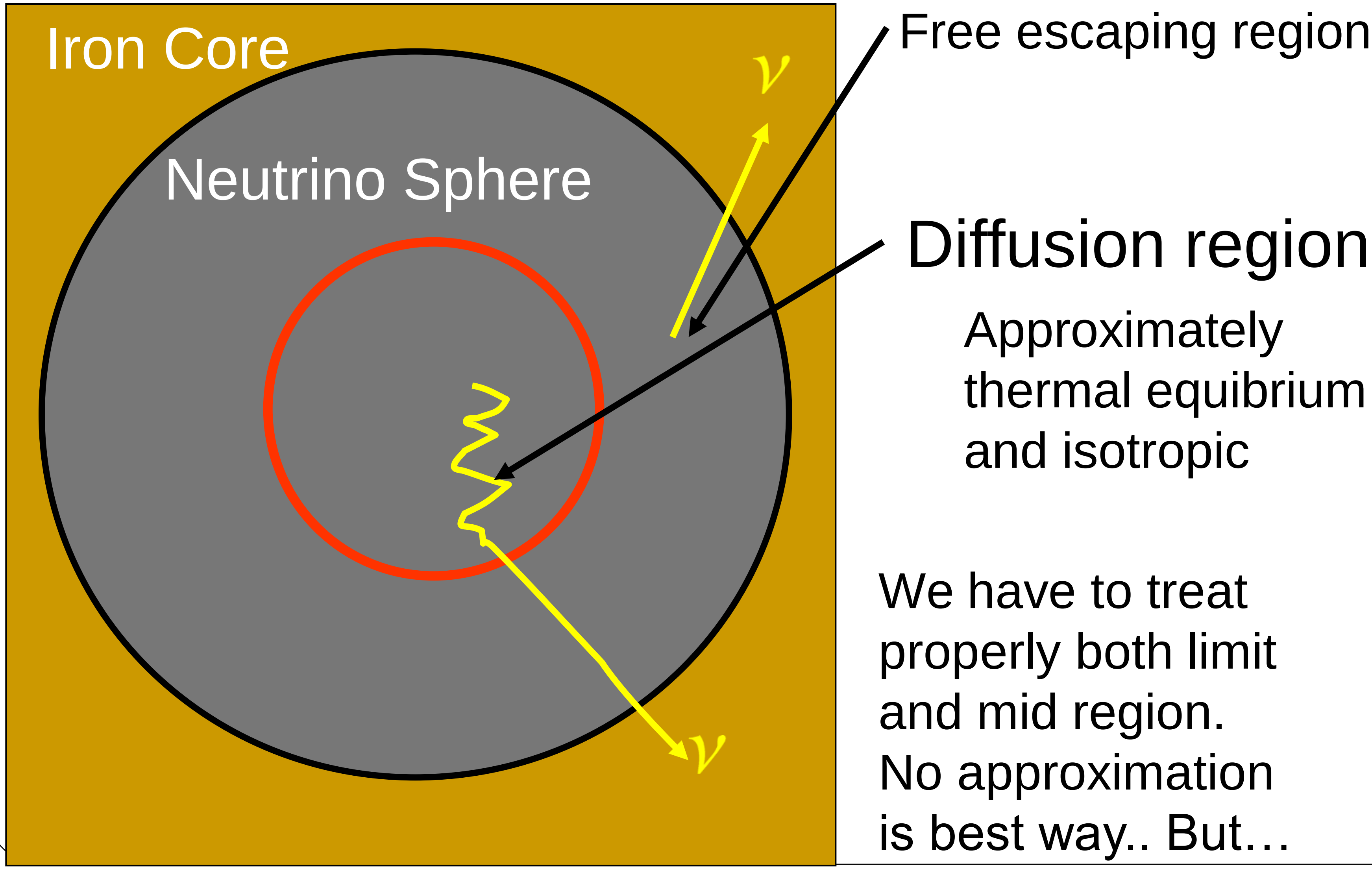
$$\rho_{\mathrm{neutron}} \times \rho_{\mathrm{neutrino}}(\propto T_\nu^3/r^2) \times \sigma(\propto T_\nu^2) \times \bar{E}(\propto T_\nu)$$

If we assume hydrostatic profile with pressure of radiation dominant.

$$\mathcal{C} \propto 1/r^9$$
$$\mathcal{H} \propto 1/r^5$$

Above gain radius, the heating is dominant.

Neutrino transport



Approximately thermal equibrium and isotropic

We have to treat properly both limit and mid region.
No approximation is best way.. But…

Solvers of Radiation Hydrodynamics

Ab initio, no assumption=>

Boltzmann equation, 6D differential equation

$$f^{\text{in}}(\underline{r, \theta, \phi,}\, t;\, \underline{\mu_\nu, \phi_\nu, \varepsilon^{\text{in}}})$$

Sn method directly solve the equation

3 space

3 phase space(momentum or velocity space)

$$\begin{aligned}
&\frac{1}{c}\frac{\partial f^{\text{in}}}{\partial t} + \frac{\mu_\nu}{r^2}\frac{\partial}{\partial r}(r^2 f^{\text{in}}) + \frac{\sqrt{1-\mu_\nu^2}\cos\phi_\nu}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f^{\text{in}}) \\
&\quad + \frac{\sqrt{1-\mu_\nu^2}\sin\phi_\nu}{r\sin\theta}\frac{\partial f^{\text{in}}}{\partial\phi} + \frac{1}{r}\frac{\partial}{\partial\mu_\nu}\left[(1-\mu_\nu^2) f^{\text{in}}\right] \\
&\quad - \frac{\sqrt{1-\mu_\nu^2}}{r}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\phi_\nu}(\sin\phi_\nu f^{\text{in}}) = \left[\frac{1}{c}\frac{\delta f^{\text{in}}}{\delta t}\right]_{\text{collision}},
\end{aligned}$$

Sumiyoshi & Yamada 2012

Computation cost is extra-ordinary high

Solvers of Radiation Hydrodynamics

$$\frac{1}{c}\frac{\partial f^{\rm in}}{\partial t} + \frac{\mu_\nu}{r^2}\frac{\partial}{\partial r}(r^2 f^{\rm in}) + \frac{\sqrt{1-\mu_\nu^2}\cos\phi_\nu}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f^{\rm in})$$
$$+ \frac{\sqrt{1-\mu_\nu^2}\sin\phi_\nu}{r\sin\theta}\frac{\partial f^{\rm in}}{\partial\phi} + \frac{1}{r}\frac{\partial}{\partial\mu_\nu}[(1-\mu_\nu^2)f^{\rm in}]$$
$$- \frac{\sqrt{1-\mu_\nu^2}}{r}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\phi_\nu}(\sin\phi_\nu f^{\rm in}) = \left[\frac{1}{c}\frac{\delta f^{\rm in}}{\delta t}\right]_{\rm collision}$$

To omit computational cost, integrate out phase space

$$E = \int \mathrm{d}\phi_\nu \int \mathrm{d}\mu_\nu f$$

$$\mathbf{F} = \int \mathrm{d}\phi_\nu \int \mathrm{d}\mu_\nu \mathbf{n} f$$

$$\mathbf{P} = \int \mathrm{d}\phi_\nu \int \mathrm{d}\mu_\nu \mathbf{n}\mathbf{n} f$$

$$\frac{\partial E}{\partial t} + \nabla\cdot\mathbf{F} = \left(\frac{\delta E}{\delta t}\right)_\nu$$
$$\frac{\partial \mathbf{F}}{\partial t} + \nabla\cdot\mathbf{P} = \left(\frac{\delta \mathbf{F}}{\delta t}\right)_\nu$$

<- 4 dimensional equations

Fundamental problem

If we solve equation for E and F, how P is determined?

Many solvers

Two moment

$$\frac{\partial E}{\partial t} + \nabla \cdot \mathbf{F} = \left(\frac{\delta E}{\delta t}\right)_\nu$$
$$\frac{\partial \mathbf{F}}{\partial t} + \nabla \cdot \mathbf{P} = \left(\frac{\delta \mathbf{F}}{\delta t}\right)_\nu$$

Variable Eddington factor:
P is computed by the simplified Boltzmann equation

M1-Closure:
P is assumed from the E and F

One moment

$$\frac{\partial E}{\partial t} + \nabla \cdot \mathbf{F} = \left(\frac{\delta E}{\delta t}\right)_\nu$$

FLD: F is assumed by E and ∇E

IDSA:
F is assumed from the neutrino sphere

No transport system

$$\frac{\partial e_{\mathrm{matter}}}{\partial t} = -\left(\frac{\delta E}{\delta t}\right)_\nu$$

Leakage: source term is only considered as the cooling term

Method : Sn > VE > M1 > FLD, IDSA > Leakage

←ab initio approximation→

←high cost low cost→

Many solvers

Sn & Variable Eddington factor

no assumptions

M1−Closure

$$f(\omega,\theta)=\frac{1}{\exp(g(\omega)+\eta\cos\theta)+1} \qquad \eta=\eta(F/E)$$

Flux limited Diffusion

$$f(\omega,\theta)=f_0(\omega)+\cos\theta f_1(\omega) \qquad f_1(\omega)\propto\nabla f_0$$

In optically thick region

Isotropic diffusion source

$$f(\omega,\theta)=\frac{1}{\exp\left(\frac{\omega-\mu}{kT}\right)+1}+f_1(\omega,\theta) \qquad \frac{df_1(\omega,\theta)}{dt}\propto\Delta f_{\mathrm{FD}}$$

Note: Method and Groups

Sn: <= Sumiyoshi, Yamada(1D)

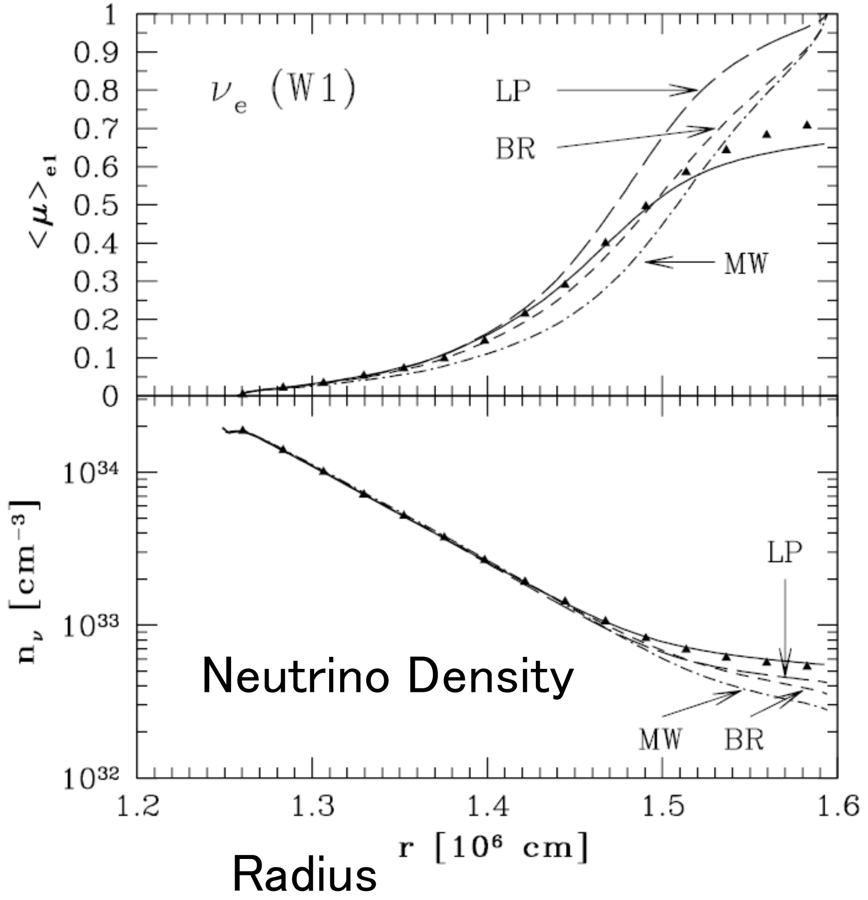
VE <= Janka, Hanke, B. Mueller(3D)

M1 <=Ott O'cconnor(1D); Obergaulinger, Janka(2D)

FLD <=Bruenn(2D), Burrows(2D)

IDSA<=Takiwaki, Suwa, Kotake(3D)

Method の比較



- Yamada et al 1999

Density in FLD is lower than that of Sn and Monte Calro

solid：Boltzmann

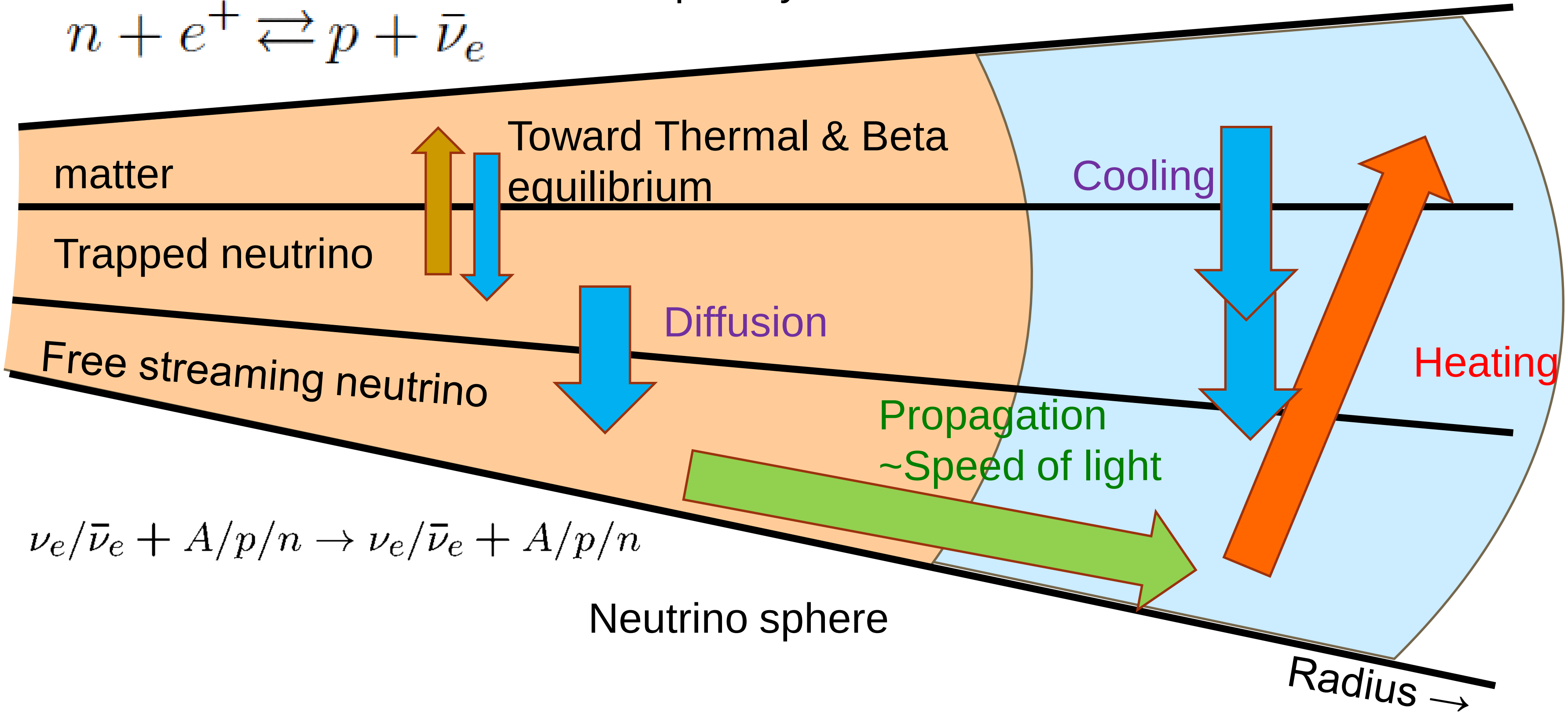
▲：Monte Carlo

These small difference could be important.

IDSA Neutrino transport, concept

$$p + e^- \rightleftarrows n + \nu_e$$
$$n + e^+ \rightleftarrows p + \bar{\nu}_e$$

IDSA(isotropic diffusion source approximation)
Developed by Liebendoerfer 2005



$$\nu_e/\bar{\nu}_e + A/p/n \rightarrow \nu_e/\bar{\nu}_e + A/p/n$$

Dividing neutrino into two parts. Trapped and free streaming.
For v_X, simple leakage scheme is used.

IDSA: trapped part

$$\frac{df}{cdt} + \mu\frac{\partial f}{\partial r} + \left[\mu\left(\frac{d\ln\rho}{cdt} + \frac{3v}{cr}\right) + \frac{1}{r}\right](1-\mu^2)\frac{\partial f}{\partial \mu}$$
$$+ \left[\mu^2\left(\frac{d\ln\rho}{cdt} + \frac{3v}{cr}\right) - \frac{v}{cr}\right]E\frac{\partial f}{\partial E}$$
$$= j(1-f) - \chi f + \frac{E^2}{c(hc)^3}$$
$$\times \left[(1-f)\int Rf'd\mu' - f\int R(1-f')d\mu'\right].$$

f(x,y,z,E,theta,phi)
6 dimensional variable

Trapped Particle

↓Ray-by-Ray

Angular integration

Diffusion term
(To free streaming part)

$$\frac{df^{t}}{cdt} + \frac{1}{3}\frac{d\ln\rho}{cdt}E\frac{\partial f^{t}}{\partial E} = j - (j+\chi)f^{t} - \Sigma.$$

$$\Sigma = \min\left\{\max\left[\alpha + (j+\chi)\frac{1}{2}\int f^{s}d\mu, 0\right], j\right\}$$
$$\alpha = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{-r^2}{3(j+\chi+\phi)}\frac{\partial f^{t}}{\partial r}\right).$$

Energy integration

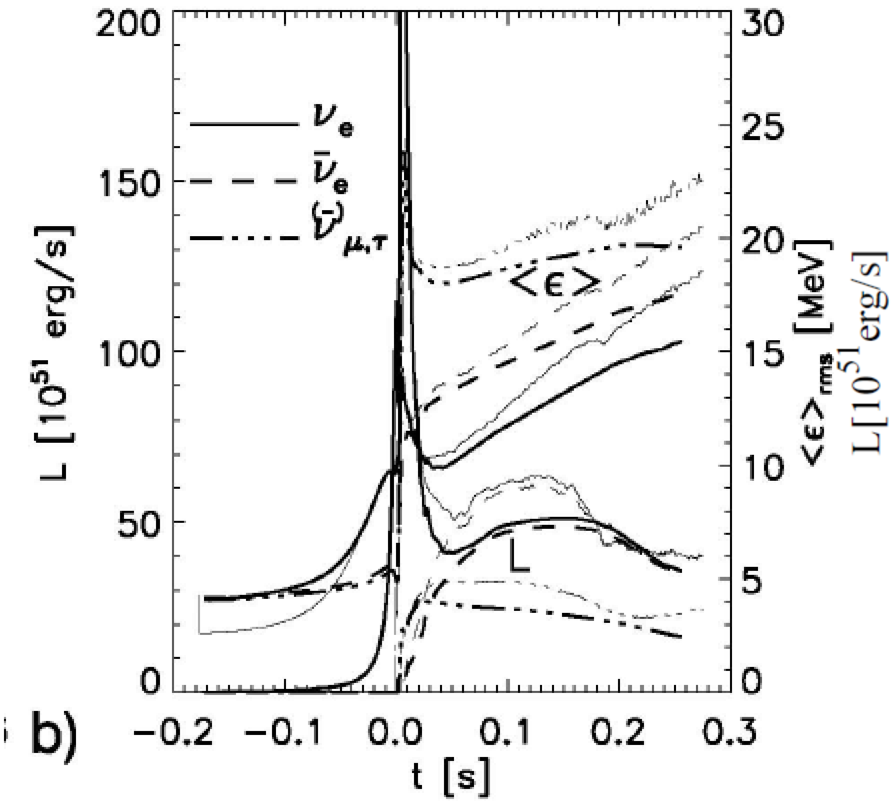
$$Y^{t} = \frac{m_b}{\rho}\frac{4\pi}{(hc)^3}\int f^{t}E^2dEd\mu$$
$$Z^{t} = \frac{m_b}{\rho}\frac{4\pi}{(hc)^3}\int f^{t}E^3dEd\mu,$$

Determine temperature and chemical potential for Fermi-Dirac distribution by Y and Z

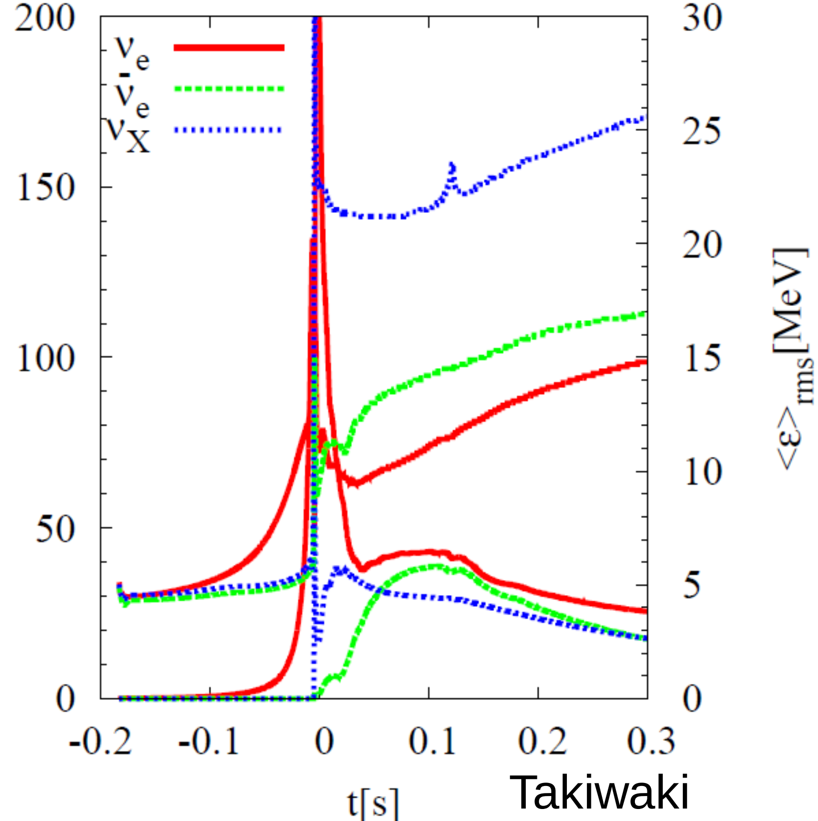
$$\frac{\partial}{\partial t}(\rho Y^{t}) + \frac{\partial}{r^2\partial r}(r^2 v\rho Y^{t})$$
$$= m_b\frac{4\pi c}{(hc)^3}\int[j - (j+\chi)f^{t} - \Sigma]E^3dE.$$

$$f_l^{t}(E) = \{\exp[\beta_l(E - \mu_l)] + 1\}^{-1},$$

Comparison of Method



Liebendoerfer et al 2005



Takiwaki

IDSA

ecp,aecp,eca,csc,nsc,pap,nes,nbr

Newtonian Gravity

Sn andVE

General relativistic simulation

For simple spherical computation, the result is rather consistent.

Quantitatively small difference found

Numerical cost

Not proportional to product of the grid, Nr Nt Np Ne Ntn Npn

Nr Nt Np :space

Ne Ntn Npn : phase space

Neutrino reactions are usually solved implicitly.

matrix inversion and iteration is necessary.

$$\boldsymbol{F}(\boldsymbol{X}) = 0$$

Guess X

$$\frac{\partial \boldsymbol{F}\left(\tilde{\boldsymbol{X}}\right)}{\partial \tilde{\boldsymbol{X}}} \cdot \delta \boldsymbol{X} = -\boldsymbol{F}\left(\tilde{\boldsymbol{X}}\right)$$

$\delta X \sim 0$

Correct X is found!

X: neutrino distribution function

$$\tilde{X}^{n+1} = \tilde{X}^{n} + \delta X$$

Cost of matrix inversion: N^3 or N^2 *O(10)

For example: Sumiyoshi 2005 (N_r N_e N_tn)^3

Numerical Cost

Not proportional to product of the grid, Nr Nt Np Ne Ntn Npn

Nr Nt Np :space

Ne Ntn Npn : phase space

Sumiyoshi+ 2005 (N_r N_e N_tn)^3
Peta Scale
Takiwaki+ in prep. N_r N_t N_p N_e

N_r N_e N_tn
= 255, 14, 7

N_r N_t N_p N_e
=384,128, 256, 20

In IDSA, neutrino is assumed to fermi dirac distribution
and the deviation from that is computed explicitly

Exascale
6D Sn: Nr Nt Np Ne (Ntn Npn)^2

Nr Nt Np Ne Ntn Npn
=512,64,128,24, 24, 24

5D Sn: Nr Nt Np (Ne Ntn)^2

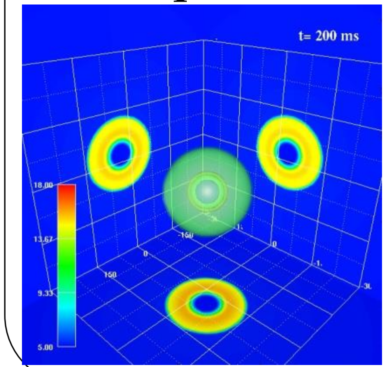
Nr Nt Np Ne Ntn
=512,128, 256, 24, 24

Summary

2005

(1+2)D

No explosion

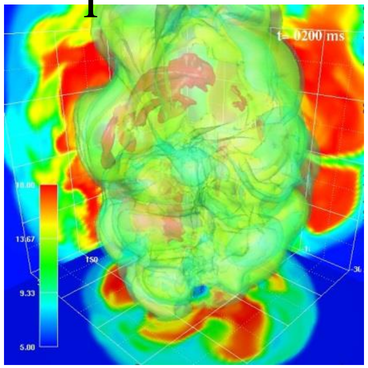


2013 Petascale

(3+1)D

Approximate transport

Explosion



2018~ Exascale

(3+3)D <= no assumption

Full Boltzmann

Explosion or No explosion?

Super computer opens physics of Supernovae